Exhibit A

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STATE OF NORTH CAROLINA COUNTY OF WAKE	IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION __ CVS ____

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR.; DANDRIELLE LEWIS; TIMOTHY CHARTIER; TALIA FERNÓS; KATHERINE NEWHALL; JASON PARSLEY; EDNA SCOTT; ROBERTA SCOTT; YVETTE ROBERTS; JEREANN KING JOHNSON; REVEREND REGINALD WELLS; YARBROUGH WILLIAMS, JR.; REVEREND DELORIS L. JERMAN; VIOLA RYALS FIGUEROA; and COSMOS GEORGE, Plaintiffs, v. REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting; SENATOR WARREN DANIEL, in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; SENATOR RALPH E. HISE, JR., in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; SENATOR PAUL NEWTON, in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; REPRESENTATIVE TIMOTHY K. MOORE, in his official capacity as Speaker of the North Carolina House of Representatives; SENATOR PHILIP E. BERGER, in his official capacity as President Pro Tempore of the North Carolina Senate; THE STATE OF NORTH CAROLINA; THE NORTH CAROLINA STATE BOARD OF ELECTIONS; DAMON CIRCOSTA, in his official capacity as Chairman of the North Carolina State Board of Elections; STELLA ANDERSON, in her official capacity as Secretary of the North Carolina State Board of Elections; JEFF CARMON III, in his official capacity as Member of the North Carolina State Board of Elections; STACY EGGERS IV, in his official capacity as Member of the North Carolina State Board of Elections; TOMMY TUCKER, in his official capacity as Member of the North Carolina State Board of Elections; and KAREN BRINSON BELL, in her official capacity as Executive Director of the North Carolina State Board of Elections, Defendants.	**<u>AFFIDAVIT OF DR. MOON DUCHIN</u>**

I, Dr. Moon, Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.

2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.

4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.

5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.

6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.

7. A current copy of my full CV is attached to this report.

8. I am compensated at the rate of $400 per hour.

Analysis of 2021 enacted redistricting plans in North Carolina

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

November 16, 2021

1 Introduction

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. This affidavit contains a brief summary of my evaluation of the properties of these plans. My focus will be on the egregious partisan imbalance in the enacted plans, following a brief review of the traditional districting principles.

Because redistricting inevitably involves complex interactions of rules, which can create intricate tradeoffs, it will be useful to employ a direct comparison to an alternative set of plans. These demonstrative plans illustrate that it is possible to *simultaneously maintain or improve* metrics for all of the most important redistricting principles that are operative in North Carolina's constitution and state and federal law. Crucially, this shows that nothing about the state's political geography compels us to draw a plan with a massive and entrenched partisan skew.

To this end, I will be comparing the following plans: the enacted plans SL-174, SL-173, and SL-175 and a corresponding set of alternative plans labeled NCLCV-Cong, NCLCV-Sen, and NCLCV-House (proposed by plaintiffs who include the North Carolina League of Conservation Voters).

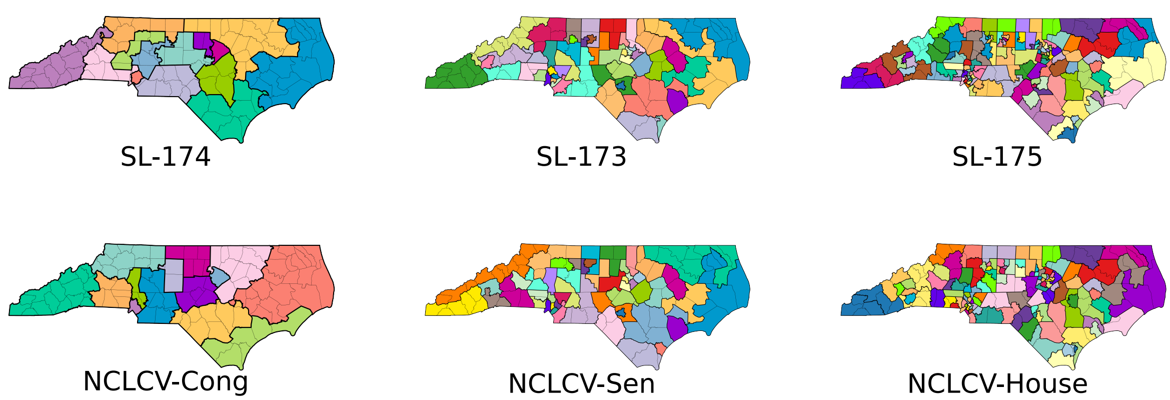


Figure 1: The six plans under discussion in this affidavit.

2 Traditional districting principles

Principles that are relevant to North Carolina redistricting include the following.

- **Population balance.** The standard interpretation of *One Person, One Vote* for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.

  There is more latitude with legislative districts; they typically vary top-to-bottom by no more than 10% of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that 5% deviation must be tolerated if it means preserving more counties intact.

  All six plans have acceptable population balance.

Population deviation

	Max Positive Deviation	District	Max Negative Deviation	District
SL-174	0	(eight districts)	−1	(six districts)
NCLCV-Cong	0	(eight districts)	−1	(six districts)
SL-173	10,355 (4.960%)	5	−10,434 (4.997%)	13,18
NCLCV-Sen	10,355 (4.960%)	5	−10,427 (4.994%)	15
SL-175	4250 (4.885%)	18	−4189 (4.815%)	112
NCLCV-House	4341 (4.990%)	82	−4323 (4.969%)	87

Table 1: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.

- **Minority electoral opportunity.** Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must hinge not on the demographics of the districts but on electoral history and an assessment of polarization patterns. That is not the focus of the current affidavit. Instead we make the brief note that it is important to avoid the conflation of *majority-minority districts* with *effective districts* for a minority group. An involved analysis of voting patterns—necessarily incorporating both primary and general elections to ensure that candidates of choice can be successfully nominated and elected—will frequently reveal that districts can be effective at demographic levels well below 50% of voting-age population or citizen voting-age population (VAP and CVAP, respectively). For instance, in [3], my co-authors and I drew an illustrative plan for Texas congressional districting in which some parts of the state had districts that were shown to reliably elect Black candidates of choice with BCVAP as low as 28.6%; by contrast, there are other parts of Texas where a 40% BCVAP district is less consistently effective. In a Louisiana case study, we found somewhat different patterns of human and political geography, producing numerous examples of Congressional-sized districts with 55% BCVAP in some parts of the state that are nonetheless marginal in terms of opportunity for Black voters to elect candidates of choice.

