STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,

REBECCA HARPER, et al.,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

SECOND AFFIDAVIT OF DR. MOON DUCHIN ON REMEDIES

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.

2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.

4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.

5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.

6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.

7. A current copy of my full CV is attached to this report.

8. I am compensated at the rate of $400 per hour.

Second Report on Remedial Districting Plans in North Carolina

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

February 20, 2022

1 Introduction

Below, I will execute the analytical framework for evaluating remedial plans outlined in my report of February 18. The newly-passed plans SL-3 (new Congressional), SL-2 (new Senate), and SL-4 (new House) will be compared to the earlier proposals by the Legislature, and to the plaintiffs' alternative maps.

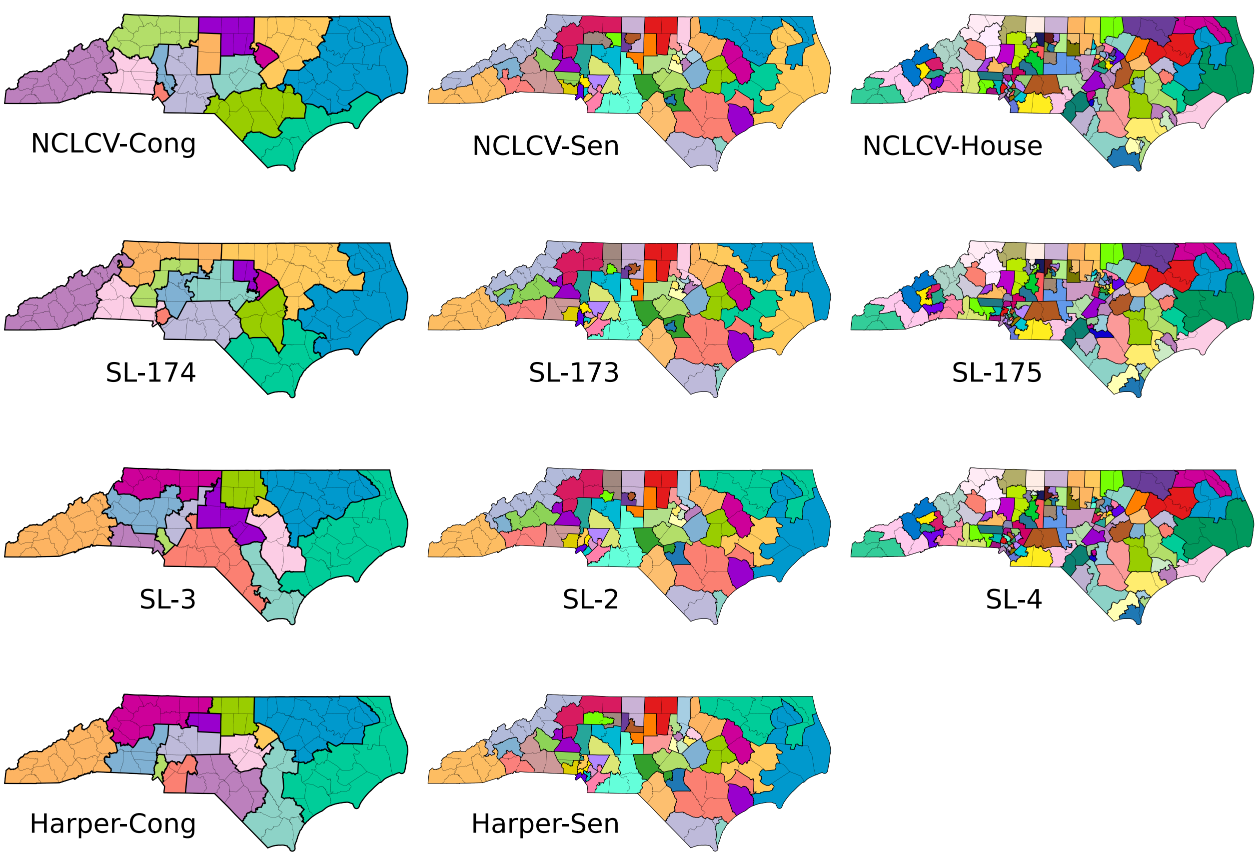


Figure 1: The eleven plans being compared in this report.

2 Close-Votes-Close-Seats

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. I use the full set of 52 general elections that occurred in North Carolina with a partisan ID in the last census cycle. This is a powerful tool to understand the performance of plans without the use of any vote index or counterfactuals.

First, Table 1 overviews the overlays with numbers, then Figures 2-4 illustrate the same data.[1]

The seats-votes scatterplots show all 52 data points for each map: one for each election, plotted as vote share for Democrats (x axis) against seat share for Democrats (y axis).

The northwest and southeast quadrants of these plots are the zones where anti-majoritarian outcomes fall. In each plot, I've marked the number of these outcomes in the associated quadrant. (I have excluded the JS120 race, which was so close to a 50-50 partisan outcome that its majoritarian properties are less meaningful.)

Out of 35 elections with a Republican vote advantage, the NCLCV-Cong plan has three instances where Democrats get more seats. Out of 16 elections with a Democratic vote advantage, the LCV plans have 0, 5, and 8 anti-majoritarian outcomes favoring Republicans.

In those 16 contests, the previous generation of plans from the legislature had 12, 12, and 14 anti-majoritarian outcomes (for Congress, Senate, and House, respectively). The new remedial proposals from the Legislature have 7, 7, and 8. And the Harper plaintiffs' Congressional and Senate plans have 1 and 2. (Note that the Harper plaintiffs did not submit a House plan.)

[1] Codes for reading Table 1: AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN = Senator; SOS = Secretary of State; SUP = Superintendent of Public Instruction; TRS = Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals. The two-digit suffix designates the election year. Where there was more than one judicial candidate from a given party on the ballot, they candidates from that party were combined for this analysis, so that there is a total Republican vote and a total Democratic vote in that contest.

