# Remedial Report : Congressional and NC Senate Plans 

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## 1 Introduction and summary

We have been asked by the Harper Plaintiffs and the Common Cause Plaintiffs to analyze two redistricting maps for both the North Carolina Congressional districts and the North Carolina Senate districts. Specifically, we will examine the Congressional and Senate maps that were recently passed by the General Assembly in laws 2022-3 (Congressional, S745), 2022-2 (Senate, S744), as well as alternative maps put forward by the Harper plaintiffs. The comments and analysis addressing the Harper Plaintiffs' proposed map were done solely at the request of the Harper Plaintiffs and not by the Common Cause Plaintiffs.

Because of the language in the court ruling, our primary tool of analysis is to examine partisan symmetry, which is the idea that a specific vote share should translate into a specific seat share, independent of which party received that vote.[1] The exact translation of votes to seats need not be known ahead of time; the important aspect of symmetry is that the translation is the same for both parties. As one example, under a map that has partisan symmetry, if the Republicans receive $55 \%$ of the vote and $70 \%$ of the seats, then when the Democrats receive $55 \%$, they will also receive $70 \%$ of the seats. Prioritizing symmetry does not translate into any proportionality standard. However under a symmetric map, the party that wins the majority of the vote should win the majority of the seats (or at least not be in the minority).

The Supreme Court's order also mentioned other metrics that can give some insight into the symmetry properties (as well as other properties) of a map, including the mean-median difference and the efficiency gap. We prefer to report directly on measures of partisan symmetry and focus on those in this report, but we also report mean-median difference and efficiency gaps.

We examine partisan symmetry characteristics of the four maps under 16 historic elections from 2016 and 2020: 2016 Attorney General, 2016 Governor, 2016 Lieutenant Governor, 2016 Presidential, 2016 U.S. Senate, 2020 State Auditor, 2020 Attorney General, 2020 Commissioner of Agriculture, 2020 Commissioner of Insurance, 2020 Commissioner of Labor, 2020 Governor, 2020 Lieutenant Governor, 2020 Presidential, 2020 Secretary of State, 2020 Treasurer, and 2020 U.S. Senate.

We find that the plaintiff maps show significantly greater amounts of symmetry than the recently passed maps put forward by the North Carolina legislature. We also demonstrate that if twenty maps were drawn from our original ensemble, which was constructed without regard to partisan symmetry, it would be extremely likely to find a map with significantly superior partisan symmetry when compared with the legislature's enacted remedial maps. In other words, even drawing maps at random, it is not difficult to draw maps that achieve significantly better partisan symmetry than the legislature's proposed remedial maps.

## 2 Qualifications

We are Professors of Mathematics at Duke University. Dr. Mattingly is also a Professor of Statistical Science at Duke University. His degrees are from the North Carolina School of Science and Math (High School Diploma), Yale University (B.S.), and Princeton University (Ph.D.). He grew up in Charlotte, North Carolina, and currently lives in Durham, North Carolina. Dr. Herschlag's degrees are from Taylor Allderdice (High School Diploma), University of Chicago (B.S.), and the University of North Carolina at Chapel Hill (Ph.D.). He has lived in North Carolina since 2007.

Both of us lead a group at Duke University that conducts non-partisan research to understand and quantify gerrymandering. This report grows out of aspects of our group's work around the current North Carolina legislative districts which are relevant to the case being filed.

Dr. Mattingly previously submitted an expert report in Common Cause v. Rucho, No. 18-CV-1026 (M.D.N.C.), Diamond v. Torres, No. 17-CV-5054 (E.D. Pa.), Common Cause v. Lewis (N.C. Sup. Ct No. 18-cvs-014001), and Harper v. Lewis
(No. 19-cv-012667) and was an expert witness for the plaintiffs in Common Cause v Rucho and Common Cause v. Lewis. Dr. Herschlag previously submitted an affidavit in North Carolina v. Covington, No. 1:15-cv-00399. We are being paid at a rate of $\$ 400 /$ per hour for this work. Much of the work, including the randomly generated maps, derives from an independent research effort, unrelated to this lawsuit, to understand gerrymandering nationally and in North Carolina specifically. Some of the analysis described in this report was previously released publicly as part of a non-partisan effort to inform the discussion around the redistricting process.

## 3 Methods

We evaluate the proposed plans using a partisan symmetry metric described below. We also report the the mean-median difference and the efficiency gap. Each of these metrics was calculated using the results of sixteen recent statewide elections: 2016 Attorney General, 2016 Governor, 2016 Lieutenant Governor, 2016 Presidential, 2016 U.S. Senate, 2020 State Auditor, 2020 Attorney General, 2020 Commissioner of Agriculture, 2020 Commissioner of Insurance, 2020 Commissioner of Labor, 2020 Governor, 2020 Lieutenant Governor, 2020 Presidential, 2020 Secretary of State, 2020 Treasurer, and 2020 U.S. Senate. In many analyses, we also consider the uniform swing of the elections under consideration which allows us to consider a varied range of statewide partisan vote fractions over multiple plausible voting patterns.

In line with the classic definition of partisan symmetry, the North Carolina Supreme Court explained, "voters are entitled to have substantially the same opportunity to electing a supermajority or majority of representatives as the voters of the opposing party would be afforded if they comprised" a given percentage "of the statewide vote share in that same election." Harper v. Hall, No. 413PA21, slip op. $\mathbb{1} 169$ (N.C. Feb. 14, 2022). To implement this directive, we measure the partisan symmetry by calculating the number of seats awarded to the party winning the majority of votes in pairs of elections that have total statewide partisan vote shares which are symmetric about the $50 \%$ level. Examples of symmetric pairs are $49 \%$ and $51 \%$ or $48 \%$ and $52 \%$. We then report the absolute difference in the number of seats awarded. If both parties were treated symmetrically, this difference would be zero.

To take an example: we begin with the results of the 2016 Governor election and apply a "uniform swing" to reflect a $48 \%$ Democratic statewide vote share for that election. We calculate how many Republican representatives would be elected with this $48 \%$ Democratic vote share. We then apply a uniform swing to the election so that it reflects the corresponding, reciprocal Democratic vote share-i.e., $52 \%$. We then compute the number of Democratic representatives that would be elected with that $52 \%$ Democratic vote share. We then calculate the absolute difference between the number of Republican representatives elected with $48 \%$ Democratic vote share and the number of Democratic representatives elected with a $52 \%$ Democratic vote share. Thus, if 8 Republicans were elected with $48 \%$ Democratic vote share, and 7 Democrats were elected with $52 \%$ vote share, the absolute difference would be 1 seat. (Because the figure is absolute, the value is always positive. It does not reflect which party benefits from the asymmetry; it captures only the degree of asymmetry.) We repeat this process using several sets of vote fractions which are equidistant from the majority line of $50 \%$. Namely, we consider $45 \%$ and $55 \%$, $46 \%$ and $54 \%, 47 \%$ and $53 \%$, and $49 \%$ and $51 \%$.