  In North Carolina, taking the crossover voting patterns of White, Latino, and Asian voters into account, I note that a district with BCVAP in the low to mid 30s can often be effective for Black voters—but there is no demographic shortcut to a full examination of primary and general election history.

- **Contiguity.** All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.

- **Compactness.** The two compactness metrics most commonly appearing in litigation are the *Polsby-Popper score* and the *Reock score*. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4\pi A/P^2$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1. Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1, which is achieved only by circles.

  These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the *cut edges score*, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.

  The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
SL-174	5194	0.303	0.381
NCLCV-Cong	4124	0.383	0.444
SL-173	9702	0.342	0.402
NCLCV-Sen	9249	0.369	0.423
SL-175	16,182	0.351	0.419
NCLCV-House	13,963	0.414	0.456

Table 2: Comparing compactness scores via one discrete and two contour-based metrics.

- **Respect for political subdivisions.** For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows.

  - First, if any county is divisible into a whole number of districts that will be within ±5% of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
  - Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
  - Repeat for groupings of three, and so on, until all counties are accounted for.

  A complete set of solutions is described in detail in the white paper of Mattingly et al.—though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten single-district fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multi-district fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. Again, it is important to note that VRA compliance may present a compelling reason to select some clusterings and reject others.

Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the ±5% population standard for districts. To address this, I have counted the *county traversals* in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.

Table 3 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13, but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

The alternative plans are comparable to the enacted plans, or sometimes far superior, in each of these key metrics regarding preservation of political boundaries.

County and municipality preservation

	# county pieces
SL-174	25
NCLCV-Cong	26

	# traversals
SL-173	97
NCLCV-Sen	89
SL-175	69
NCLCV-House	66

	# municipal pieces
SL-174	90
NCLCV-Cong	58
SL-173	152
NCLCV-Sen	125
SL-175	292
NCLCV-House	201

Table 3: Comparing the plans’ conformance to political boundaries.

I will briefly mention several additional redistricting principles.

- **Communities of interest.** In North Carolina, there was no sustained effort by the state or by community groups to formally collect community of interest (COI) maps, to my knowledge. Without this, it is difficult to produce a suitable metric.
- **Cores of prior districts.** In some states, there is statutory guidance to seek districting plans that preserve the cores of prior districts. In North Carolina, this is not a factor in the constitution, in statute, or in case law. In addition, attention to core preservation would be prohibitively difficult in the Senate and House because of the primacy of the Whole County Provisions, which forces major changes to the districts simply as a consequence of fresh population numbers.

- **Incumbent pairing.** In 2017, the North Carolina legislative redistricting committee listed "incumbency protection" as a goal in their itemization of principles. In 2021, this was softened to the statement that "Member residence may be considered" in the drawing of districts. I have counted the districts in each plan that contain more than one incumbent address; these are sometimes colorfully called "double-bunked" districts. For this statistic, it is not entirely clear whether a high or low number is preferable. When a plan remediates a gerrymandered predecessor, we should not be surprised if it ends up pairing numerous incumbents.

Double-bunking

	# districts pairing incumbents
SL-174	3
NCLCV-Cong	1
SL-173	6
NCLCV-Sen	9
SL-175	7
NCLCV-House	15

Table 4: For Congress and Senate, the enacted and alternative plans are comparable; at the House level, the alternative plan has more double-bunking. *Note: These numbers were calculated using the most accurate incumbent addresses that have been provided to me.*

3 Partisan fairness

3.1 Abstract partisan fairness

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address *how a certain share of the vote should be translated to a share of the seats* in a state legislature or Congressional delegation.

The numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a 50-50 split in partisan preference should produce a roughly 50-50 representational split. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. But the new enacted plans, like the plans from ten years ago, are not conducive to even representation.

3.2 Geography and fairness

However, some scholars have argued that this ideal (that even vote preferences should translate to even representation) ignores the crucial *political geography*—the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [5], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The alternative plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, the geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives giving a seat share for each party that falls, in aggregate, within a few percentage points of the vote share across a large set of elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats the parties equally and fairly.

3.3 Translating votes to seats

The enacted plans behave as though they are built to resiliently safeguard electoral advantage for Republican candidates. We can examine this effect without invoking assumptions like "uniform partisan swing" that impose counterfactual voting conditions; instead, we will use the rich observed dataset of 52 statewide party-ID general elections in North Carolina in the last ten years. 29 of these are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three presidential races, three for U.S. Senate, and 17 judicial races since mid-decade, when those became partisan contests. See Table 6 for more detail on the election dataset.

I will sometimes focus on the smaller set of better-known "up-ballot" races: in order, the first five to appear on the ballot are the contests for President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. Together these occurred 14 times in the last Census cycle.

	Up-ballot generals (14)		**All generals (52)**	
	D vote share	D seat share	D vote share	D seat share
SL-174	.4883	.2908	.4911	.3118
NCLCV-Cong		.4796		.4931
SL-173	.4883	.3957	.4911	.4065
NCLCV-Sen		.4557		.4592
SL-175	.4883	.3994	.4911	.4080
NCLCV-House		.4649		.4684

Table 5: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are reported with respect to the major-party vote total.

To understand how the enacted plans create major shortfalls for Democratic representation, we will overlay the plans with voting patterns from individual elections in the past Census cycle. As we will see, the enacted Congressional plan (SL-174) shows a remarkable lack of responsiveness, giving 10–4 partisan outcomes across a wide range of recent electoral conditions, meaning that 10 Republicans and only 4 Democrats would represent North Carolina in Congress. The alternative plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes.

The top of Figure 2 shows this dynamic in the three Presidential contests in the last Census cycle, with a Democratic vote share (pink box) between 48% and 50% of the major-party total each time. For a contest that is so evenly divided, we would expect a fair map to have 6, 7, or 8 out of 14 districts favoring each party. The alternative Congressional map NCLCV-Cong does just that, while the enacted plan SL-174 has just 4 out of 14 Democratic-majority districts each time (green and maroon circles). The alternative plan is far more successful at reflecting the even split of voter preferences. Below the initial explainer, simplified versions of the same type of graphic are presented for all five up-ballot races. Figure 3 compares legislative maps in the same fashion. Next, Figure 4 returns to the full 52-election dataset to give the big picture of entrenched partisan advantage in the enacted plans.

