Expert Report on the North Carolina State Legislature and Congressional Redistricting (Corrected Version)

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1 Introduction

I am a Professor of Mathematics and Statistical Science at Duke University. My degrees are from the North Carolina School of Science and Math (High School Diploma), Yale University (B.S.), and Princeton University (Ph.D.). I grew up in Charlotte, North Carolina and currently live in Durham, North Carolina.

I lead a group at Duke University which conducts non-partisan research to understand and quantify gerrymandering. This report grows out of aspects of our group's work around the current North Carolina legislative districts which are relevant to the case being filed.

I previously submitted an expert report in Common Cause v. Rucho, No. 18-CV-1026 (M.D.N.C.), Diamond v. Torres, No. 17-CV-5054 (E.D. Pa.), Common Cause v. Lewis (N.C. Sup. Ct No. 18-cvs-014001), and Harper v. Lewis (No. 19-cv-012667) and was an expert witness for the plaintiffs in Common Cause v Rucho and Common Cause v. Lewis. I am being paid at a rate of $400/per hour for the work on this case. Much of the work derives from an independent research effort, unrelated to this lawsuit, to understand gerrymandering nationally and in North Carolina specifically. Much of the core analysis described in this report was previously released publicly as part of a non-partisan effort to inform the discussion around the redistricting process.

2 General Overview

I was asked in this case to analyze whether the enacted Congressional, state House, and state Senate redistricting plans for North Carolina were drawn intentionally for partisan advantage. In summary, to conduct our analysis, we used historic voting data to compare election results under the enacted plans with elections results under a collection of non-partisan maps generated using Markov Chain Monte Carlo methods, referred to throughout this report as an "ensemble." No partisan information is used to construct this ensemble of maps; only the generally accepted districting criteria of approximately equal population per district, contiguous and relatively compact districts, reducing traversals, and keeping counties, precincts, and possibly municipalities whole. One strength of the ensemble method is that it makes no assumptions in advance about what structure an election should have such as a relation to proportional representation or some type of symmetry considerations. Rather it shows what results would naturally occur, and the structure of those results, because of political geography of the state when non-partisan maps are used. We examine both the number of seats that would have been won under these vote counts, along with the expected margins of victory.

We see that each of the enacted plans is an extreme outlier with respect to its partisan properties in comparison to the ensemble. The Congressional, House, and Senate plans each systematically favor the Republican Party to an extent which is rarely, if ever, seen in the non-partisan collection of maps. Under many historic elections considered, each of the enacted maps elects significantly fewer Democrats than the typical number of Democrats found in the collection of maps. Specifically, the enacted Congressional plan produces 10 Republican seats and 4 Democratic seats across a wide range of historic elections, spanning roughly a 6-point differential in the statewide two-party vote share. In other words, Republicans win 10 congressional seats despite large shifts in the statewide vote fraction and across a variety of election structures. Over the statewide vote Democratic partisan vote range of 46.59% to 52.32%, the enacted map only twice changes the number of Republicans elected. The outcome of the election is largely stuck at 4 Democrats. Our non-partisan ensemble plans, by contrast, are far more responsive to changes in the election structure and the statewide vote fraction.

Under the enacted Senate and House plans, at times the Democratic Party is either denied a majority of seats or denied breaking a Republican supermajority when the overwhelming majority of maps in our ensemble would have resulted in either a Democratic majority or a simple Republican majority. In the Senate, we find instances in which the Republicans would have gained a supermajority under the enacted plan, but would have lost a supermajority in nearly every map in our collection. In the House, we find instances in which the Republicans won the supermajority of seats under the enacted plan but they would have not won the supermajority in the majority of maps in our collection.

In the House and Senate plans, the extreme statewide tilt towards the Republican Party is the result of a significant number of truly independent choices at the level of the county-clusters into which the state is divided. The chance of making so many independent choices which bias the results towards the Republican Party unintentionally, without corresponding choices favoring the Democratic party, is astronomically small.

In addition to this systematic bias towards the Republican Party which when aggregated produces highly atypical results, the enacted House and Senate plans also have highly atypical results in a number of county clusters even when viewed alone. Beyond often creating atypical results in terms of the number of seats won in a given cluster, our results also show a durability in the results in certain clusters under the enacted plans. By durable, we mean that the results remain atypically unchanged over a wide range of elections. This unresponsiveness to changes in vote counts is another problematic feature revealed by our analysis of the enacted plans.

Our analysis show that each of the three enacted plans is an extreme gerrymander over a range of voter behavior seen historically in North Carolina. The effect of these extreme gerrymanders is to prevent the Democrats from winning as many seats in Congress, the House, and the Senate as they would have had the maps been drawn in a neutral way without political considerations. In many cases, the enacted maps reduce the extent to which the results of an election respond to the changing options of the electorate as expressed at the ballot box.

3 Discussion on Interpreting The Ensemble Method

3.1 The Political Geography

In redistricting conversations, there are often discussions of the urban versus rural divide and natural packing. These points demonstrate the need for a methodology that accounts for this political geography; ensemble methods precisely capture it. The distribution on redistricting plans can distinguish between typical plans and atypical plans. This determination is fundamentally informed by the geometry of the state, its political geography, and the spatial structure of the elections used to probe the redistricting plan.