Reciprocity in a single election does not speak to possible variations in the spatial voting patterns seen across the state in different elections. Therefore, we repeat this procedure across the 16 historic statewide elections listed above, and then calculate an average of the absolute difference between the number of Republican seats elected (under the lower Democratic vote share) and the number of Democratic seats elected (under the higher Democratic vote share). The metric thus captures the average, absolute deviation, across elections and across vote shares, between the number of seats that the two parties are expected to elect at the same given vote share. Lower numbers reflect greater partisan symmetry, and in particular, reflect a more "equal opportunity to electing a supermajority or majority of representatives as the voters of the opposing party would be afforded if they comprised" a given percentage "of the statewide vote share in that same election." Harper slip op. 『169.

We emphasize that we consider the average deviation across 16 different elections, thereby capturing the degree of partisan symmetry exhibited by the map across a variety of different election climates. This is very different from considering a single electoral vote pattern constructed by averaging elections to create a different, possibly unobserved, vote pattern, and only then assessing the deviation.

In addition to examining the averaged deviation from partisan symmetry, we also examine the mean-median difference and the efficiency gap. The mean-median is defined to be the difference between the average Democratic vote share and the median Democratic vote share. ${ }^{1}$ The efficiency gap is defined to be the difference in wasted votes across the two parties

[^0]divided by the total vote for the two parties. Wasted votes are found by summing overall votes in losing districts and all votes in winning districts that are more than half the total votes; for example, if $D$ and $R$ are the Democratic and Republican votes in a district, and $D<R$ then the Democrats would have wasted $D$ votes and the Republicans would have wasted $R-(D+R) / 2$ votes. When computing the efficiency gap we uniformly swing each election to range from $45 \%$ to $55 \%$ of the vote in increments of $1 \%$, which provides greater diversity to the elections considered. ${ }^{2}$

## 4 Congressional Districts

Using the set of statewide elections listed in Section 3, the partisan symmetry of the Harper Plaintiffs' proposed congressional map - as measured using the metric described below, which reflects the average deviation in seats won between the parties given a particular vote share - is 0.36875 seats. In practical terms, this means that for any given statewide election, the number of Democratic and Republican seats elected at a given party vote fraction will more often than not be the same number; and the expected difference averaged across a range of sixteen statewide elections is only 0.36875 seats. Only 96 of the 80,000 sampled congressional plans both accounted for incumbency and had a partisan symmetry score of less than 0.40 seats.

The legislature's 2022 remedial congressional plan has an average partisan symmetry deviation of 1.575 seats - meaning the average seat deviation between the parties given the same vote share is 4 times as high as it is in Harper plaintiffs proposed plan. This reflects that, under the enacted plan, Republicans win 8 or 9 seats when they get $51 \%$ of the vote, while Democrats win 7 or 8 seats when they get $51 \%$ of the vote. If the map makers would have examined just 20 random plans from our ensemble, they would have found a plan with higher partisan symmetry than the S745 plan with a $99.998 \%$ chance. Furthermore, there would be a $98.56 \%$ chance that at least one of those plans would have a seat deviation of less than 1 . The 2022 enacted remedial Congressional plan has a mean-median gap of $1.01 \%$. The average efficiency gap calculated by conducting uniform swings on these election results, ranging from $45 \%$ to $55 \%$ Democratic vote share, is $7.312 \%$.

As to other partisan fairness metrics identified in the Supreme Courts order and opinion: The average mean-median difference for the Harper Plaintiffs' proposed map is $0.4504 \%$. The average efficiency gap calculated by conducting uniform swings on these election results, ranging from $45 \%$ to $55 \%$ Democratic vote share, is $2.7180 \%$.


Figure 1: We show the number of seats (horizontal axis) compared with the statewide vote (vertical axis) in our 16 historic elections under the Harper Plaintiffs' map (left), and the enacted map (S745; middle). We also directly compare the two maps (right)

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Figure 2: We show the statewide vote percentage won by the party in the majority of the vote (horizontal axis) compared with the statewide seats won by the majority party (vertical axis) in our 16 historic elections under the enacted map ( S 745 ; left), and the Harper Plaintiffs' plan (right). In a perfectly symmetric map, the blue line would always coincide with the red line.

To better illuminate the extent to which the two maps treat the parties symmetrically, we plot in Figure 1 what would be results of congressional elections run with historical elections mentioned in Section 3. We begin by noticing that the Harper Plaintiffs' proposed map always gives at at least half of the seats to the party which wins the majority of the votes. In contrast, the Legislature's S745 map only gives the Democrats at least half the seat in three of the six elections where they win the majority while always giving the Republicans at least half the seats in the elections where they win the majority of the votes. One can also understand the degree to which the maps produce seat counts which are symmetric. In a symmetric map, the behavior in the bottom half of these plots should "mirror" the behavior in the top half.

To better examine this, we calculate the seats won by the party with the majority of the vote under the sixteen specified elections when they are shifted, using the uniform swing hypothesis, to have a statewide Democratic share ranging from $45 \%$ to $55 \%$. We then average these 16 seat counts over each of the statewide vote fractions. We plot this average in Figure 2 as a function of the statewide majority vote fraction. When the Democrats are in the Majority (Democratic vote shares of $50 \%-55 \%$ ) we use a blue curve and plot the Democratic seat share. When the Republicans are in the Majority (Democratic vote shares of $45 \%-50 \%$ ), we use a red curve and plot the Republican seat share. If the map is symmetric, the seats elected in response to Democratic majority votes will be the same as the seats elected in response to Republican majority votes, and the two curves will lie on top of each other. The gray shaded region emphasizes the deviation from ideal partisan symmetry.

Looking at Figure 2, we see that there is a significant deviation from symmetry in the legislature's proposed 2022 remedial Congressional plan while the Harper Plaintiffs' proposed plan shows a high degree of symmetry, particularly between $49 \%$ and $51 \%$. Both maps favor the Republicans with respect to their deviation from partisan symmetry, as shown by the fact that the red curve is above the blue curve.