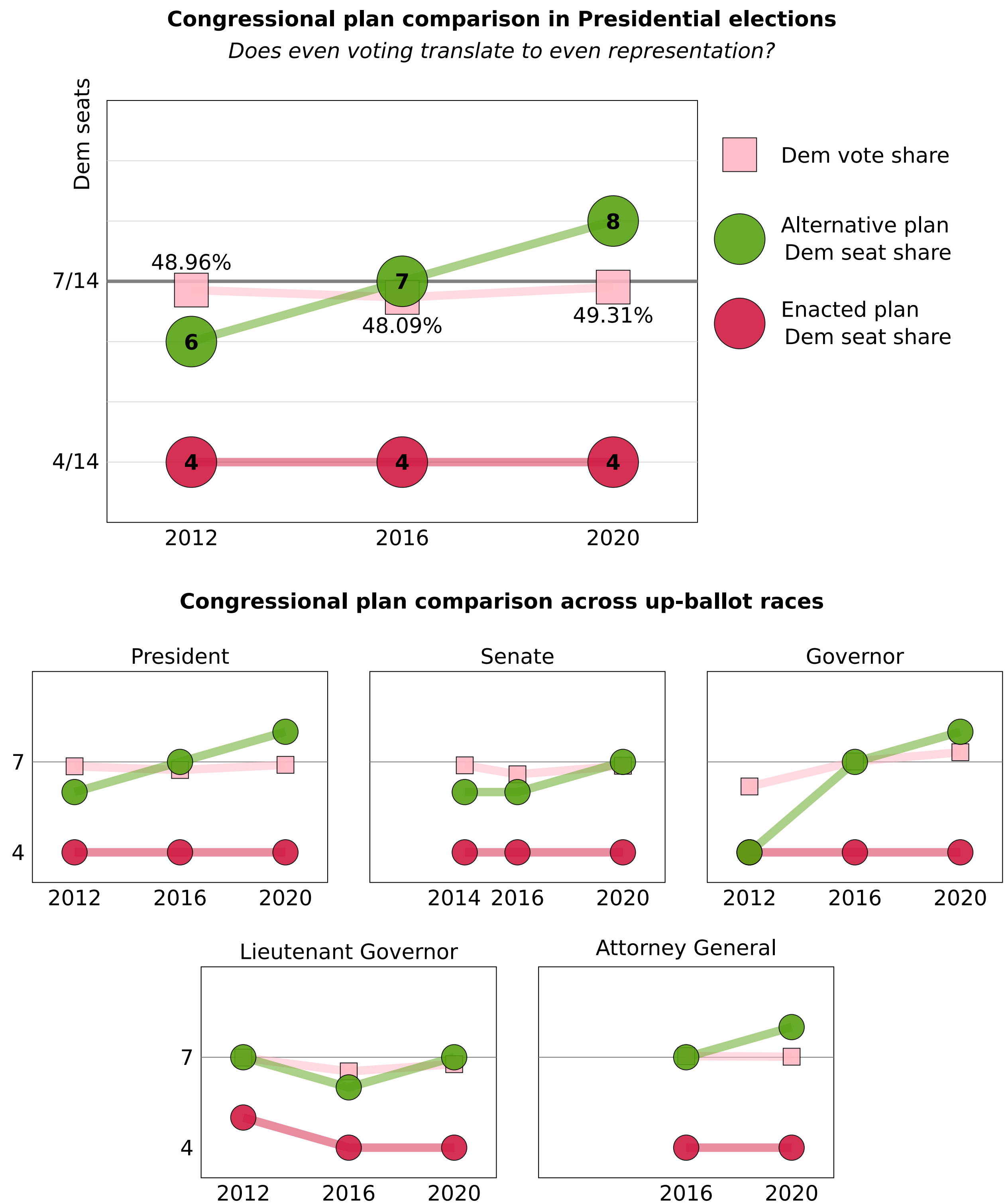


Figure 2: For up-ballot general election contests across the previous Census cycle, we can compare the seat share under the enacted Congressional plan SL-174 (maroon) and the seat share under the alternative Congressional plan NCLCV-Cong (green) to the vote share (pink) for Democratic candidates. At top is a detailed look at the presidential contests; this is repeated below, alongside the other four up-ballot offices. The 50% line is marked each time.