	D Vote Share	NCLCV-Cong	SL-174	SL-3	Harper-Cong	NCLCV-Sen	SL-173	SL-2	Harper-Sen	NCLCV-House	SL-175	SL-4
GOV12	0.4418	4	4	4	6	18	16	16	15	44	41	39
AGC16	0.4444	4	4	4	6	17	17	17	16	42	40	40
LAC16	0.4475	5	4	4	5	20	18	18	17	45	42	43
JHU16	0.4563	5	4	4	6	19	18	19	17	49	42	44
AGC20	0.4615	4	3	4	5	19	17	19	19	51	40	44
JZA16	0.4619	5	4	4	6	21	19	20	18	50	43	46
JDI16	0.4653	6	4	4	6	21	19	20	19	53	44	47
LTG16	0.4665	6	4	4	6	21	19	20	21	54	44	47
LAC12	0.4674	5	4	5	6	20	20	16	15	51	44	43
AGC12	0.4678	5	4	5	6	18	18	16	16	50	43	42
SEN16	0.4705	6	**4**	**4**	6	**21**	**19**	**20**	**22**	55	**43**	**47**
TRS16	0.473	6	**4**	**4**	6	**21**	**19**	**20**	**19**	**53**	**45**	**49**
TRS20	0.4743	6	**4**	**4**	6	**20**	**17**	**19**	**21**	**51**	**45**	**49**
JA620	0.4806	7	**4**	**4**	6	**21**	**17**	**19**	**21**	55	**46**	**53**
PRS16	0.4809	7	**4**	**4**	7	**22**	**19**	**21**	23	56	**48**	**52**
JA420	0.4822	7	**4**	**4**	6	**22**	**17**	**19**	**21**	56	**47**	**54**
INC20	0.4823	7	**4**	**4**	7	23	**18**	**20**	**22**	56	**47**	**53**
LTG20	0.4836	7	**4**	**4**	6	**21**	**18**	**21**	**21**	55	**46**	**54**
JA720	0.4842	7	**4**	**4**	6	**22**	**17**	**21**	**21**	56	**48**	55
SUP20	0.4862	7	**4**	**4**	6	23	**19**	**22**	**22**	56	**49**	57
JA520	0.4874	7	**4**	**4**	6	**22**	**18**	**21**	**21**	57	**49**	57
JA218	0.4876	7	**4**	**4**	7	**22**	**18**	**20**	**22**	55	**45**	**49**
JS420	0.4879	7	**4**	**5**	7	24	**19**	**22**	23	56	**49**	57
J1320	0.4885	7	**4**	**4**	7	23	**19**	**22**	**22**	56	**49**	57
PRS12	0.4897	6	**4**	6	6	**21**	**20**	**21**	**19**	55	**46**	**48**
SEN20	0.491	7	**4**	6	6	24	**20**	**22**	23	56	**48**	56
LAC20	0.4918	8	**4**	**5**	7	25	**21**	23	23	58	**51**	56
SEN14	0.4919	6	**4**	6	6	**22**	**20**	**20**	**21**	**52**	**46**	**49**
PRS20	0.4932	8	**4**	**5**	6	25	**20**	**22**	**22**	60	**50**	59
JS220	0.4934	8	**4**	6	7	24	**21**	**22**	24	59	**51**	58
SUP16	0.4941	6	**4**	6	7	23	**22**	23	25	57	**49**	**53**
JS118	0.4955	7	**4**	**5**	7	25	**20**	**22**	23	58	**50**	**54**
INC16	0.496	6	**4**	**5**	7	**22**	**22**	**22**	25	57	**50**	**53**
JST16	0.4976	7	**4**	6	7	23	**21**	**22**	25	58	**50**	**54**
LTG12	0.4992	7	**5**	6	6	**22**	**22**	**22**	**22**	58	**50**	**53**
JS120	0.5	8	**4**	6	7	27	**22**	25	27	60	**52**	60
AUD16	0.5007	8	**5**	6	7	23	**22**	23	26	56	**51**	**51**
GOV16	0.5011	7	**4**	6	7	27	**20**	23	26	58	**50**	**54**
ATG20	0.5013	8	**4**	6	7	25	**21**	23	24	58	**51**	59
ATG16	0.5027	7	**4**	6	7	23	**20**	23	24	57	**50**	**54**
JA118	0.5078	8	**4**	7	7	26	**22**	24	25	58	**51**	59
AUD20	0.5088	8	**4**	7	7	**28**	24	26	**28**	61	**54**	62
JA318	0.5091	8	**4**	6	7	26	**21**	25	25	59	**52**	58
SOS20	0.5116	8	**5**	8	7	**28**	24	26	**28**	62	**53**	61
JGE16	0.5131	8	**5**	6	7	25	**22**	25	**28**	59	**52**	**54**
INC12	0.5186	8	**5**	6	6	**22**	**22**	**22**	25	61	55	57
SOS16	0.5226	**9**	**5**	7	7	24	24	24	27	62	57	60
GOV20	0.5229	8	**4**	8	8	27	23	25	27	63	58	64
AUD12	0.5371	9	8	7	7	28	27	27	29	65	61	64
SOS12	0.5379	9	7	8	7	26	26	25	29	63	59	62
TRS12	0.5383	9	7	10	7	24	25	25	28	65	59	63
SUP12	0.5424	9	8	9	8	28	28	28	31	66	61	64

Table 1: Do close votes translate to close seats? I have identified, for each plan, the elections with a partisan margin of closer than six points, but where the outcome falls outside of the range of 6-8 Congressional seats, 23-27 Senate seats, or 55-65 House seats.