Table 1: We summarize Figure 2 on the congressional two maps with the above table. Pay particular attention to the number of times which map fails to give a party the majority of seats when they win the majority of the votes. Notice that this only occurs for the Democrats.



Figure 3: We show the number of seats (horizontal axis) compared with the statewide vote (vertical axis) in our 16 historic elections under the Harper Plaintiffs' map (left), and the NC Legislature's enacted map (S744; middle). We also directly compare the two maps (right).

## 5 Senate Districts

Using the set of statewide elections listed in Section 3, the partisan symmetry of the Harper Plaintiffs' proposed senate map - as measured using the metric described above for the congressional plans, which reflects the average deviation in seats won between the parties given a particular vote share - is 1.04375 seats. ${ }^{3}$

The legislature's 2022 enacted remedial senate plan has an average partisan symmetry deviation of 4.0125 seats - meaning the average seat deviation between the parties given the same vote share is again 4 times as high as it is in Harper plaintiffs proposed plan. This reflects that, under the enacted plan, Republicans win 29 or 30 seats when they get $52 \%$ of the vote, while Democrats win 25 or 26 seats when they get $52 \%$ of the vote. This is enough to potentially grant the Republicans a supermajority, whereas the Democrats either split the chamber or gain the smallest possible majority. If the map makers would have examined just 1 random plan from our ensemble, they would have found a plan with higher partisan symmetry than the S 744 plan with a $99.6 \%$ chance. Furthermore, there would be a $92.5 \%$ chance that at least one of those plans would have a symmetry deviation of less than 3 seats. If they had considered 20 plans, they would have been essentially guaranteed to find one with a symmetry deviation of less than 3 seats. The 2022 enacted remedial Senate plan has a mean-median gap of $1.304 \%$. The average efficiency gap calculated by conducting uniform swings on these election results, ranging from $45 \%$ to $55 \%$ Democratic vote share, is $4.072 \%$.

As to other partisan fairness metrics identified in the Supreme Courts order and opinion: The average mean-median difference for the Harper Plaintiffs' proposed senate map is $0.228 \%$. The average efficiency gap calculated by conducting uniform swings on these election results, ranging from $45 \%$ to $55 \%$ Democratic vote share, is $1.955 \%$.

In Figure 3, we plot what would be results of North Carolina Senate elections run with historical elections mentioned in Section 3. We begin by noticing that both the Harper Plaintiffs' proposed NC Senate map and the Legislature's S744 map always give at least half of the seats to the Republican Party when they win the majority. The Harper Plaintiffs' proposed NC Senate map gives the majority of the seats to the Democrats in four out of six elections where they win the majority of the votes while the Legislature's S744 map does so in three out of six elections. More telling, the Legislature's S744 map gives the Republicans the supermajority of seats or close to it, when they receive between $51 \%$ and $52 \%$ of the votes while the Democrats barely get or share the majority when they receive between $51 \%$ and $52 \%$ of the votes.

To better understand the extent to which the two plans respond symmetrically to swings in the Democratic or Republican

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Figure 4: We show the statewide vote percentage won by the party with the majority of the vote (horizontal axis) compared with the statewide won seats by the majority party (vertical axis) in our 16 historic elections under the enacted map (S744; left), and the Harper Plaintiffs' plan (right). In a perfectly symmetric map, the blue line would always coincide with the red line
direction, we calculate the seats won by the party with the majority of the vote under the sixteen specified elections when they are shifted, using the uniform swing hypothesis, to have statewide Democratic share ranging from $45 \%$ to $55 \%$. We then average these 16 seat counts over each of the statewide vote fractions. We plot this average in Figure 4 as a function of the statewide majority vote fraction. When the Democrats are in the Majority (Democratic vote shares of $50 \%-55 \%$ ) we use a blue curve. When the Republicans are in the Majority (Democratic vote shares of $45 \%-50 \%$ ), we use a red curve and plot the Republican seat share. If the response to Democratic majority votes is the same as Republican majority votes the two curves will be on top of each other. The gray shaded region emphasizes the deviation from ideal partisan symmetry.

It is clear from Figure 4 that the Legislature's S744 map is significantly less symmetric than the Harper Plaintiffs' plan. It is particularly striking that Harper Plaintiffs' plan shows almost perfect symmetry for deviations immediately around $50 \%$. Beyond that range the Harper Plaintiffs' plan actually treats Republicans more favorably than Democrats.

## References

[1] Bernard Grofman and Gary King. The future of partisan symmetry as a judicial test for partisan gerrymandering after LULAC v. Perry. Election Law Journal, 6(1):457-472, 2007.

We declare under penalty of perjury under the laws of the state of North Carolina that the foregoing is true and correct to the best of our knowledge.


Greg Herschlag 2/21/2022


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High School, NC School of Science and Math, Durham NC 1988

## APPOINTMENT HISTORY

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Professor in the Department of Statistical Science, Statistical Science 2012-2015
Associate Professor, Statistical Science 2008-2011
Associate Professor of Mathematics, Mathematics 2006-2012
Assistant Professor of Mathematics, Mathematics 2002-2005
Member special year in SPDE/Tubulence, Institute for Advance Study, Princeton. 2002-2003
NSF Post-Doctoral Fellow, Stanford University. 1999-2002
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Contractor, AT\&T Shannon Labs. 1999-1999 Summer Intern, Bell Labs, Lucent. 1996-1996