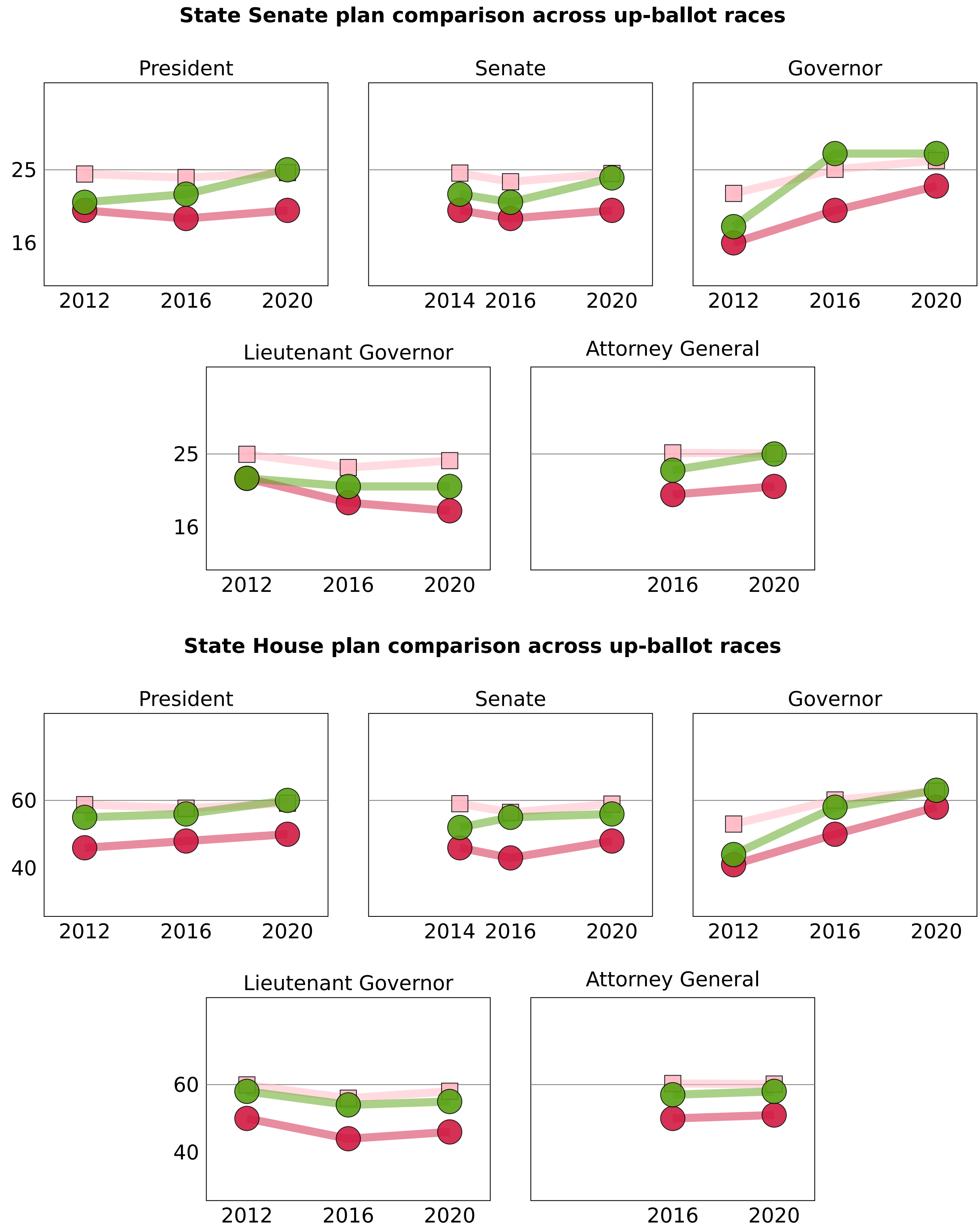


Figure 3: Legislative plans tested against voting patterns from up-ballot elections. The enacted plans SL-173 and SL-175 are shown in maroon. The alternative plans NCLCV-Sen and NCLCV-House, in green, have seat shares tracking much closer to the nearly even voting preferences.

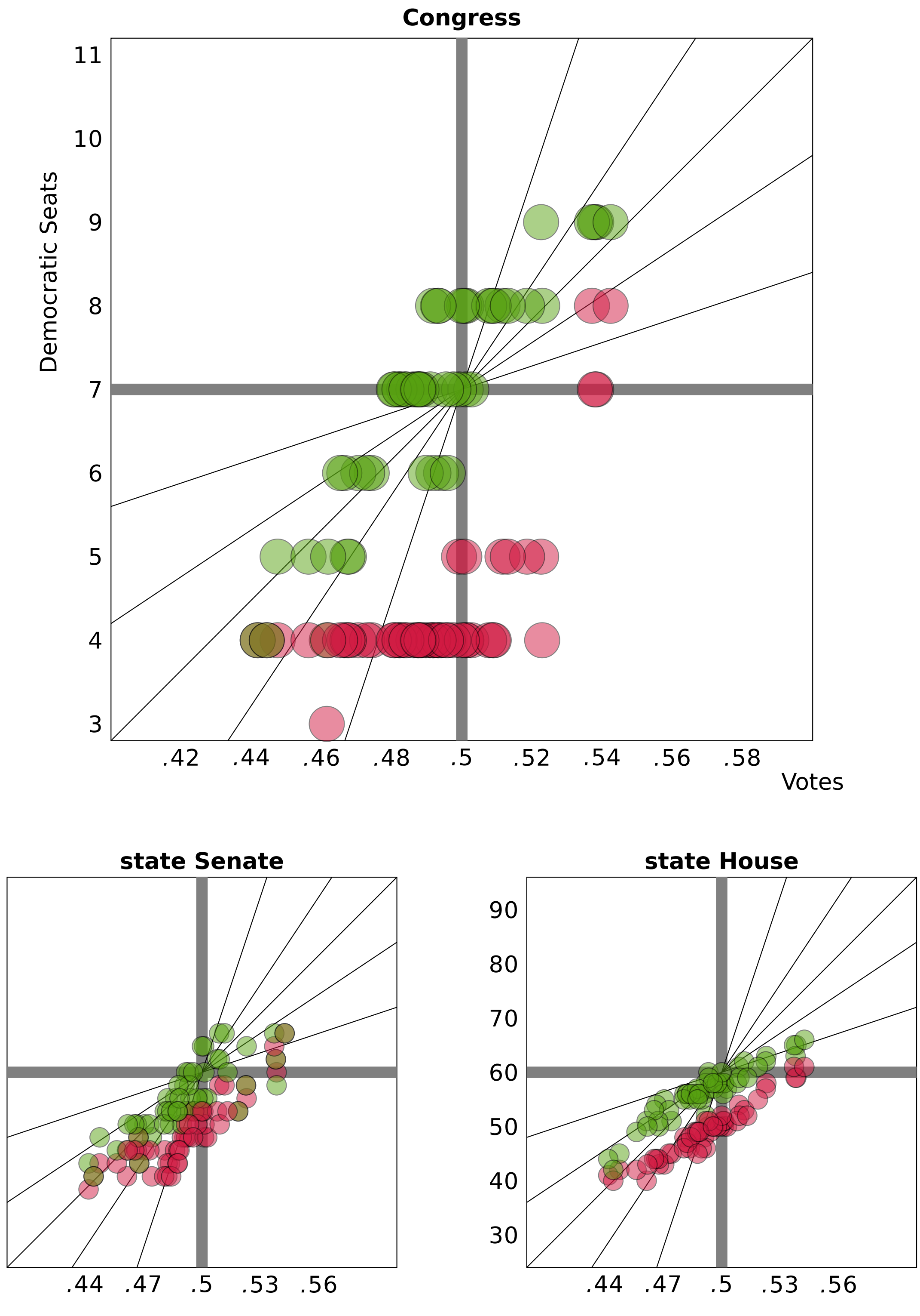


Figure 4: On a seats-vs.-votes plot, the election results for the six maps are shown for 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share). The diagonals show various lines of *responsiveness* that pivot around the central point of fairness: half of the votes securing half of the seats. The Congressional comparison is at top, followed by Senate and House. The enacted plans are shown in maroon and the alternative plans in green.