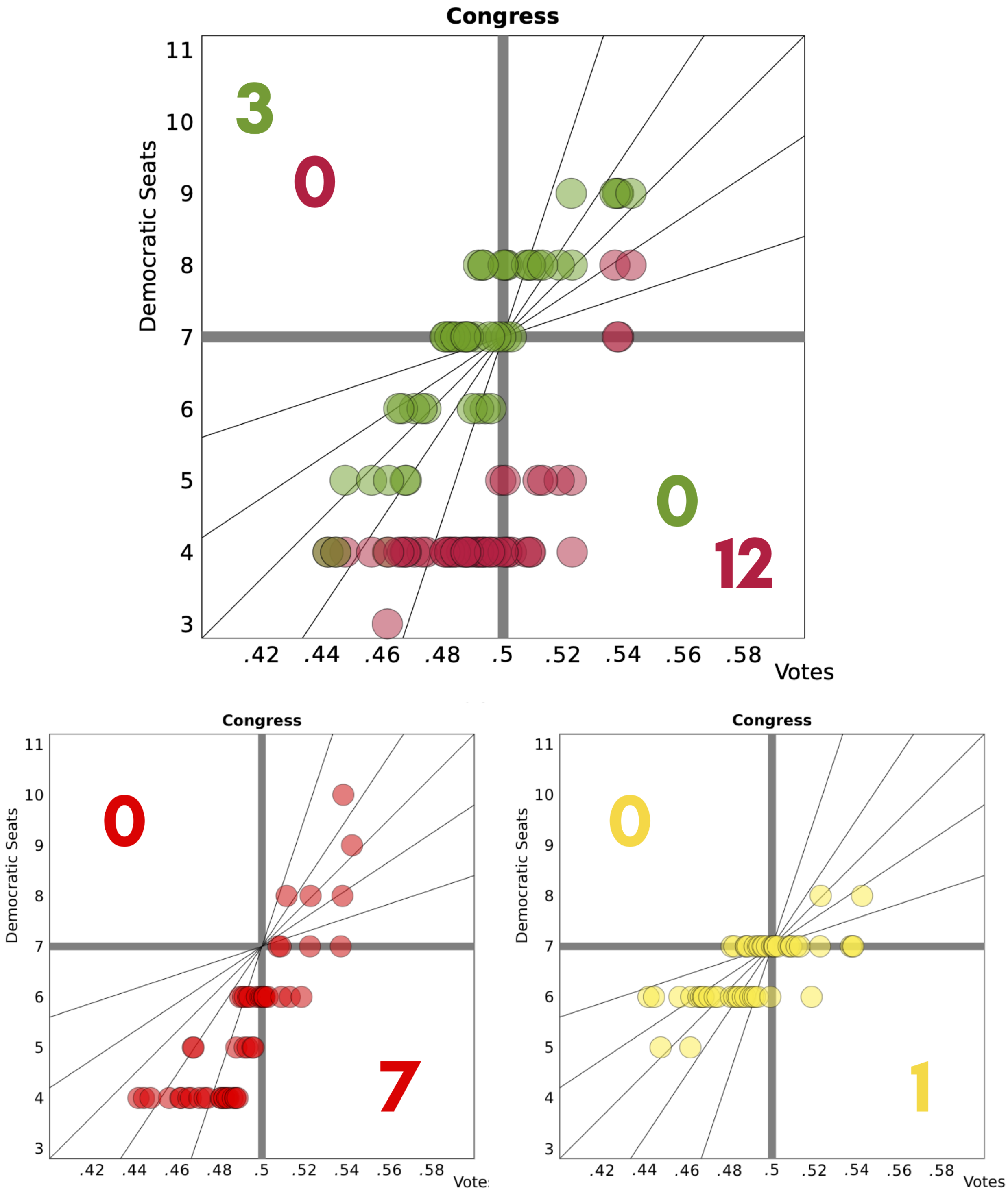


Figure 2: Congressional comparison. Top figure shows votes and seats for NCLCV-Cong (green) and the now-invalidated SL-174 (maroon); below that are SL-3 (red) and Harper-Cong (yellow). The number of anti-majoritarian outcomes for each map is noted.

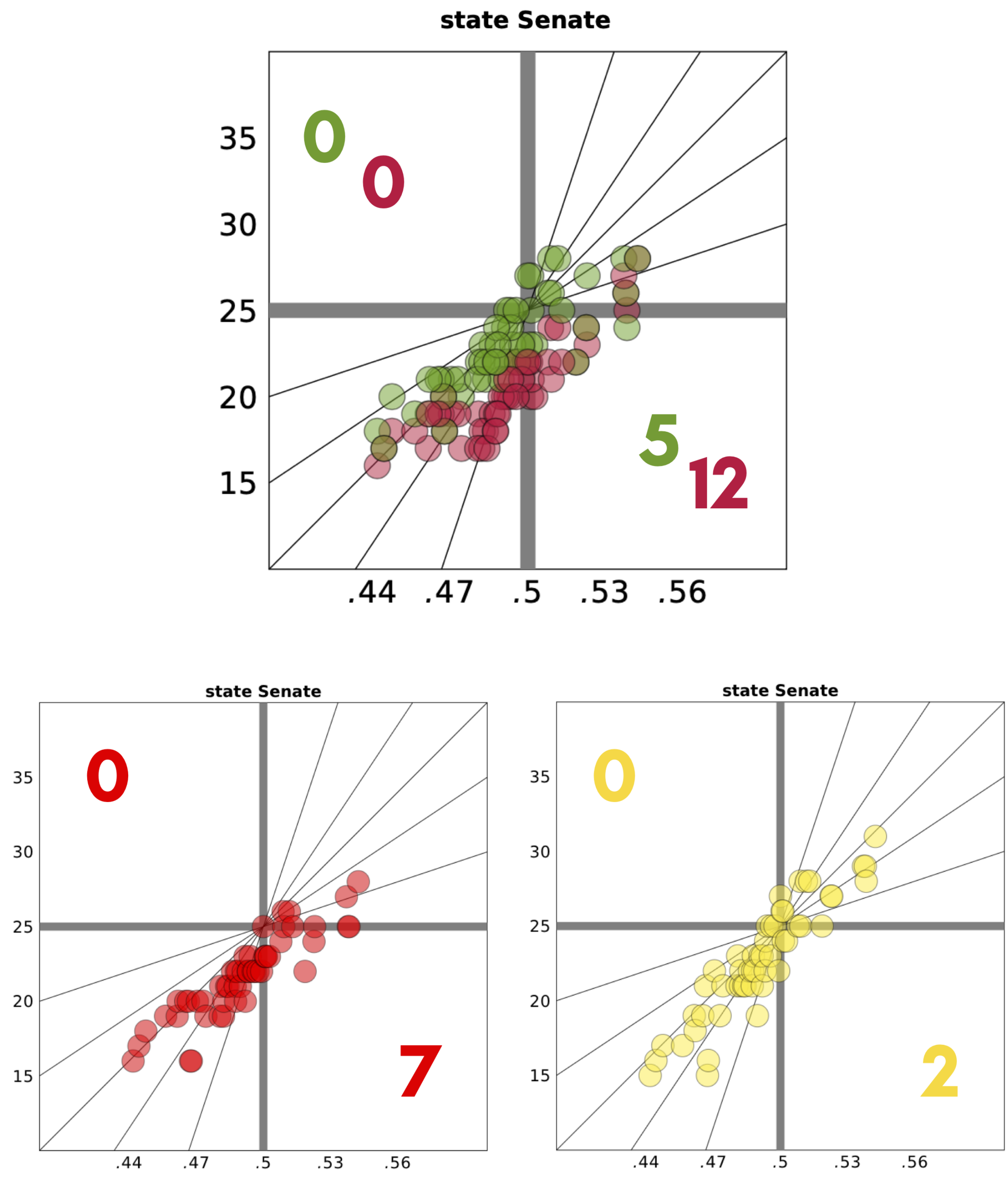
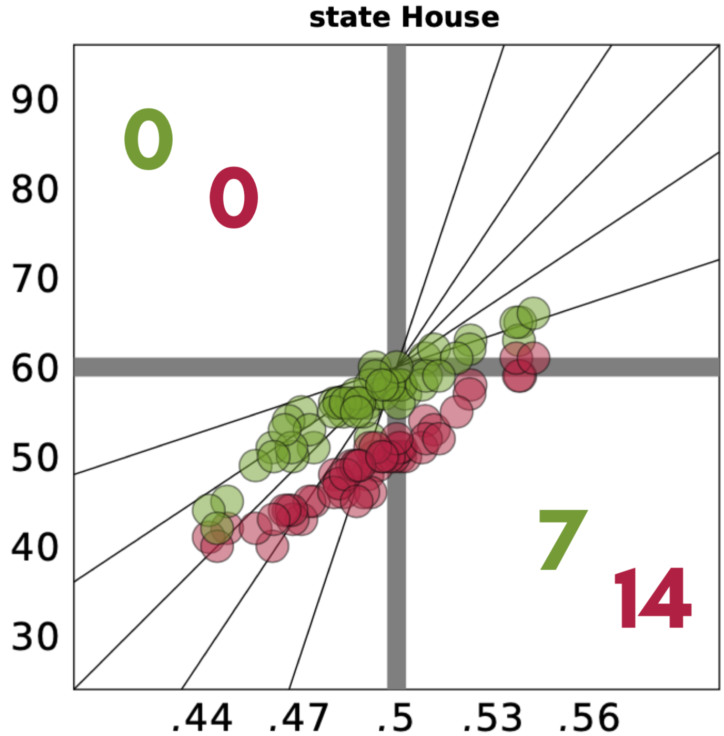


Figure 3: Senate comparison. Top figure shows votes and seats for NCLCV-Sen (green) and the now-invalidated SL-173 (maroon); below that are SL-2 (red) and Harper-Sen (yellow). The number of anti-majoritarian outcomes for each map is noted.