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Hairer, M., and J. C. Mattingly. "Ergodicity of the 2D Navier-Stokes equations with degenerate stochastic forcing." Annals of Mathematics, vol. 164, no. 3, 2006, pp. 993-1032.
Mattingly, J. C., and É. Pardoux. "Malliavin calculus for the stochastic 2D Navier-Stokes equation." Communications on Pure and Applied Mathematics, vol. 59, no. 12, 2006, pp. 1742-90. Scival, doi:10.1002/cpa.20136.
Bakhtin, Y., and J. C. Mattingly. "Stationary solutions of stochastic differential equations with memory and stochastic partial differential equations." Communications in Contemporary Mathematics, vol. 7, no. 5, Oct. 2005, pp. 553-82. Scopus, doi:10.1142/S0219199705001878.
Mattingly, J. C., and T. M. Suidan. "The small scales of the stochastic Navier-Stokes equations under rough forcing." Journal of Statistical Physics, vol. 118, no. 1-2, Jan. 2005, pp. 343-64. Scopus, doi:10.1007/s10955-004-8787-3.
Hairer, Martin, et al. "Malliavin calculus and ergodic properties of highly degenerate 2D stochastic Navier-Stokes equation." Arxiv Preprint Math/0409057, 2004.
Hairer, Martin, et al. "Malliavin calculus for highly degenerate 2D stochastic Navier-Stokes equations." Comptes Rendus Mathématique. Académie Des Sciences. Paris, vol. 339, 2004, pp. 793-96. Manual, doi:10.1016/j.crma.2004.09.002.
Hairer, Martin, and Jonathan C. Mattingly. "Ergodic properties of highly degenerate 2D stochastic Navier-Stokes equations." Comptes Rendus Mathématique. Académie Des Sciences. Paris, vol. 339, 2004, pp. 879-82. Manual, doi:10.1016/j.crma.2004.09.035.
Mattingly, Jonathan C. On recent progress for the stochastic Navier Stokes equations. Univ. Nantes, Nantes, 2003, p. Exp.No.XI-52.

Mattingly, Jonathan C. "The dissipative scale of the stochastics Navier-Stokes equation: regularization and analyticity." Journal of Statistical Physics, vol. 108, 2002, pp. 1157-79. Manual, doi:10.1023/A:1019799700126.
Mattingly, J. C., et al. "Ergodicity for SDEs and approximations: locally Lipschitz vector fields and degenerate noise." Stochastic Processes and Their Applications, vol. 101, 2002, pp. 185-232. Manual, doi:10.1016/S0304-4149(02)00150-3.
Mattingly, J. C. "Contractivity and ergodicity of the random map \$x\mapstolvert x- $\theta \backslash$ vert $\$$." RossilU\I Skaya Akademiya Nauk. Teoriya VeroyatnostelU\I I Ee Primeneniya, vol. 47, 2002, pp. 388-97. Manual, doi:10.1137/S0040585X97979767.
Mattingly, Jonathan C. "Exponential convergence for the stochastically forced Navier-Stokes equations and other partially dissipative dynamics." Communications in Mathematical Physics, vol. 230, 2002, pp. 421-62. Manual, doi:10.1007/s00220-002-0688-1.

Mattingly, J. C., and A. M. Stuart. "Geometric ergodicity of some hypo-elliptic diffusions for particle motions." Markov Processes and Related Fields, vol. 8, 2002, pp. 199-214.
Mattingly, Jonathan Christopher. "Contractivity and ergodicity of the random map \$x\mapsto|x-\theta|\$." Teoriya Veroyatnostei I Ee Primeneniya, vol. 47, no. 2, Steklov Mathematical Institute, 2002, pp. 388-97. Crossref, doi:10.4213/tvp3671.
E, Weinan, et al. "Gibbsian dynamics and ergodicity for the stochastically forced Navier-Stokes equation." Communications in Mathematical Physics, vol. 224, 2001, pp. 83-106. Manual, doi:10.1007/s002201224083.
E, Weinan, and Jonathan C. Mattingly. "Ergodicity for the Navier-Stokes equation with degenerate random forcing: finite-dimensional approximation." Communications on Pure and Applied Mathematics, vol. 54, 2001, pp. 1386402. Manual, doi:10.1002/cpa. 10007.

Mattingly, Jonathan C. "Ergodicity of \$2\$D Navier-Stokes equations with random forcing and large viscosity." Communications in Mathematical Physics, vol. 206, 1999, pp. 273-88. Manual, doi:10.1007/s002200050706.
Mattingly, J. C., and Ya G. Sinai. "An elementary proof of the existence and uniqueness theorem for the NavierStokes equations." Communications in Contemporary Mathematics, vol. 1, 1999, pp. 497-516. Manual, doi:10.1142/S0219199799000183.
Holmes, Philip J., et al. "Low-dimensional models of coherent structures in turbulence." Physics Reports. a Review Section of Physics Letters, vol. 287, 1997, pp. 337-84. Manual, doi:10.1016/S0370-1573(97)00017-3.
Johndrow, James E., et al. Optimal approximating Markov chains for Bayesian inference.
Bangia, Sachet, et al. Redistricting: Drawing the Line.
Johndrow, James E., and Jonathan C. Mattingly. Error bounds for Approximations of Markov chains used in Bayesian Sampling.
Wang, Chuang, et al. Scaling Limit: Exact and Tractable Analysis of Online Learning Algorithms with Applications to Regularized Regression and PCA.
Carter, Daniel, et al. A Merge-Split Proposal for Reversible Monte Carlo Markov Chain Sampling of Redistricting Plans.
Herschlag, Gregory, et al. Non-reversible Markov chain Monte Carlo for sampling of districting maps.
Autry, Eric A., et al. Multi-Scale Merge-Split Markov Chain Monte Carlo for Redistricting.
Leimbach, Matti, et al. Noise-induced strong stabilization.
Mattingly, Jonathan C., et al. The Gaussian Structure of the Singular Stochastic Burgers Equation.
Herzog, David P., et al. Gibbsian dynamics and the generalized Langevin equation.
Earle, Gabriel, and Jonathan Mattingly. Convergence of Stratified MCMC Sampling of Non-Reversible Dynamics. Mattingly, Jonathan C., et al. "Diffusion limits of the random walk Metropolis algorithm in high dimensions."
Annals of Applied Probability, vol. 22, no. 3, pp. 881-930. Arxiv, doi:10.1214/10-AAP754.
Heymann, Matthias, et al. Rare Transition Events in Nonequilibrium Systems with State-Dependent Noise: Application to Stochastic Current Switching in Semiconductor Superlattices.

## Theses and Dissertations

Mattingly, Jonathan. The Stochastic Navier-Stokes Equation: Energy Estimates and Phase Space Contraction, under Yakov Sinai.