3.4 Swing districts and competitive contests

Another way to understand the electoral properties of districting plans is to investigate how many districts always give the same partisan result over a suite of observed electoral conditions, and how many districts can "swing" between the parties. Figure 5 compares the six plans across the up-ballot elections. The enacted plans lock in large numbers of always-Republican seats. In the Senate and House, nearly half the seats are locked down for Republicans. In the Congressional plan, it's well *over* half. This provides another view from which the NCLCV plans provide attractive alternatives.

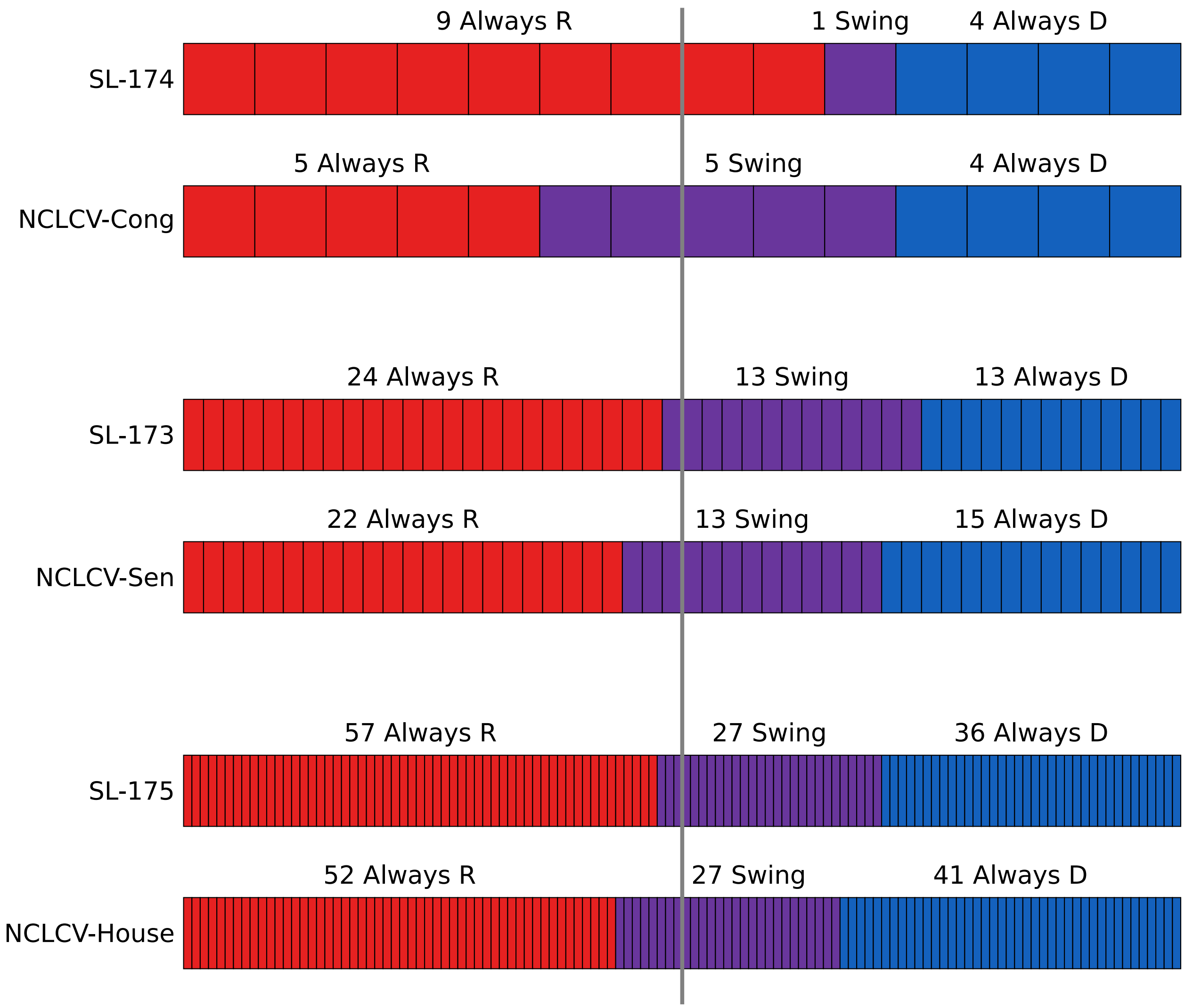
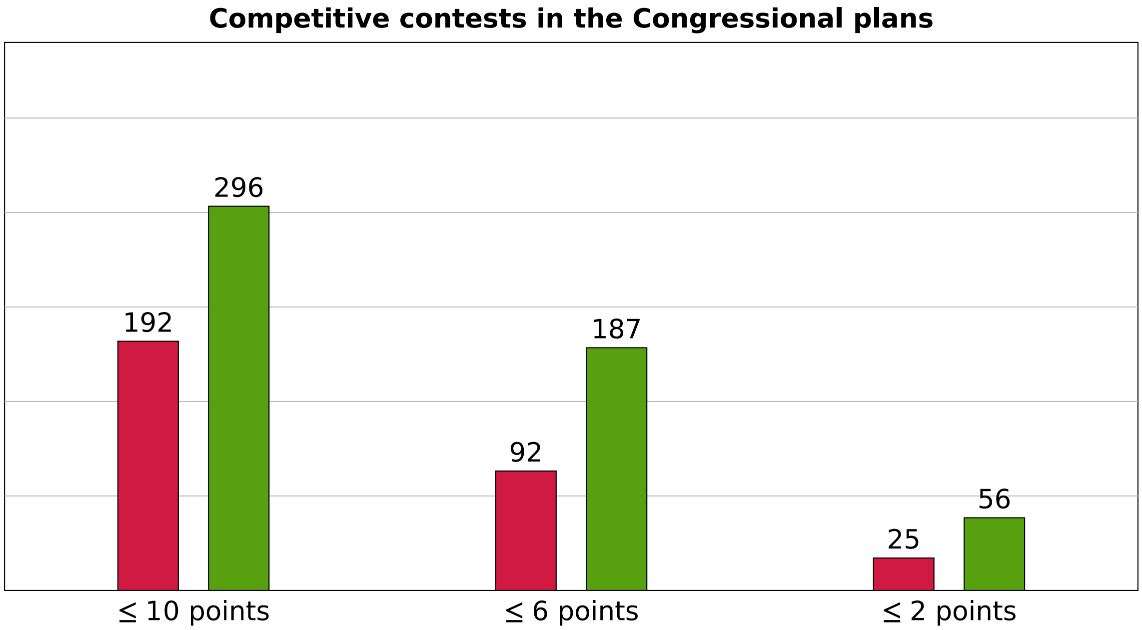


Figure 5: These visuals show the breakdown of seats that always have a Republican winner, always have a Democratic winner, or are sometimes led by each party across the 14 up-ballot elections over the previous Census cycle. The 50-50 split is marked.