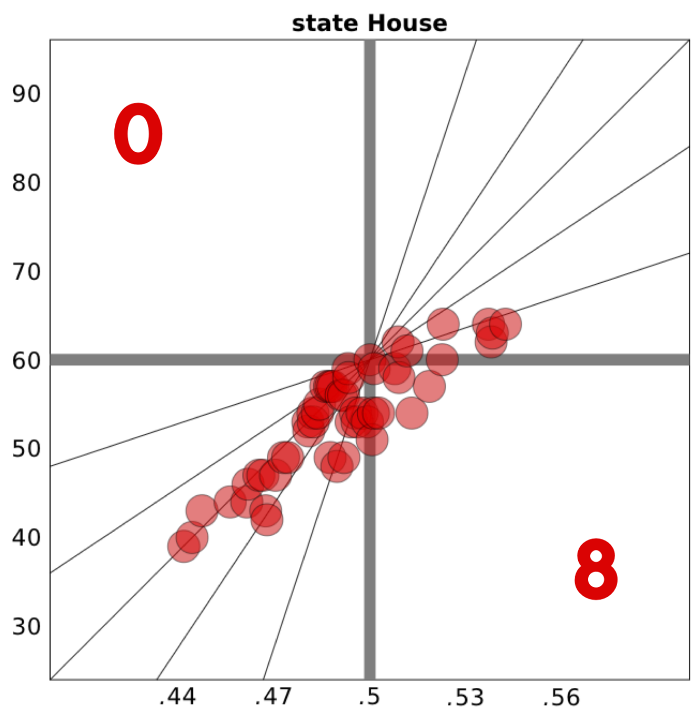


Figure 4: House comparison. Top figure shows votes and seats for NCLCV-House (green) and the now-invalidated SL-175 (maroon); below that is SL-4 (red). The number of anti-majoritarian outcomes for each map is noted.

3 Summary scores of partisan fairness

3.1 Recap of metrics

Recall the following metrics of partisan fairness, to be presented in Tables 2-4.

- *Efficiency gap* (*EG*) is the difference in "wasted" votes for the two parties, across the state, as a share of votes cast [10]. The authors of the paper that popularized efficiency gap (Stephanopoulos–McGhee) later advocated for a *simplified efficiency gap* formula $EG = 2V - S - \frac{1}{2}$, where V is the vote share in an election and S is the seat share. Original efficiency gap and simplified efficiency gap would be exactly equal if the districts had equal turnout; it's the simplified formula that was invoked, for example, in the language for the Freedom To Vote act. The authors proposed .08, later refined to .07, as the flag for a presumptive gerrymander.[2]

- *Partisan symmetry* is a family of scores based on the principle of table-turning: if the votes for the parties were reversed, would the representation also be reversed? An asymmetric plan is one in which one party fares better with its portion of support than the other party would with the same portion. Scores in this group include the *mean-median gap* (*MM*), the *partisan bias* score (*PB*), and the *partisan Gini* (*PG*). The mean-median gap literally takes the difference between the average vote share in a district and the median, or middle, district (or the average of the two middle districts when the number of districts is even). The gap is zero when the middle district looks like the state as a whole, so that half the districts are more favorable to one party and half are more favorable to the other. Partisan bias is described in the literature as measuring how much "extra" representation each party would secure in a hypothetical 50-50 election. Finally, *partisan Gini* is a summary statistic for all of the various kinds of symmetry measures in the political science literature. The "Partisan Symmetry Standard" of King and his co-authors asks that a seats-votes curve be literally symmetric about the center point, meaning that it predicts exactly the same representation for either party at any share of the vote [8]. The partisan Gini, first proposed by Bernard Grofman in 1983, takes this literally, measuring the area between the curve and its mirror image [9]. This is an unsigned metric, with zero as an ideal. (When the *PG* score is zero, all other symmetry scores, like mean-median and partisan bias, are necessarily zero as well.)

- The metric I have called *Eguia county skew* (*ECS*) is based on economist Jon Eguia's "jurisdictional partisan advantage" [7]. Eguia built a metric based on comparing the actual representation secured by a party under a vote pattern to the representation if cities and counties played the role of districts. I have applied it here only to counties, because of the fundamental importance of counties in North Carolina redistricting in particular. A simple way to explain this Eguia-style metric is as follows: in a particular election, what percentage of North Carolinians live in counties that favored Republicans? That is the benchmark for Republican representation; if their seat share is higher, the map is tilted Republican, and if lower, the map is tilted Democratic.

[2] In paragraph 167 of the North Carolina Supreme Court's recent decision in this case, it is noted that "With regard to the efficiency gap measure, courts have found "that an efficiency gap above 7% in any districting plan's first election year will continue to favor that party for the life of the plan."" (Quoting the U.S. Supreme Court, from *Whitford v. Gill*).

From these three types, I have chosen five signed scores to present in Tables 2-3: *EG*, simplified *EG*, *MM*, *PB*, and *ECS*. For all five scores, zero is ideal.

After that, I will use a second table, Table 4 to present the seat average for each party, the size of disproportionality for each election set, and the partisan Gini *PG*.

In both of these tables, I will use three sets of elections: first, the full set of 52 general elections. Next, the 35 non-judicial contests. And finally, the 14 "up-ballot" contests, which are the first five to appear on the ballot: President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. (These each occurred three times in the previous cycle, except for Attorney General, which was only contested twice.)

3.2 Comparison of metrics

We will see a phenomenon clearly visible in the following tour of the metrics (which was actually already apparent in Table 1 and Figures 2-4): when given a chance to re-draw maps, the Legislature produced maps that *split the difference* between the partisan properties of the original proposals and the properties observed in the plaintiffs' maps.

At the Congressional level, this brings the mean-median scores down substantially, but leaves all the other scores at extremely elevated levels.