## PROFESSIONAL AWARDS AND SPECIAL RECOGNITION

IE Block Community Lecture. SIAM. 2021
Defenders of Democracy. National Common Cause. 2018
Fellow of the American Mathematical Society. American Mathematical Society. 2015
Simons Visiting Professor . MSRI. 2015
Institute of Mathematical Statistics Fellow. Institute of Mathematical Statistics. 2012
Faculty Early Career Development (CAREER) Program. National Science Foundation. 2005

Presidential Early Career Awards for Scientists and Engineers. National Science Foundation. 2005
Sloan Research Fellowship-Mathematics. Alfred P. Sloan Foundation. 2005
School of Mathematics/ Members. Institute for Advanced Study. 2002

## PRESENTATIONS AND APPEARANCES

Sampling to Understand Gerrymandering and Influence Public Policy. MIT. January 1, 2021
Panel on Qunatifying Gerrymandering. Democracy in America. October 1, 2021
Hearing the Will of the People. ISM. August 1, 2021
Non-rversible samplers for Gerrymandering. Netherlands. August 1, 2021
The Gaussian Structure of the Stochastically Forced Burgers Equation. Berlin. May 1, 2021
The Mathematics and Policy of Gerrymandering. IAS. December 1, 2021
Gaussian Structure of Burgers Equation. India (online). January 1, 2021
A new model of randomly forced Fluid equations. Princeton Fluids Seminar. November 1, 2021
A new model of randomly forced Fluid equations. ICEM. October 1, 2021
A new model of randomly forced Fluid equations. IAS. December 1, 2021
Gaussian Structure of Stochastic Burgers. February 1, 2021
New Sampling Methods of Quantifying Gerrymandering. Brown Applied Math Colloquium . October 1, 2020
Interactions between noise and instabilities.. IHP, Paris. July 1, 2018
Quantifying Gerrymandering: A Mathematician Goes to Court. July 1, 2018
Ergodicity of Singular SPDEs. Columbia. May 1, 2018
Approximate/exact controllability and ergodicity for (additive noise) SPDEs/SODEs. CIRM, Marseilles 2018
Discovering the geopolitical structure of the United States through Markov Chain Monte Carlo sampling. The Alan
Turing Institute, UK. May 1, 2018
Drawing the line in redistricting (A mathematician's take). Stanford University. March 1, 2018
Ergodic and global solutions for singular SPDEs. Corvallis, Oregon. March 1, 2018
A mathematician Goes to Court. October 1, 2017
Stabilization of Stochastic Dynamics . UCLA. IPAM. January 1, 2017
Stabilization and noise. Berekey Mathematics Department. November 12, 2015
Stochastic PDEs. October 1, 2015
Ergodicity Finite and Infinite dimentional Markov Chains. McGill University. July 1, 2015

## Lectures

New Sampling Methods to Quantify Gerymandering. IID. Duke Law and TRIPODS. March 1, 2020
Anatomy of an ergodic theorem. Summer School. June 1, 2018
Dynamics Days 2014. Atlanta GA. January 4, 2014
Stabilization by Noise. November 19, 2013
Uniqueness of the inviscid limit in a simple model damped/driven system.. Probability and Mathematical Physics Seminar. November 5, 2013
Stochastic stabilization of OEDs.. Applied Math Seminar, NYU. September 6, 2013
Stochastic partial differential equations. SPA2013. August 1, 2013
Stabilization by noise. University of Maryland. May 1, 2013
Stablization by Noise. Conférence en l'honneur d'Etienne Pardoux, CIRM, Marseillais France.. February 14, 2013
Perspectives on Ergodicity. Conference on SPDEs, IMA, Minnesota. January 14, 2013

A Numerical Method for the SDEs from Chemical Equations. Probability and Biology section, 2012 Canadian Mathematical society (winter meeting). December 1, 2012
Minerva Lectures: Erodicity of Markov Processes: From Chains to SDEs to SPDEs. Mathematics Department, Columbia University. November 1, 2012
Stochastic Stabilization. Inria - Sophia Antipolis. July 1, 2012
A Menagerie of Stabilization. Joint Probability and Analysis Seminar, Nice, France. July 1, 2012
Building Lyapunov Functions (4 lectures). EPSRC Symposium Workshop - Easter Probability Meeting. March 1, 2012

Noise Induced Stability. MBI. February 1, 2012
A Menagerie of Stochastic Stabilization. CAMP/Probability Seminar, University of Chicago. October 18, 2011
A menagerie of stochastic stabilization. Equadiff 2011, Loughborough University. August 1, 2011
Coarse-graining of many-body systems: analysis, computations and applications. July 1, 2011
Ergodicity of systems with singular interaction terms. Stochastic Dynamics Transition Workshop, SAMSI. November 18, 2010
Oberwolfach Seminar: The Ergodic Theory of Markov Processes. Oberwolfach, Germany. October 1, 2010
Malliavin Calculus to prove ergodic theorems for SPDEs. ICM Satellite Conference on Probability and Stochastic Processes Indian Statistical Institute, Bangalore. August 13, 2010
SPDE scaling limits of an Markov chain Montecarlo algorithm. Stochastic Partial Differential Equations:
Approximation, Asymptotics and Computation, Newton Institute. June 28, 2010
The spread of randomness. German-American Frontiers of Science, Potsdam Germany. June 1, 2010
How to prove an ergodic theorem. oberwolfach. May 1, 2010
Coupling at infinity. Seminar on Stochastic Processes. March 30, 2010
Long time stochastic simiulation. Imperial College. March 15, 2010
Spectral Gaps in Wasserstien Distance. Ergodic Theory Seminary, Princeton Mathematics. March 4, 2010
Trouble with a chain of stochastic oscillators. PACM, Princeton University. March 2, 2010
Hypo-ellipticity for SPDEs. SPDE program , Newton Institute. March 1, 2010
Numerics of SDEs. Warwick University, UK. February 24, 2010
Long Time Behavior of Stochastically Forced PDEs.. AMS Joint Meeting, San Francisco. January 14, 2010
Ellipticity and Hypo-ellipticity for SPDEs *or* What is ellipticity in infinite dimensions anyway?. Stochastic Partial Differential Equations, Newton Institute. January 8, 2010
SPDE Limits of the Random Walk Metropolis Algorithm in High Dimensions. SIAM PDE Meeting. December 7, 2009
Stochastic fluctuations in bio chemical networks. MBI: Mathematical Developments Arising from Biology. November 9, 2009
What makes infinite dimensional Markov processes different ?. Stochastic Process and Applications, Berlin. July 1, 2009
Introduction to Ergodicity in Infinite Dimentions. TU Berlin. July 1, 2009
Stochastically forced fluid equations: Transfer between scales and ergodicity.. AMS Sectional Meeting (invited talk). April 4, 2009
Trouble with a chain of stochastic oscillators. Princeton University. PACM. April 3, 2009
What makes the ergodic theory if Markov Chains in infinite dimensions different (and dificult) ?. Princeton Ergodic theory seminar. March 3, 2009
Ergodicity, Energy Transfer, and Stochastic Partial Differential Equations. Columbia University. Columbia University. December 15, 2008
The Spread of Randomness: Ergodicity in Infinite Dimensions. Mathematisches Forschungsinstitut Oberwolfach. December 15, 2008
The spread of randomness through dimensions. IPAM. November 1, 2008