One more measure of partisan fairness, frequently referenced in the public discourse, is the tendency of a districting plan to promote close or competitive contests. We close with a comparison of the enacted and alternative plans that displays the number of times across the full dataset of 52 elections that a contest had a partisan margin of closer than 10 points, 6 points, or 2 points, respectively. This can occur up to $14 \cdot 52 = 728$ times in Congressional maps, $50 \cdot 52 = 2600$ times in state Senate maps, and $120 \cdot 52 = 6240$ times in state House maps. The figures below show horizontal rules at every 10% interval of the total number of possible competitive contests; we can see, for instance, that the alternative Congressional plan has contests within a 10-point margin more than 40% of the time.



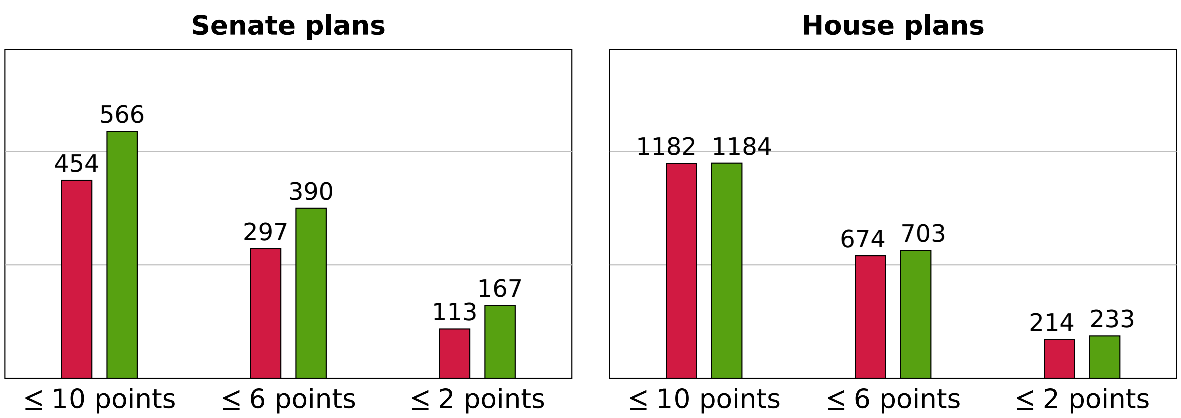


Figure 6: These bar graphs show the number of competitive contests for the enacted plans (maroon) and the alternative plans (green). In each plot, we consider increasingly restrictive definitions of "competitive" from left to right, counting districts in which the major-party vote split is closer than 45-55, 47-53, and 49-51, respectively.

4 Conclusion

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer. We can make a striking observation by laying our six plans over the vote patterns.

	D Vote Share	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
GOV12	0.4418	4	4	16	18	41	44
AGC16	0.4444	4	4	17	17	40	42
LAC16	0.4475	4	5	18	20	42	45
JHU16	0.4563	4	5	18	19	42	49
AGC20	0.4615	3	4	17	19	40	51
JZA16	0.4619	4	5	19	21	43	50
JDI16	0.4653	4	6	19	21	44	53
LTG16	0.4665	4	6	19	21	44	54
LAC12	0.4674	4	5	20	20	44	51
AGC12	0.4678	4	5	18	18	43	50
SEN16	0.4705	4	6	19	21	43	55
TRS16	0.4730	4	6	19	21	45	53
TRS20	0.4743	4	6	17	20	45	51
JA620	0.4806	4	7	17	21	46	55
PRS16	0.4809	4	7	19	22	48	56
JA420	0.4822	4	7	17	22	47	56
INC20	0.4823	4	7	18	23	47	56
LTG20	0.4836	4	7	18	21	46	55
JA720	0.4842	4	7	17	22	48	56
SUP20	0.4862	4	7	19	23	49	56
JA520	0.4874	4	7	18	22	49	57
JA218	0.4876	4	7	18	22	45	55
JS420	0.4879	4	7	19	24	49	56
J1320	0.4885	4	7	19	23	49	56
PRS12	0.4897	4	6	20	21	46	55
SEN20	0.4910	4	7	20	24	48	56
LAC20	0.4918	4	8	21	25	51	58
SEN14	0.4919	4	6	20	22	46	52
PRS20	0.4932	4	8	20	25	50	60
JS220	0.4934	4	8	21	24	51	59
SUP16	0.4941	4	6	22	23	49	57
JS118	0.4955	4	7	20	25	50	58
INC16	0.4960	4	6	22	22	50	57
JST16	0.4976	4	7	21	23	50	58
LTG12	0.4992	5	7	22	22	50	58
JS120	0.5000	4	8	22	27	52	60
AUD16	0.5007	5	8	22	23	51	56
GOV16	0.5011	4	7	20	27	50	58
ATG20	0.5013	4	8	21	25	51	58
ATG16	0.5027	4	7	20	23	50	57
JA118	0.5078	4	8	22	26	51	58
AUD20	0.5088	4	8	24	28	54	61
JA318	0.5091	4	8	21	26	52	59
SOS20	0.5116	5	8	24	28	53	62
JGE16	0.5131	5	8	22	25	52	59
INC12	0.5186	5	8	22	22	55	61
SOS16	0.5226	5	9	24	24	57	62
GOV20	0.5229	4	8	23	27	58	63
AUD12	0.5371	8	9	27	28	61	65
SOS12	0.5379	7	9	26	26	59	63
TRS12	0.5383	7	9	25	24	59	65
SUP12	0.5424	8	9	28	28	61	66

Table 6: 52 general elections, sorted from lowest to highest Democratic share. Election codes have a three-character prefix and a two-digit suffix designating the office and the election year, respectively. AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; PRS = President; SEN = Senator; SOS = Secretary of State; SUP = Superintendent of Schools; TRS -=Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), those beginning with JS* refer to elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals.