The fundamental power of the ensemble method is that it begins with a clear set of redistricting criteria as an input. It then creates a representative ensemble of redistricting plans which accounts for the geometry of the state and the geography of where people live and how they vote. Any collection of voting data can then be applied to this ensemble of restricting plans to obtain a collection of election results. The election results give a benchmark against which a particular redistricting may be compared under the same set of voting data. It is only the relative difference between the ensemble and the enacted plan which matters. Our ensemble of restricting plans naturally incorporates how nonpartisan redistricting criteria interact with the political geography and geometry of the state. It naturally adapts to natural packing in urban areas and other effects. It is capable of separating these natural effects from those of partisan gerrymandering. Because of this, this mode of analysis can separate bias that natural packing might induce from other effects.

Additionally, none of these analyses rely on any forms of partisan symmetry or ideas of proportional representation. The ensemble method does not impose any idea of fairness nor does it select for a particular seats-to-votes curve. Rather it illuminates what the result would have typically been had only the stated redistricting criteria been utilized. It is quite possible, and often happens, that the results from the ensemble method do not yield proportional representation and one party has a natural advantage relative to the statewide vote fraction. One can then use this natural advantage as a benchmark to detect when a particular plan is biased beyond the neutral standard the ensemble establishes.

3.2 Different Elections have Different Voting Patterns

Elections differ both in the statewide partisan vote fraction and the spatial patterns of voting across the state. Hence, it is not at all surprising that a given map can act differently under different voting patterns; even those that share the same statewide partisan vote fractions. For instance, a map could be designed to neutralize the effectiveness of a particular set of coalitions, and hence would only be a statistical outlier in elections when those coalitions are active.

On a number of occasions, we have seen maps that particularly show the effect of the Gerrymander when there is a danger that the majority or supermajority are lost. To better understand why this is natural, consider the following example. Let us assume that a region has three varieties of people who always vote as a block and are spatially contiguous. For definiteness, let us call them red, purple, and blue people. We will assume that red always vote for the red candidate and blue for the blue candidate. Sometimes the purple vote for the red candidate and sometimes for the blue candidate. Hence, sometimes red wins two seats, and sometimes three seats, depending on how the purple people vote. Let us assume that most redistricting plans that one would naturally draw (without knowing where the red, purple, and blue people lived) would produce 2 majority red districts, 2 majority blue districts, and one majority purple district. We will call these neutral plans. Now let us consider a plan which is carefully drawn so that the purple people are never a majority but rather the purple people are split such that there are three majority blue districts and two majority red. We will call this the gerrymandered plan.

Under the gerrymandered plan the red candidates always win two of the five seats, but never more. This is typical of elections where the purple people vote with the blue people. It is typical because the majority purple district in the neutral plans would vote for the blue candidate to elect three blue candidates. On the other hand, in elections where the purple people vote with the red people, the outcome would be highly atypical as the neutral maps would have always produced three red winners but the gerrymandered plan only produces two red winners. In summary, atypical maps may lead to a typical split of elected officials under some vote counts, but not under others. It is not unusual for gerrymandered maps to sometimes produce typical results.

3.3 Collected Seat Histograms and Uniform Swing Analysis

It is a misconception that a gerrymandered map will behave atypically under all different types of elections. Gerrymandered maps can behave atypically under some types of elections and typically under other types of elections. For example, a map may only become atypical when a party is in danger of losing the majority. We demonstrate this through a type of plot we call *Collected Seat Histograms*. The election data use can either be historical elections or data generated using a uniform swing hypothesis.[1]

In both cases, we plot the histograms tabulating the fraction of the ensemble maps which produce a particular number of Democratic seats under a particular choice of statewide votes (tabulated at the precinct level). We then collect these histograms on a single plot where they are arranged on the vertical axis according to their statewide vote fractions, with the most Republican at the bottom and the most Democratic at the top. On each of the individual histograms, we also place a mark corresponding to the number of seats the enacted map would produce using those votes. Using these plots, one can identify trends and types of elections were the enacted maps products outlier results. When considering the NC State House and Senate, we also place vertical lines on each plot to mark where the supermajorities are in effect and where the simple majority in the chamber changes hand.

In addition to using historical statewide votes to produce our Collected Seat Histograms, we also create a collection of Collected Seat Histograms built from a single historical vote which is shifted using the Uniform Swing Hypothesis to produce a collection of votes which preserve the relative voting pattern across the state while seeing the effect of shifting the partisan tilt of the election.

Both kinds of Collected Seat Histograms are effective at identifying maps that are non-responsive to changing voter opinions or under-respond to those changes. A district map that results in different representation when the number of votes for a particular party changes sufficiently is a minimal requirement of a democratic process that is responsive to the changing will of the people. The Collected Seat Histograms can be used to determine the level of responsiveness to changes in the votes one should expect of the maps that were drawn without a partisan bias. The Rank Ordered Boxplots in the next section can help illuminate the structure of the map which is responsible for any systematic bias or lack of responsiveness relative to the nonpartisan benchmark embodied in the ensemble.