The spread of randomness through dimensions. IPAM- Mathematical Frontiers in Network Multi-Resolution Analysis. November 1, 2008
Troubles with oscillators. Stanford: JBK85, Workshop on Applied Mathematics IN HONOR OF JOSEPH B. KELLER. October 1, 2008
What is different about the ergodic theory of stochastic PDEs (vs ODEs). UC Irvine, PDE and Probabilty Seminar. October 1, 2008
Trouble with a chain of stochastic oscillators. Stochastic Seminar, GaTech. September 1, 2008
Troubles with oscillators. East Midlands Stochastic Analysis Seminars. August 1, 2008
Troubles with chains of anharmonic oscillators. Statisical Mechaniques working group. June 1, 2008
The spread of randomness in infinite dimensions and ergodicity for SPDEs. Stochastic Analysis, Random Fields and Applications, Asscona IT. June 1, 2008
Ergodicity of Degenerately forced SPDEs. Séminaire de Probabilités, Laboratoire de Probabilités et Modèles Aléatoires des Universités Pierre et Marie Curie et Denis Diderot. May 27, 2008
Ergodicity of Degenerately forced SPDEs. ETH, Zurich. May 1, 2008

## Named Lectures

Barton Lectures in Computational Mathematics. UNCG. November 1, 2021
IE Block Community Lecture . SIAM Annual Meeting. SIAM. July 1, 2021
Quantifying and Understanding Gerrymandering - How a quest to understand his state's political geography led a mathematician to court. ICERM . October 1, 2020

AMS Regional Meeting Plenary Speaker. Gainesville . AMS. January 1, 2019
Long Time Numerical Simulation of SDEs. Insbruk. SciCADE2019 . January 1, 2019
Quantifying Gerrymandering: A mathematician goes to court. UBC. May 1, 2018
Quantifying Gerrymandering: a mathematician goes to court. Stanford Mathematics Department. March 1, 2018
Stochastic PDEs. July 1, 2016

## Event/Org Administration

Co-Organizer . Quantifying Gerrymandering. SAMSI. October 2018
Co-Organizer . Regional Gerrymandering Conference. November 2017
Co-Organizer . Interacting particle systems WITH APPLICATIONS IN BIOLOGY, ECOLOGY, AND STATISTICAL PHYSICS. SEPC 2017. May 2017
Organiser Special Term. MSRI, Berkeley CA. August 2015 - December 2015
Organized invited session at SPA2013. August 2013
Co Organizer (with Amarjit Budhiraja ) : Seminar on Stochastic Processes 2013. March 2013
Local Orgnaizer (with Rick Durrett) : Woman in Probability III. October 2012
SAMSI Stochastic Dynamics tradition workshop. November 2010
MFO week long school on ergodic theory. October 2010
SAMSI Opening Workshop for Stochastic Dynamics. August 2009
local liaison/Organizer SAMSI year on stochastic dynamics. 2009-2010
Organiser Special Term. MSRI, Berkeley CA. August 2007 - December 2007

## CURRICULUM VITAE

Gregory Joseph Herschlag, Ph.D.
Assistant Research Professor
gjh@math.duke.edu

CURRENT APPOINTMENTS AND AFFILIATIONS

Assistant Research Professor of Mathematics

## EDUCATION, TRAINING AND CERTIFICATIONS

Ph.D., Department of Mathematics, University of North Carolina - Chapel Hill, 2013

- Thesis supervisor: Prof. Sorin Mitran.
- Thesis: Multiple Scale Algorithm Design for Advancing Fronts

BS with Honors, University of Chicago, 2007

## DUKE APPOINTMENT HISTORY

Phillip Griffiths Assistant Research Professor 2018-2019
Visiting Assistant Professor of Mathematics, Mathematics 2013-2018

## PUBLICATIONS

## Academic Articles

Autry, Eric A., Daniel Carter, Gregory J. Herschlag, Zach Hunter, and Jonathan C. Mattingly. "Metropolized Multiscale Forest Recombination for Redistricting." Multiscale Modeling \& Simulation 19, no. 4 (January 2021): 1885-1914. https://doi.org/10.1137/21m1406854.
G. Herschlag, S. Lee, J. Vetter and A. Randles, "Analysis of GPU Data Access Patterns on Complex Geometries for the D3Q19 Lattice Boltzmann Algorithm," in IEEE Transactions on Parallel and Distributed Systems, 2021, doi: 10.1109/TPDS.2021.3061895.

Herschlag, G., Kang, H. S., Luo, J., Graves, C. V., Bangia, S., Ravier, R., \& Mattingly, J. C. (2020). Quantifying gerrymandering in north carolina. Statistics and Public Policy, 7(1), 30-38. doi:10.1080/2330443X.2020.1796400.

Carter, D., Hunter, Z., Teague, D., Herschlag, G., \& Mattingly, J. (2020). Optimal Legislative County Clustering in North Carolina. Statistics and Public Policy, 7(1), 19-29. doi:10.1080/2330443X.2020.1748552.

Herschlag, G., J. Gounley, S. Roychowdhury, E. Draeger, and A. Randles. "Multi-physics simulations of particle tracking in arterial geometries with a scalable moving window algorithm." Proceedings Ieee International Conference on Cluster Computing, Iccc, vol. 2019-September, 2019. Scopus, doi:10.1109/CLUSTER.2019.8891041.

Chin, A., Herschlag, G., \& Mattingly, J. (2018). The Signature of Gerrymandering in Rucho v. Common Cause. SCL Rev., 70, 1241.

Herschlag, G., Lee, S., Vetter, J. S., \& Randles, A. (2018, May). GPU data access on complex geometries for D3Q19 lattice Boltzmann method. In 2018 IEEE International Parallel and Distributed Processing Symposium (IPDPS) (pp. 825-834). IEEE, doi:10.1109/IPDPS.2018.00092.

Cao, Y., Feng, Y., Ryser, M. D., Zhu, K., Herschlag, G., Cao, C., ... \& You, L. (2017). Programmable assembly of pressure sensors using pattern-forming bacteria. Nature biotechnology, 35(11), 1087-1093. PMID: 28991268. PMCID: 28991268.