	NCLCV-Cong			SL-174 (old Cong plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	0.006	0.001	−0.001	−0.167	−0.159	−0.181
Simplified *EG*	0.011	0.005	0.003	−0.17	−0.163	−0.186
Mean-median	0.007	0.006	0.007	−0.047	−0.044	−0.045
Partisan Bias	0.036	0.029	0.031	−0.192	−0.184	−0.204
Eguia County Skew	−0.006	−0.009	−0.006	−0.188	−0.176	−0.195
	SL-3 (new Cong plan)			**Harper-Cong**		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.093	−0.078	−0.088	−0.016	−0.021	−0.016
Simplified *EG*	−0.098	−0.083	−0.094	−0.017	−0.022	−0.017
Mean-median	−0.015	−0.017	−0.016	−0.009	−0.011	−0.009
Partisan Bias	−0.066	−0.063	−0.061	−0.014	−0.018	−0.020
Eguia County Skew	−0.115	−0.097	−0.103	−0.034	−0.035	−0.027

Table 2: Five simplified scores of partisan fairness, averaged over different sets of elections. These five metrics are all *signed*, meaning that they can take positive or negative values; positive and negative scores are intended to flag an advantage to Democrats and Republicans, respectively. *EG* and *MM* are computed as a share of votes; *PB* and the Eguia score are computed as a share of seats. Colors are intended for ease of comparisons and are consistent within each score.

For the Senate plan, the split-the-difference approach leaves significantly inferior scores on all metrics of partisan fairness than the ones, very near zero, in the plaintiffs' maps. For the House, on the other hand, the new plan is now down to a level that is markedly better in several of the metrics.

	NCLCV-Sen			SL-173 (old Senate plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.020	−0.024	−0.017	−0.075	−0.068	−0.080
Simplified *EG*	−0.023	−0.028	−0.021	−0.076	−0.070	−0.081
Mean-median	−0.009	−0.012	−0.009	−0.036	−0.036	−0.037
Partisan Bias	−0.015	−0.023	−0.016	−0.072	−0.069	−0.08
Eguia County Skew	−0.040	−0.041	−0.030	−0.093	−0.083	−0.09

	SL-2 (new Senate plan)			Harper-Sen		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.045	−0.048	−0.046	−0.022	−0.023	−0.029
Simplified *EG*	−0.048	−0.051	−0.050	−0.027	−0.028	−0.034
Mean-median	−0.020	−0.022	−0.021	−0.003	−0.005	−0.003
Partisan Bias	−0.044	−0.045	−0.049	−0.013	−0.018	−0.002
Eguia County Skew	−0.065	−0.064	−0.059	−0.044	−0.041	−0.043

	NCLCV-House			SL-175 (old House plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.020	−0.022	−0.017	−0.076	−0.075	−0.078
Simplified *EG*	−0.014	−0.016	−0.012	−0.074	−0.074	−0.077
Mean-median	−0.015	−0.015	−0.017	−0.039	−0.039	−0.04
Partisan Bias	−0.018	−0.019	−0.018	−0.082	−0.082	−0.086
Eguia County Skew	−0.031	−0.030	−0.021	−0.091	−0.088	−0.086

	SL-4 (new House plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.039	−0.043	−0.039
Simplified *EG*	−0.037	−0.042	−0.039
Mean-median	−0.019	−0.021	−0.019
Partisan Bias	−0.042	−0.045	−0.044
Eguia County Skew	−0.054	−0.056	−0.048

Table 3: The same scores, now assessed for state Senate and state House maps. Across the board, the new maps from the Legislature split the difference between the invalidated plans and the LCV remedial proposals. Colors are intended for ease of comparisons and are consistent within each score.

When we turn to seats by party and the partisan Gini, the story is quite similar (Table 4).

	NCLCV-Cong			SL-174 (old Cong plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	6.9	6.9	6.7	4.4	4.5	4.1
R Seats	7.1	7.1	7.3	9.6	9.5	9.9
Disproportionality	0.0	0.0	−0.1	−2.5	−2.4	−2.8
Partisan Gini	0.021	0.020	0.021	0.078	0.073	0.080

	SL-3 (new Cong plan)			Harper-Cong		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	5.4	5.6	5.4	6.5	6.5	6.4
R Seats	8.6	8.4	8.6	7.5	7.5	7.6
Disproportionality	−1.5	−1.3	−1.5	−0.4	−0.4	−0.4
Partisan Gini	0.032	0.032	0.032	0.014	0.015	0.014

	NCLCV-Sen			SL-173 (old Senate plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	23.0	22.9	22.8	20.3	20.8	19.8
R Seats	27.0	27.1	27.2	29.7	29.2	30.2
Disproportionality	−1.6	−1.7	−1.6	−4.2	−3.9	−4.6
Partisan Gini	0.026	0.026	0.026	0.051	0.049	0.054

	SL-2 (new Senate plan)			Harper-Sen		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	21.7	21.7	21.4	22.8	22.9	22.1
R Seats	28.3	28.3	28.6	27.2	27.1	27.9
Disproportionality	−2.9	−2.9	−3.1	−1.8	−1.7	−2.3
Partisan Gini	0.036	0.035	0.038	0.027	0.027	0.028

	NCLCV-House			SL-175 (old House plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	56.2	56.3	55.8	49.0	49.3	47.9
R Seats	63.8	63.7	64.2	71.0	70.7	72.1
Disproportionality	−2.7	−2.9	−2.8	−10.0	−9.8	−10.7
Partisan Gini	0.032	0.031	0.034	0.050	0.050	0.053

	SL-4 (new House plan)		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	53.4	53.2	52.5
R Seats	66.6	66.8	67.5
Disproportionality	−5.5	−5.9	−6.1
Partisan Gini	0.037	0.037	0.039

Table 4: Average seat totals and the distance from proportionality. The partisan Gini score measures how far the seats-votes curve is from perfect symmetry. Across the board, the "splits the difference" trend is apparent.

Finally, for another way of slicing the same data:

	Up-ballot generals (14)		**All generals (52)**	
	D vote share	D seat share	D vote share	D seat share
NCLCV-Cong	.4883	.4796	.4911	.4931
SL-174 (old Cong plan)		.2908		.3118
SL-3 (new Cong plan)		.3857		.3857
Harper-Cong		.4571		.4643
NCLCV-Sen	.4883	.4557	.4911	.4592
SL-173 (old Sen plan)		.3957		.4065
SL-2 (new Sen plan)		.4280		.4340
Harper-Sen		.4420		.4560
NCLCV-House	.4883	.4649	.4911	.4684
SL-175 (old House plan)		.3994		.4080
SL-4 (new House plan)		.4375		.4450

Table 5: Comparing overall fidelity of representation to the voting preferences of the electorate. As from every other point of view, the new plans from the Legislature split the difference from their original proposal to the LCV plans, which score better on all metrics of partisan fairness.

4 Comparison to Barber report

I have described the scores on a range of metrics that result from overlaying eleven plans with 52 elections, and I've also presented several more selective subsets of the elections, to make it clear the that findings are robust.

Dr. Michael Barber filed a report on February 18 in which he obtains systematically less severe bias indicators for the Legislature's new proposed maps.

For instance, consider the reported efficiency gaps.

	Barber method (12 elections)	current method (Barber elections)	current method (14 "up-ballot")
SL-174 (old)	−.195	−.195	−.181
SL-3 (new)	−.053	−.093	−.088
SL-173 (old)	−.080	−.078	−.080
SL-2 (new)	−.040	−.036	−.046
SL-175 (old)	−.072	−.079	−.078
SL-4 (new)	−.008	−.024	−.039

Table 6: Efficiency gap in each election using the wasted votes method (which is described above as the "original" *EG*).