The three enacted plans combine with those 38 relatively even vote patterns to produce 114 outcomes. Every single pairing of an enacted plan with a close statewide contest—a complete sweep of 114 opportunities—gives an *outright Republican majority* of seats. All three enacted plans will lock in an extreme, resilient, and unnecessary advantage for one party.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the alternative plans meet or exceed the performance of the enacted plans. It is therefore demonstrated to be possible, without any cost to the redistricting principles in play, to select maps that are far fairer to the voters of North Carolina.

References

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[2] Richard Barnes and Justin Solomon, *Gerrymandering and Compactness: Implementation Flexibility and Abuse.* Political Analysis, Volume 29, Issue 4, October 2021, 448–466.

[3] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal.
Available at https://www.liebertpub.com/doi/epdf/10.1089/elj.2020.0704

[4] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.*
https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

[5] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 16th day of November, 2021.

Moon Duchin

Moon Duchin

Sworn and subscribed before me

this the 16 of November, 2021.

Notary Public

Name: GREGORY J. AGNEW
Notary Public
COMMONWEALTH OF MASSACHUSETTS
My Commission Expires
June 28, 2024

My Commission Expires:

Moon Duchin

moon.duchin@tufts.edu - mduchin.math.tufts.edu
Mathematics · STS · Tisch College of Civic Life | Tufts University

Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017—

Senior Fellow | Tisch College of Civic Life 2017—

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005–2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019–2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009–2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

Conjugation curvature for Cayley graphs
Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

A computational approach to measuring vote elasticity and competitiveness
Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics **346** (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in *Office Hours with a Geometric Group Theorist*, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society **63**, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics **10**, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society **143** (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups
Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. **590** (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infinity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology **12** (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of flat metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Rafi)

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. **432** (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

Clustering propensity: A mathematical framework for measuring segregation
Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.
reprinted from Scientific American, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the efficiency gap
Notices of the American Mathematical Society **64** No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society **64**, No. 7 (2017), 682–683.

Graduate Advising in Mathematics

Nate Fisher (PhD 2021), Sunrose Shrestha (PhD 2020), Ayla Sánchez (PhD 2017),
Kevin Buckles (PhD 2015), Mai Mansouri (MS 2014)

Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

Principal supervisor Thomas Weighill (2019–2020)

Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

Teaching

Courses Developed or Customized

Mathematics of Social Choice | sites.tufts.edu/socialchoice
Voting theory, impossibility theorems, redistricting, theory of representative democracy, metrics of fairness.

History of Mathematics | sites.tufts.edu/histmath
Social history of mathematics, organized around episodes from antiquity to present. Themes include materials and technologies of creation and dissemination, axioms, authority, credibility, and professionalization. In-depth treatment of mathematical content from numeration to cardinal arithmetic to Galois theory.

Reading Lab: Mathematical Models in Social Context | sites.tufts.edu/models
One hr/wk discussion seminar of short but close reading on topics in mathematical modeling, including history of psychometrics; algorithmic bias; philosophy of statistics; problems of model explanation and interpretation.

Geometric Literacy
Module-based graduate topics course. Modules have included: p-adic numbers, hyperbolic geometry, nilpotent geometry, Lie groups, convex geometry and analysis, the complex of curves, ergodic theory, the Gauss circle problem.

Markov Chains (graduate topics course)
Teichmüller Theory (graduate topics course)
Fuchsian Groups (graduate topics course)
Continued Fractions and Geometric Coding (undergraduate topics course)
Mathematics for Elementary School Teachers

Standard Courses

Discrete Mathematics, Calculus I-II-III, Intro to Proofs, Linear Algebra, Complex Analysis, Differential Geometry, Abstract Algebra, Graduate Real Analysis, Mathematical Modeling and Computation

Weekly Seminars Organized

- Geometric Group Theory and Topology
- Science, Technology, and Society Lunch Seminar

Selected Talks and Lectures

Distinguished Plenary Lecture June 2021
75th Anniversary Meeting of Canadian Mathematical Society, Ottawa, Ontario *online (COVID)*

BMC/BAMC Public Lecture April 2021
Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland *online (COVID)*

AMS Einstein Public Lecture in Mathematics [March 2020]
Southeastern Sectional Meeting of the AMS, Charlottesville, VA *postponed*

Gerald and Judith Porter Public Lecture
AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA January 2018

Mathematical Association of America Distinguished Lecture
MAA Carriage House, Washington, DC October 2016

American Mathematical Society Invited Address
AMS Eastern Sectional Meeting, Brunswick, ME September 2016

Named University Lectures

- Parsons Lecture | UNC Asheville — October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis — [March 2020]
- Math, Stats, CS, and Society | Macalester College — October 2019
- MRC Public Lecture | Stanford University — May 2019
- Freedman Memorial Colloquium | Boston University — March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy — January 2019
- Barnett Lecture | University of Cincinnati — October 2018
- School of Science Colloquium Series | The College of New Jersey — March 2018
- Kieval Lecture | Cornell University — February 2018
- G. Milton Wing Lectures | University of Rochester — October 2017
- Norman Johnson Lecture | Wheaton College — September 2017
- Dan E. Christie Lecture | Bowdoin College — September 2017

Math/Computer Science Department Colloquia

- Reed College — Dec 2020
- Georgetown (CS) — Sept 2020
- Santa Fe Institute — July 2020
- UC Berkeley — Sept 2018
- Brandeis-Harvard-MIT-NEU — Mar 2018
- Northwestern University — Oct 2017
- University of Illinois — Sept 2017
- University of Utah — Aug 2017
- Wesleyan — Dec 2016
- Worcester Polytechnic Inst. — Dec 2016
- Université de Neuchâtel — Jun 2016
- Brandeis University — Mar 2016
- Swarthmore College — Oct 2015
- Bowling Green — May 2015
- City College of New York — Feb 2015
- Indiana University — Nov 2014
- the Technion — Oct 2014
- Wisconsin–Madison — Sept 2014
- Stony Brook — March 2013

Minicourses

- Integer programming and combinatorial optimization (two talks) | Georgia Tech — May 2021
- Workshop in geometric topology (main speaker, three talks) | Provo, UT — June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA — August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble — May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton — May 2016
- Nilpotent groups (three talks) | Seoul National University — October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul — April 2014

Science, Technology, and Society

- The Mathematics of Accountability | Sawyer Seminar, Anthropology, Johns Hopkins — February 2020
- STS Circle | Harvard Kennedy School of Government — September 2019
- Data, Classification, and Everyday Life Symposium | Rutgers Center for Cultural Analysis — January 2019
- Science Studies Colloquium | UC San Diego — January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society — November 2018