Herschlag, G., Liu, J. G., \& Layton, A. T. (2016). Fluid extraction across pumping and permeable walls in the viscous limit. Physics of Fluids, 28(4), 041902, doi:10.1063/1.4946005.

Herschlag, G. J., Mitran, S., \& Lin, G. (2015). A consistent hierarchy of generalized kinetic equation approximations to the master equation applied to surface catalysis. The Journal of chemical physics, 142(23), 234703. doi:10.1063/1.4922515. PMID: 26093569. PMCID: 26093569.

Herschlag, G., Liu, J. G., \& Layton, A. T. (2015). An exact solution for stokes flow in a channel with arbitrarily large wall permeability. SIAM Journal on Applied Mathematics, 75(5), 2246-2267, doi:10.1137/140995854.
G. Herschlag, T. C. Elston, M. G. Forest, G. Garcia, B. Reinhardt, B. Button, R. Tarran and B. Lindley. A mechanochemical model for auto-regulation of lung airway surface layer volume. Journal of Theoretical Biology. 325 (2013) 4251
G. Herschlag and L. A. Miller. Reynolds number limits for jet propulsion: A numerical study of simplified jellyfish. Journal of Theoretical Biology 285 (2011) 84-95

## Pre-prints

Herschlag, G., Mattingly, J. C., Sachs, M., \& Wyse, E. (2020). Non-reversible Markov chain Monte Carlo for sampling of districting maps. arXiv preprint arXiv:2008.07843.

Carter, D., Herschlag, G., Hunter, Z., \& Mattingly, J. (2019). A merge-split proposal for reversible monte carlo markov chain sampling of redistricting plans. arXiv preprint arXiv:1911.01503.

Herschlag, G., Ravier, R., \& Mattingly, J. C. (2017). Evaluating partisan gerrymandering in Wisconsin. arXiv preprint arXiv:1709.01596.

## Other work

Contributer and maintainer of the Duke Quantifying Gerrymandering Blog at https://sites.duke.edu/quantifyinggerrymandering/ (2018-present)
Aided in preparing the affidavit of Jonathan Mattingly in Harper v. Lewis https://sites.duke.edu/quantifyinggerrymandering/files/2019/12/Mattingly-Nov.-26-Declaration.pdf (2019)
Aided in preparing the expert report and rebuttal of Jonathan Mattingly in Common Cause v. Lewis. https://sites.duke.edu/quantifyinggerrymandering/files/2019/09/Report.pdf (2019)

Guy-Uriel Charles, Andrew Chin, Gregory Herschlag and Jonathan C. Mattingly. Op-Ed: "The fight against partisan gerrymandering continues." Harold Sun https://www.heraldsun.com/opinion/article217639645.html August 31, 2018 10:25 AM

Herschlag. Affidavit on Evidence of Racial Gerrymandering in Covington V. North Carolina (2017)
Aided in preparing the expert report of Jonathan Mattingly in Rucho v. Common Cause. https://s10294.pcdn.co/wp-content/uploads/2016/05/Expert-Report-of-Jonathan-Mattingly.pdf (2017)

## Code Repositories

Multi-scale merge-split; a hierarchical sampling algorithm on multi-level graph partitions:
https://git.math.duke.edu/gitlab/gjh/multiscalemergesplit_codebase
Merge-split; a sampling algorithm on graph partitions:
https://git.math.duke.edu/gitlab/gjh/mergesplitcodebase
An optimal county clustering algorithm based on legal redistricting criteria:
https://git.math.duke.edu/gitlab/gjh/countycluster.git

## Courses Taught

- MATH 493: Research Independent Study on Bayesian Methods to Evaluate School Report Cards (with Atsushi Hu; Fall 2020, Fall 2021)
- MATH 494: Research Independent Study on Bayesian Methods to Evaluate School Report Cards (with Atsushi Hu; Fall 2020, Fall 2021)
- MATH 490/790-95: Sampling: Theory and Practice (Spring 2021)
- IDS 798: Capstone Project (Spring 2020, Fall 2020, Spring 2021)
- MATH 202D: Multivariable Calculus for Economics (Fall 2020)
- MATH 230/730; STA 230: Probability (Fall 2019)
- MATH 390: Special Topics in Mathematics (Bass Connections on Gerrymandering) (Fall 2018, Spring 2019)
- MATH 393: Research Independent Study on Election Data Analysis (with Yashas Manjunatha; Spring 2019)
- MATH 353: Ordinary and Partial Differential Equations (Fall 2013, Fall 2014, Fall 2016(two sections), Fall 2017 (two sections))
- MATH 361S: Numerical Analysis (Spring 2016)
- MATH 431: Advanced calculus (Spring 2015)
- MATH 212: Multivariable calculus (Fall 2015)


## Mentoring Activities

- Post-doc in Mathematics Eric Autrey on graph partition algorithms (Summer 2019 - present)
- Organized, facilitated and ran the Master's in Interdisciplinary Data Science Capstone projects: 18 projects and 39 students in the Spring of 2020, and 52 students and 17 projects in the 2020-21 accademic year. This includes actively engaging, guiding, and mentoring project teams throughout the program.
- Organized, facilitated and ran the Data+ program in the summer of 2020 and 2021. This includes actively engaging, guiding, and mentoring project teams throughout the program.
- Three Master's students in MIDS, Jaryl Ngan, Anshupriya Srivastava, and Ishan Gupta, on understanding the history of segregation in Durham Public Schools and effects of redistricting (2020-2021)
- Master's student Evan Wyse on non-reversible sampling methods in the context of sampling graph partitions (Fall 2019 - present)
- Undergraduate math major Atsushi Hu on a project examining Simpson's Paradox and Bayesian Inference within reporting School Quality; PRUV mentor and advisor for senior thesis (Summer 2020-Spring 2021)
- Doctoral student in Biomedical Engineering Daniel Puleri on lattice Boltzmann Methods (2018 - present)
- Post-doc in Mathematics Matthias Sachs on non-reversible skew detailed balance algorithms (2018-2020)
- Master's student in Biomedical Engineering Ismael Perez on lattice Boltzmann Methods (2018-2019)
- Mentored Onuoha Odim on a Public Policy undergraduate capstone project. The project was on racially polarized voting in Dallas, Texas, and lead to an undergraduate publication "Segregation and Integration in Dallas County" in DUJPPE Fall 2020 (Spring through Fall of 2019).
- Undergraduate computer science majors Luke Farrell and Jacob Schulman on undergraduate honors thesis around stratified sampling graph partitions related to quantifying gerrymandering; Supervisor (2018-2019).
- Undergraduate math major Claire Weibe on honor thesis concerning voting patterns and representation; committee member and mentor (2018-2019)
- Lead a Bass Connections course on understanding gerrymandering spanning the 2018-2019 accademic school year; involved 18 students and 4 research projects. (2018-2019)
- Master's student in Computer Science Elizabeth Margolin, a student of Ashwin Machanavajjhala, assisted with data analysis and algorithms for evaluating the effects of differential privacy on redistricting (2018-2019)
- High School students (at NCSSM) Daniel Carter, Zach Hunter on advance sampling algorithms (Summer 2019)
- High School students (at NCSSM) Daniel Carter, Zach Hunter, Olivia Fujikawa, and Sam Ferguson on optimal clustering algorithms, modelling how spatial patterns effect district representation, and advance sampling algorithms (2018-2019)
- Master's student in Statistics Lisa Libovich on analyzing redistricting in Maryland (2017-Summer 2018)