I have made a serious attempt at replication in the very limited time available and have not been able to figure out how Dr. Barber arrives at his numbers, exactly. My conclusion is one of two things: either the discrepancy owes to the problematic way he blends elections together, which I will describe below, or he is actually using a different method from the one he describes in his report.[3]

[3] For instance, there are published methods that introduce statistical corrections into the data for fractional seats, or that randomly add noise to an election index. He has not said that he is doing either of these, but it is possible that he is employing software that does this without realizing it.

Dr. Barber describes his election index as follows: "if a district has an index value of 0.51, this would mean that 51% of the votes cast for the two major parties across these 12 elections went to Democratic candidates." This means that he is adding up the votes, rather than weighting all elections equally. I will make two observations about the problems this causes.

Weighting. The first effect is to upweight higher-turnout elections. To see the effects of the up-weighting, note that ten of 12 elections are from 2020 (see Table 7 for the list), which means that he is giving over 85% of the weight to a single election year.[4] Dr. Barber indicates that he is using the same twelve elections used by Dr. Mattingly in an earlier report—but that is a selective attribution. Mattingly uses a larger set of 15 elections for his statewide analysis. Notably dropped are ATG16 and GOV16—two elections that would counteract the dominance of 2020, and that show anti-majoritarian outcomes under the SL-3 map.

Faulty averaging: practical illustration. Consider the election-by-election efficiency gaps for Barber's 12 elections.

	PRS20	SEN20	GOV20	LTG20	ATG20	SOS20
EG	−0.1276	−0.0532	0.0225	−0.1792	−0.0742	0.0457
D seats	5	6	8	4	6	8
D votes	.4932	.4910	.5229	.4836	.5013	.5116

	TRS20	AGC20	AUD20	LAC20	PRS16	LTG16
EG	−0.1602	−0.1349	−0.0177	−0.1239	−0.1693	−0.1386
D seats	4	4	7	5	4	4
D votes	.4743	.4615	.5088	.4918	.4809	.4665

average of these *EG* values: −0.09255

Barber's reported *EG*: −.0529

Table 7: Election-by-election scores in Barber's elections for the original efficiency gap—the wasted-votes method that Barber describes in his report.

It is unreasonable on its face to take a set of actually observed elections that show such large efficiency gaps and propose a style of blending them that hides that effect.

Faulty averaging: abstract example. How is this happening? Most partisan scores are *non-linear*, meaning that if you average elections and then compute the score, this is NOT the same as reporting the average of the by-election scores.

For efficiency gap specifically, adding elections creates an unintelligible blended election from the point of view of the *meaning* of the metric. Is a vote wasted or not wasted? That depends on who wins the district. But a "wasted vote" is a property of the individual election, not of the composite.

Here is an illustrative example. Suppose that there have been ten elections in a two-district state. Nine of them had 51-49 wins for Party A in both districts. The tenth went 80-20 the other way, in favor of Party B. The nine tight elections had one wasted vote for Party A and 49 for Party B in each district, for an efficiency gap of $\frac{2(1-49)}{200}$, or −.48, indicating a huge advantage to Party A. (The largest possible magnitude of the gap is .5, so this is a truly massive gerrymander.) The last election had $EG = \frac{2(20-30)}{200} = -.1$, also indicating advantage to Party A. Let's apply Dr. Barber's method. We sum all the elections, so that now each district

[4] For instance, the total major-party cast votes in PRS20 were 5,443,067 (highest) while for LTG16 it was 4,438,769 (lowest), giving the first contest 23% more weight. Applying that factor of 1.2 to ten elections out of twelve gives them a 12/14 share of the weight, which is about 85.7%.

has 484 votes for Party A and 516 votes for Party B. Now the efficiency gap is $\frac{2(479-21)}{2000}$, or $+.458$. This looks like a single tight election, and an epic gerrymander, for Party B. That is, summing the elections gives you an uninterpretable stew. It takes a situation where one party has thin-sliced its advantage to repeatedly convert narrow preferences to a $2-0$ sweep of seats, and it obscures that pattern completely.

Let me repeat what is illustrated by this example: an application of Barber's method takes ten elections where nine had $EG = -.48$ and the last had $EG = -.1$ and, by averaging the contests into an election index, produces an overall EG of $+.458$. It is a strange method indeed if ten negative numbers can average to a positive total.

The same flaws permeate Dr. Barber's entire analysis, because each of his partisan metric calculations draws on the same problematic election index. This implicates not only his efficiency gap scores but also his mean-median scores and his partisan symmetry scores, which are likewise based on non-linear combinations of electoral data. (That is, the median of an average is not the average of the medians, and so on.) For each of his scores, he has applied an unreasonable averaging method that makes the systematic advantage for Republicans disappear.

North Carolina provides an extraordinary opportunity to base partisan determinations on a large number of actual election patterns from the last ten years, many of which were extremely close elections. We have a chance to employ methods that take advantage of this large naturalistically observed dataset rather than those that hide its systematic properties.

5 Electoral opportunity for Black voters

In my previous report, I explained how I constructed a determination of which districts are *effective* at providing Black voters with an opportunity to elect candidates of choice.

Running the same effectiveness count for the current plans, I obtain the following numbers.

Effective districts for Black voters

	NC-LCV maps	previous Leg. maps	new Leg. maps	Harper maps
Congress	4 (CD 2, 4, 9, 11)	2 (CD 2, 9)	2 (CD 1, 12)	3 (CD 1, 6, 12)
Senate	12 (SD 1, 5, 11, 14, 18, 19, 26, 27, 32, 38, 39, 40)	8 (SD 5, 11, 14, 19, 28, 38, 39, 40)	10 (SD 3, 5, 11, 14, 19, 27, 28, 38, 40, 41)	11 (SD 3, 5, 11, 13, 16, 19, 27, 28, 38, 40, 41)
House	36 (HD 2, 8, 9, 10, 23, 24, 25, 27, 31, 32, 33, 38, 39, 40, 42, 43, 44, 45, 48, 57, 58, 59, 60, 61, 63, 66, 71, 88, 92, 99, 100, 101, 102, 106, 107, 112)	24 (HD 8, 23, 24, 25, 27, 32, 38, 39, 42, 44, 48, 57, 58, 60, 66, 71, 92, 99, 100, 101, 102, 106, 107, 112)	27 (HD 8, 23, 24, 25, 27, 31, 32, 33, 38, 39, 42, 44, 45, 48, 58, 60, 61, 66, 71, 92, 99, 100, 101, 102, 106, 107, 112)	– –

Table 8: The plaintiffs' plans secure additional electoral opportunity for Black voters in North Carolina.

For comparison, Black voting age population (BVAP) levels by district can be found in Appendix A.