Data Science, Computer Science, Quantitative Social Science

- Data Science for Social Good Workshop (DS4SG) | Georgia Tech (virtual) — November 2020
- Privacy Tools Project Retreat | Harvard (virtual) — May 2020
- Women in Data Science Conference | Microsoft Research New England — March 2020
- Quantitative Research Methods Workshop | Yale Center for the Study of American Politics — February 2020
- Societal Concerns in Algorithms and Data Analysis | Weizmann Institute — December 2018
- Quantitative Collaborative | University of Virginia — March 2018
- Quantitative Social Science | Dartmouth College — September 2017
- Data for Black Lives Conference | MIT — November 2017

Political Science, Geography, Law, Democracy, Fairness

- The Long 19th Amendment: Women, Voting, and American Democracy | Radcliffe Institute — Nov–Dec 2020
- "The New Math" for Civil Rights | Social Justice Speaker Series, Davidson College — November 2020
- Math, Law, and Racial Fairness | Justice Speaker Series, University of South Carolina — November 2020
- Voting Rights Conference | Northeastern Public Interest Law Program — September 2020
- Political Analysis Workshop | Indiana University — November 2019
- Program in Public Law Panel | Duke Law School — October 2019
- Redistricting 2021 Seminar | University of Chicago Institute of Politics — May 2019
- Geography of Redistricting Conference Keynote | Harvard Center for Geographic Analysis — May 2019
- Political Analytics Conference | Harvard University — November 2018
- Cyber Security, Law, and Society Alliance | Boston University — September 2018
- Clough Center for the Study of Constitutional Democracy | Boston College — November 2017
- Tech/Law Colloquium Series | Cornell Tech — November 2017
- Constitution Day Lecture | Rockefeller Center for Public Policy, Dartmouth College — September 2017

Editorial Boards

Harvard Data Science Review
Associate Editor — since 2019

Advances in Mathematics
Member, Editorial Board — since 2018

Selected Professional and Public Service

Amicus Brief of Mathematicians, Law Professors, and Students 2019
principal co-authors: Guy-Uriel Charles and Moon Duchin
Supreme Court of the United States, in Rucho v. Common Cause - cited in dissent

Committee on Science Policy 2020–2023
American Mathematical Society

Program Committee 2020–2021
Symposium on Foundations of Responsible Computing

Presenter on Public Mapping, Statistical Modeling 2019, 2020
National Conference of State Legislatures

Committee on the Human Rights of Mathematicians 2016–2019
American Mathematical Society

Committee on The Future of Voting: Accessible, Reliable, Verifiable Technology 2017–2018
National Academies of Science, Engineering, and Medicine

Visiting Positions and Residential Fellowships

Visiting Professor Department of Mathematics Fall 2021
Boston College | Chestnut Hill, MA

Fellow Radcliffe Institute for Advanced Study 2018–19
Harvard University | Cambridge, MA

Member Center of Mathematical Sciences and Applications 2018–19
Harvard University | Cambridge, MA

Visitor Microsoft Research Lab 2018–19
MSR New England | Cambridge, MA

Research Member Geometric Group Theory program Fall 2016
Mathematical Sciences Research Institute | Berkeley, CA

Research Member Random Walks and Asymptotic Geometry of Groups program Spring 2014
Institut Henri Poincaré | Paris, France

Research Member Low-dimensional Topology, Geometry, and Dynamics program Fall 2013
Institute for Computational and Experimental Research in Mathematics | Providence, RI

Research Member Geometric and Analytic Aspects of Group Theory program May 2012
Institut Mittag-Leffler | Stockholm, Sweden

Research Member Quantitative Geometry program Fall 2011
Mathematical Sciences Research Institute | Berkeley, CA

Postdoctoral Fellow Teichmüller "project blanc" Spring 2009
Agence Nationale de la Recherche (Collège de France) | Paris, France

Exhibit B

Schauf, Zachary C.

From:	McKnight, Katherine L. <kmcknight@bakerlaw.com>
Sent:	Thursday, December 9, 2021 12:05 PM
To:	Theodore, Elisabeth; Feldman, Stephen; Burton Craige; Narendra Ghosh; Paul Smith; 'melias@elias.law'; 'abranch@elias.law'; lmadduri@elias.law; jshelly@elias.law; gwhite@elias.law; akhanna@elias.law; Jones, Stanton; Callahan, Sam; Doerr, Adam; Zimmerman, Erik; Hirsch, Sam; Amunson, Jessica Ring; Bracey, Kali N.; Schauf, Zachary C.; Mittal, Urja R.
Cc:	Phil Strach; Tom Farr; John Branch; Alyssa Riggins; Braden, E. Mark; Raile, Richard; Brennan, Stephanie; Majmundar, Amar; 'tsteed@ncdoj.gov'
Subject:	NCLCV v. Hall (21 CVS 15426)/Harper v. Hall (21 CVS 500085) -- Request for source code and related information

External Email – Exercise Caution

Dear Plaintiffs' Counsel in the *Harper* and *NCLCV* matters,

We write to request copies of the source code, source data, input parameters (i.e., the exact model specifications and input parameters given to the computer programs to perform the simulations analysis), and all data outputted from those simulations (including reporting as well as shapefiles or block-assignment files for the simulated plans) for the analyses that formed the basis for the expert reports of Drs. Chen and Pegden in the *Harper* case. We also request the data and model parameters underlying Dr. Duchin's expert report in the *NCLCV* matter. Finally, we request the source code, source data, input parameters (as defined above), and output data (as defined above) used to generate the three "Optimized" Maps/Plans that the *NCLCV* Plaintiffs asked Dr. Duchin to assess and that they produced to the Court.

Considering the tight timeframe governing these cases, we ask that Plaintiffs produce these materials **by 12pm Monday, December 13**. We are available to discuss best ways to transfer this material.

If Plaintiffs in either case plan to withhold any of these materials, we ask for notice of that refusal **by 12pm Monday, December 13**.

Thank you very much,

Kate

Katherine L. McKnight
Partner

BakerHostetler
Washington Square
1050 Connecticut Ave, N.W. | Suite 1100
Washington, DC 20036-5403
T +1.202.861.1618

kmcknight@bakerlaw.com
bakerlaw.com