## Presentations and Invited Talks

- Monte Carlo Methods for Revealing Gerrymandering. NYU Center for Data Science; Math \& Democracy Seminar, December 2022.
- Quanityfing Gerrymandering. BU Mathematics and Statistics Colloquium, Fall 2022.
- Uncovering Gerrymandering. CSU San Bernardino Mathematics Colloquium, March 2021.
- Voting: The Struggle for Voice in American Politics. Virtual. Kavli Frontiers in Science NSF. July 3, 2020
- County Preservation. TRIPODS Redistricting Conference, Durham, NC. Duke University. March 4, 2020
- Duke Law School Lunch. Duke Law School. October 2, 2019
- Supreme Court Lunch. UNC Law School. July 1, 2019
- Quantifying Gerrymandering. Florida State University Department of Mathematics. Florida State University. February 5, 2019
- Quantifying Gerrymandering: Separating Natural Bias from Political Bias. Political Science Department. University of Delaware. October 4, 2018
- Quantifying Gerrymandering: Sampling the Space of Redistricting Plans. Mathematics Department. University of Delaware. October 3, 2018
- GPU Data Access on Complex Geometries for D3Q19 Lattice Boltzmann Method. Vancouver, BC. IEEE. May 1, 2018
- GPU Data Layouts for D3Q19 Lattice Boltzmann Methods. University of North Carolina At Chapel Hill. March 4, 2018
- Using GIS tools to understand the space of political redistricting plans. Duke Computer Science Department. November 3, 2017
- Computational methods for sampling the space of redistricting plans. Duke University. November 3, 2017
- Quantifying Gerrymandering. Gross Hall. Information Initiative at Duke. October 1, 2017
- Introduction to Computing with GPUs. Physics Building. Mathematics Department at Duke University. April 6, 2017
- Continuum-atomistic computations for dendritic solidification. University of North Carolina Chapel Hill. August 1, 2013
- Continuum approximation of the chemical master equation. SIAM CSE, Boston. March 5, 2013
- Simulation of Solidification by Coupling of Phase Field and Microscopic Computations. ICIAM Vancouver. December 6, 2011
- Memory access patterns for Lattice Boltzmann methods on GPUs. Poster session at the Duke Research Computing Symposium. Duke University. January 2017


## Public Appearances and Outreach

- Lecture on Gerrymandering in Ellen Veomett's undergraduate seminar. January 2021
- Claiming the Power of the Vote. Virtual. STEMEMPOWER; middle and high school students. July 3, 2020
- Quantifying Gerrymandering. Raleigh Charter High School. November 5, 2019
- Gerrymandering on trial: The case for fair maps. May 3, 2019
- Panelist at Measures of Gerrymandering. Tucson, AZ. University of Arizona. October 5, 2018
- Quantifying Gerrymandering Public Lecture. San Francisco, CA. University of San Francisco. March 4, 2018


## Service to Profession

## Event/Org Administration

Organizer. TRIPODS Redistricting Conference 2020. Duke University. March 2020

Organizer. TRIPODS Quantifying Gerrymandering 2019. Duke University. November 2019

Organizer. Minisymposium at SIAM-SEAS. University of North Carolina at Chapel Hill. March 2018

Organizer. Triangle Research Group Meetings (meets roughly once per month since 2018)
Member. Industrial Affiliates Coordinator between Pratt and iID Practicum.
Participant. DCI Math Cicles; meet weekly over Spring 2021 with a group of 5 th grade students.

## Academic and Administrative Activities of the University

Organizer of Data + (2020 to present)
Masters in Interdisciplinary Data Science Capstone director (2020 to present)

Journals in which provided peer review since 2019
Applied Math Modeling
Physics of Fluids
Computer Physics Communications
Election Law Journal
Communications in Statistics - Theory and Methods
Statistics and Public Policy

## Submitted Grant Proposals

Submitted NSF grant for Computational Mathematics titled "Sampling Graph Partitions: Algorithms, Geospatial Structure, and Fairness" in November of 2020 as a co-PI

Submitted NSF grant on Harnessing the Data Revolution (HDR): Institutes for Data-Intensive Research in Science and Engineering in November of 2020 as senior personnel


[^0]:    ${ }^{1}$ Here we define Democratic/Republican vote share to be the fraction of the vote that went to one party compared with the vote going to both parties, i.e. $D /(R+D)$ where $D$ and $R$ are the Democratic and Republican votes in a district.

[^1]:    ${ }^{2}$ When performing a uniform swing analysis, it is more efficient to estimate the efficiency gap using the Democratic/Republican vote fractions as opposed to the vote. Under equal votes in each district, the use of the fractions gives the exact same result, however, it will provide a slight difference if this is not true. When employing uniform swings, we use the vote fractions. In our experience, this sightly different formulation creates little difference in the values because the populations are balanced across districts.

[^2]:    ${ }^{3}$ We remark that the coarse averaging of the measure we use is a rough approximation for the area of the gray regions shown in Figure 4 In this case, the $45 \%, 55 \%$ vote pairing is over-weighted and drives the average up (there are only 4 other number we are averaging with). If we would have instead averaged the seat deviation across all vote fractions between $50 \%-55 \%$, the deviation would have been closer to 0.5 .