6 Conclusion

At a high level, the situation with the Legislature's new maps of all three types is clear throughout all of the analysis presented here: they chose maps with intermediate partisan properties between the now-invalidated original proposals and a truly even-handed map. This is quite evident in Table 4, where the number of R Congressional seats was 7.1 in the LCV maps and 9.6 in the invalidated plans; the new plans average to 8.6. For Senate, the new plans split the difference between 27.0 and 29.7 seats, giving 28.3. And in the House, they split the difference between 56.2 seats and 49.0, giving 53.4.

I find the Legislature's new Congressional and Senate plans to be particularly problematic from a Close-Votes-Close-Seats perspective, often giving four out of 14 Congressional seats (28%) or twenty out of 50 Senate seats (40%) to Democrats even when Democrats poll at better than 48% of the major-party vote. This is borne out in the partisan fairness scores, which show the new proposals splitting the difference from the now-invalidated maps to the plaintiffs' alternatives.

The plaintiffs' proposed remedial plans simply perform far better on the Close-Votes-Close-Seats norm and on the full suite of partisan fairness scores. For the scores, there are 63 opportunities to compare the plans numerically: seven metrics (*EG*, simplified *EG*, *MM*, *PB*, disproportionality, and *PG*) times three election sets (all, non-judicial, up-ballot) times three maps (Congress, Senate, House). The newly enacted plans improve on their predecessors all 63 times, but they likewise fall significantly short of the LCV maps all 63 times (and fall short of the Harper maps in 42 of 42 available comparisons). It is as consistent and robust of a finding as can be.

The LCV plans are also superlative on the traditional districting principles (recalling previous reports) and contain a large number of districts that provide effective electoral opportunity—but not a guarantee—for Black voters. In sum, they are an excellent choice of remedial plans for adoption by the Court.

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A BVAP across the districts of the proposed remedial plans

NCLCV-Cong		
CD	B1VAP	APBVAP
1	0.289	0.304
2	0.332	0.347
3	0.118	0.131
4	0.319	0.344
5	0.226	0.245
6	0.227	0.242
7	0.115	0.128
8	0.123	0.132
9	0.277	0.298
10	0.232	0.25
11	0.271	0.289
12	0.121	0.132
13	0.114	0.124
14	0.032	0.039

SL-3		
CD	B1VAP	APBVAP
1	0.403	0.42
2	0.205	0.224
3	0.17	0.185
4	0.249	0.266
5	0.156	0.168
6	0.239	0.257
7	0.23	0.252
8	0.176	0.19
9	0.182	0.195
10	0.071	0.079
11	0.033	0.04
12	0.317	0.339
13	0.162	0.175
14	0.196	0.211

Harper-Cong		
CD	B1VAP	APBVAP
1	0.396	0.412
2	0.225	0.243
3	0.173	0.187
4	0.247	0.263
5	0.08	0.089
6	0.316	0.336
7	0.166	0.178
8	0.111	0.121
9	0.181	0.197
10	0.127	0.137
11	0.032	0.039
12	0.312	0.334
13	0.127	0.141
14	0.297	0.321

Table 9: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for Congress.

NCLCV-Sen			SL-2			Harper-Sen		
SD	B1VAP	APBVAP	SD	B1VAP	APBVAP	SD	B1VAP	APBVAP
1	0.408	0.423	1	0.165	0.175	1	0.165	0.175
2	0.165	0.175	2	0.253	0.267	2	0.253	0.267
3	0.253	0.267	3	0.408	0.423	3	0.408	0.423
4	0.334	0.35	4	0.334	0.35	4	0.334	0.35
5	0.385	0.403	5	0.385	0.403	5	0.385	0.403
6	0.13	0.153	6	0.13	0.153	6	0.13	0.153
7	0.125	0.138	7	0.105	0.117	7	0.1	0.112
8	0.12	0.128	8	0.139	0.148	8	0.142	0.152
9	0.228	0.239	9	0.228	0.239	9	0.228	0.239
10	0.154	0.167	10	0.154	0.167	10	0.154	0.167
11	0.352	0.366	11	0.352	0.366	11	0.352	0.366
12	0.189	0.206	12	0.189	0.206	12	0.189	0.206
13	0.175	0.188	13	0.181	0.199	13	0.246	0.267
14	0.312	0.332	14	0.406	0.43	14	0.115	0.131
15	0.136	0.152	15	0.128	0.143	15	0.124	0.138
16	0.08	0.092	16	0.094	0.107	16	0.382	0.405
17	0.091	0.104	17	0.102	0.115	17	0.087	0.099
18	0.323	0.347	18	0.215	0.23	18	0.169	0.181
19	0.439	0.481	19	0.356	0.392	19	0.363	0.397
20	0.22	0.237	20	0.256	0.273	20	0.39	0.41
21	0.176	0.195	21	0.259	0.284	21	0.252	0.278
22	0.364	0.382	22	0.326	0.344	22	0.195	0.211
23	0.155	0.167	23	0.154	0.167	23	0.154	0.167
24	0.278	0.296	24	0.278	0.296	24	0.278	0.296
25	0.165	0.178	25	0.165	0.179	25	0.17	0.184
26	0.332	0.35	26	0.207	0.221	26	0.283	0.3
27	0.297	0.317	27	0.272	0.29	27	0.249	0.266
28	0.282	0.303	28	0.43	0.456	28	0.376	0.399
29	0.171	0.18	29	0.169	0.178	29	0.169	0.178
30	0.084	0.092	30	0.084	0.092	30	0.084	0.092
31	0.122	0.135	31	0.207	0.222	31	0.222	0.239
32	0.329	0.35	32	0.234	0.252	32	0.224	0.24
33	0.14	0.149	33	0.14	0.149	33	0.14	0.149
34	0.184	0.202	34	0.184	0.201	34	0.184	0.201
35	0.105	0.116	35	0.106	0.117	35	0.1	0.112
36	0.04	0.046	36	0.039	0.045	36	0.04	0.046
37	0.104	0.115	37	0.104	0.114	37	0.105	0.116
38	0.354	0.377	38	0.411	0.437	38	0.422	0.448
39	0.4	0.426	39	0.212	0.231	39	0.203	0.223
40	0.376	0.402	40	0.361	0.387	40	0.341	0.365
41	0.116	0.131	41	0.374	0.396	41	0.371	0.394
42	0.224	0.24	42	0.11	0.125	42	0.127	0.143
43	0.181	0.194	43	0.173	0.186	43	0.179	0.192
44	0.129	0.138	44	0.123	0.131	44	0.129	0.138
45	0.065	0.074	45	0.066	0.076	45	0.067	0.076
46	0.054	0.06	46	0.042	0.049	46	0.056	0.063
47	0.028	0.035	47	0.028	0.034	47	0.029	0.035
48	0.046	0.054	48	0.048	0.055	48	0.044	0.051
49	0.044	0.052	49	0.063	0.072	49	0.046	0.054
50	0.014	0.02	50	0.014	0.02	50	0.014	0.02

Table 10: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for state Senate.

NCLCV-House		
HD	B1VAP	APBVAP
1	0.266	0.277
2	0.335	0.351
3	0.189	0.203
4	0.219	0.23
5	0.369	0.386
6	0.216	0.24
7	0.221	0.235
8	0.333	0.353
9	0.343	0.362
10	0.349	0.37
11	0.112	0.13
12	0.373	0.385
13	0.078	0.088
14	0.112	0.134
15	0.173	0.202
16	0.106	0.116
17	0.178	0.192
18	0.13	0.144
19	0.055	0.06
20	0.04	0.048
21	0.084	0.096
22	0.272	0.285
23	0.519	0.534
24	0.371	0.386
25	0.383	0.398
26	0.173	0.189
27	0.502	0.519
28	0.158	0.171
29	0.325	0.345
30	0.243	0.26
31	0.404	0.427
32	0.42	0.434
33	0.321	0.343
34	0.093	0.104
35	0.093	0.105
36	0.058	0.069
37	0.109	0.122
38	0.305	0.324
39	0.311	0.332
40	0.316	0.339
41	0.085	0.096
42	0.384	0.415
43	0.348	0.379
44	0.365	0.411
45	0.378	0.417
46	0.282	0.295
47	0.209	0.223
48	0.346	0.371
49	0.153	0.171
50	0.174	0.185
51	0.102	0.111
52	0.199	0.212
53	0.142	0.154
54	0.137	0.149
55	0.255	0.268
56	0.096	0.111
57	0.369	0.392
58	0.363	0.386
59	0.351	0.371
60	0.286	0.304

NCLCV-House		
HD	B1VAP	APBVAP
61	0.457	0.486
62	0.115	0.127
63	0.277	0.295
64	0.114	0.126
65	0.184	0.194
66	0.31	0.336
67	0.126	0.134
68	0.072	0.081
69	0.093	0.105
70	0.065	0.072
71	0.323	0.35
72	0.371	0.393
73	0.179	0.198
74	0.108	0.12
75	0.18	0.194
76	0.199	0.21
77	0.052	0.058
78	0.081	0.089
79	0.073	0.081
80	0.099	0.108
81	0.083	0.09
82	0.183	0.2
83	0.119	0.132
84	0.154	0.166
85	0.029	0.034
86	0.057	0.064
87	0.045	0.053
88	0.32	0.341
89	0.069	0.077
90	0.032	0.039
91	0.129	0.139
92	0.319	0.345
93	0.028	0.035
94	0.049	0.055
95	0.071	0.081
96	0.089	0.1
97	0.052	0.058
98	0.075	0.086
99	0.292	0.314
100	0.29	0.316
101	0.475	0.502
102	0.302	0.323
103	0.069	0.082
104	0.092	0.103
105	0.146	0.164
106	0.451	0.481
107	0.445	0.474
108	0.107	0.116
109	0.223	0.238
110	0.169	0.18
111	0.171	0.182
112	0.469	0.493
113	0.061	0.069
114	0.035	0.042
115	0.08	0.091
116	0.046	0.055
117	0.031	0.037
118	0.011	0.015
119	0.021	0.029
120	0.008	0.013

SL-4		
HD	B1VAP	APBVAP
1	0.172	0.182
2	0.292	0.307
3	0.188	0.202
4	0.244	0.255
5	0.369	0.386
6	0.222	0.246
7	0.221	0.235
8	0.361	0.381
9	0.313	0.332
10	0.323	0.344
11	0.121	0.136
12	0.373	0.385
13	0.079	0.088
14	0.121	0.144
15	0.164	0.191
16	0.107	0.117
17	0.099	0.107
18	0.188	0.203
19	0.047	0.054
20	0.07	0.081
21	0.085	0.096
22	0.272	0.285
23	0.519	0.534
24	0.369	0.385
25	0.385	0.4
26	0.165	0.181
27	0.502	0.518
28	0.158	0.17
29	0.29	0.31
30	0.288	0.307
31	0.434	0.456
32	0.419	0.434
33	0.32	0.34
34	0.105	0.117
35	0.17	0.187
36	0.073	0.086
37	0.111	0.124
38	0.416	0.439
39	0.314	0.336
40	0.097	0.11
41	0.07	0.083
42	0.376	0.42
43	0.342	0.369
44	0.4	0.438
45	0.354	0.392
46	0.251	0.264
47	0.241	0.256
48	0.346	0.371
49	0.142	0.16
50	0.174	0.185
51	0.154	0.167
52	0.218	0.231
53	0.147	0.16
54	0.106	0.116
55	0.248	0.261
56	0.094	0.109
57	0.233	0.251
58	0.456	0.484
59	0.306	0.325
60	0.328	0.347

SL-4		
HD	B1VAP	APBVAP
61	0.465	0.493
62	0.152	0.166
63	0.264	0.282
64	0.128	0.141
65	0.184	0.194
66	0.309	0.335
67	0.126	0.134
68	0.082	0.093
69	0.095	0.106
70	0.066	0.074
71	0.322	0.348
72	0.383	0.404
73	0.217	0.239
74	0.118	0.13
75	0.189	0.205
76	0.199	0.21
77	0.052	0.058
78	0.052	0.058
79	0.165	0.174
80	0.09	0.098
81	0.092	0.1
82	0.191	0.209
83	0.079	0.088
84	0.155	0.167
85	0.03	0.034
86	0.057	0.064
87	0.045	0.052
88	0.228	0.247
89	0.063	0.07
90	0.032	0.038
91	0.104	0.112
92	0.318	0.344
93	0.028	0.035
94	0.049	0.055
95	0.071	0.081
96	0.092	0.105
97	0.052	0.058
98	0.074	0.085
99	0.459	0.488
100	0.334	0.36
101	0.506	0.534
102	0.309	0.33
103	0.087	0.1
104	0.086	0.098
105	0.126	0.141
106	0.351	0.376
107	0.562	0.592
108	0.137	0.147
109	0.178	0.191
110	0.187	0.198
111	0.157	0.167
112	0.308	0.331
113	0.065	0.073
114	0.077	0.086
115	0.051	0.06
116	0.033	0.04
117	0.03	0.036
118	0.011	0.015
119	0.021	0.029
120	0.008	0.013

Table 11: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for the state House.

I declare under penalty of perjury that the foregoing is true and correct.

Executed: Feb 20, 2022

Prof. Moon Duchin

Sworn and subscribed before me

this the 20 of February, 2022.

Notary Public

Name: Simarjit Manhas

My Commission Expires: 02/02/2024

A notary public or other officer completing this certificate verifies only the identity of the individual who signed the document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.

State of California, County of Alameda

Subscribed and sworn to (or affirmed) before me

on this 20 day of February, 2022,

by Moon Duchin,

proved to me on the basis of satisfactory evidence to be the person~~(s)~~ who appeared before me.

Signature:

SIMARJIT MANHAS
COMM. #2317737
Notary Public - California
Contra Costa County
My Comm. Expires Feb. 2, 2024