[1] The uniform swing hypothesis takes a single election and then uniformly increases (or decreases) the percentage for a given party across all the predicts. This creates a new set of voting data with the same spatial structure but a different statewide partisan percentage for each party.

3.4 Structure of Maps and Rank-Ordered Marginal Boxplots and Histograms

While the partisan seat count is clearly a quantity of interest, it can be less effective at illuminating the structure of a map that also explores how the elections are won. To this end, we introduce the *Rank-Ordered Marginal Boxplots and Histograms*. These are formed by considering the partisan vote fraction for one of the political parties (say the Democrats, or equally the Republicans) in each of the districts for a given redistricting plan. These marginal vote fractions are then ordered from smallest to largest, that is to say; from most Republican district to most Democratic district. These ordered numbers are then tabulated over all of the plans in the ensemble.

The Rank-Ordered Marginal Boxplots plot the typical range of the most Republican district to most Democratic district. Ranges are represented by box-plots. In these box-plots, 50% of all plans have corresponding ranked districts that lie within the box; the median is given by the line within the box; the ticks mark the 2.5%, 10%, 90% and 97.5% quartiles; the extent of the lines outside of the boxes represent the range of results observed in the ensemble. The number of boxes is the same as the number of seats. That is 120 seats for the NC House, 50 seats for the NC Senate, and 14 seats for the NC Congressional Delegation. Any box that lies above the 50% line on the vertical axis will elect (or typically elect) a Democrat; any box that lies below the 50% line will elect (or typically elect) a Republican.

We take the enacted plan with each set of votes and plot the ordered district returns over the box plots. If the districts of an enacted plan lie either far above or far below the ensemble at a particular ranking, this can indicate that the district was either packed or cracked to provide an atypical result.

4 State Legislature

Using historic voting data, we compare election results under the enacted districting plans for the North Carolina House and North Carolina Senate with election results under a collection of non-partisan maps. One strength of this method is that it makes no assumptions in advance about what structure an election should have such as a relation to proportional representation or some type of symmetry considerations. We examine both the number of seats that would have been won under these vote counts, along with the expected margins of victory.

4.1 State Legislature: Overview of Findings

4.2 State Legislature: Overview of Method

We generate a collection of alternative restricting maps using Markov Chain Monte Carlo methods, and used this collection to characterize what would be expected if only non-partisan redistricting criteria where used. We have described this method in detail in our academic work. See [7, 3, 8, 10, 1, 2]. (References in this report to numbers in brackets are to articles cited in a numbered bibliography at the end of this report). No partisan information is used to construct this ensemble of maps; only the generally accepted districting criteria of approximately equal population per district, contiguous and relatively compact districts, reducing traversals, and keeping counties, precincts, and municipalities whole.

For both the NC House and NC Senate, we generate a *Primary Ensemble* whose non-partisan properties are close to those of the enacted plan. Because of this, we sometimes label this plan as the *Matched Ensemble*. For both the NC Senate and NC House, we produce a *Secondary Ensemble* which makes different policy choices concerning the preservation of municipalities. In a third ensemble built, we also consider the pairing of incumbents.

The ensembles are generated by using the Metropolis-Hasting Markov Chain Monte Carlo Algorithm in a parallel tempering framework which employs proposal from the Multiscale Forest RECOM algorithm [2, 1] and the single-node flip algorithm [7]. Using these proposals, the Metropolis-Hasting algorithm is then used to produce samples from the desired policy-informed, non-partisan distribution on redistrictings; such algorithms are widely accepted for sampling high-dimensional distributions. The Markov Chain Monte Carlo and Metropolis-Hasting algorithms are a cornerstone of modern computational statistics, protein folding and drug discovery, and weather prediction. They date back to at least the Manhattan Project in Los Alamos are used in a huge range of mathematical and statistical applications.

The distributions we use are defined to be concentrated on districting plans that contain districts near the ideal district population based on the one-person-one-vote principle (including the 5% population deviation acceptable for legislative districts). They are also designed to produce contiguous districts that are relatively compact and to reduce the number of counties and, in some cases, the number of people split out of a municipality. For the Primary Ensemble, the distribution on redistricting plans is tuned so that these non-partisan qualities, including the number of counties, municipalities, and precincts which are split, are similar to the enacted plan. We also respect the county-clustering requirement for State Legislative maps.

We will see that the enacted NC Senate preserves municipalities to a high degree; in a way consistent with the most municipality preserving distributions we could produce. Hence, we also provide a *Secondary Ensemble* for the NC Senate which does not explicitly preserve municipalities (thought compactness and the county preservation lead to a degree of municipality preservation.) It coincides with the primary ensemble properties in other resects.

For the NC house, we will see that the enacted plan is not as stringent in its municipality preservation, and that respecting the other criteria could naturally create many plans that better preserve municipalities than the enacted plan. Since we have tuned our primary ensemble to match the level of municipality preservation in the enacted plan, which include a Secondary Ensemble for the NC house we is better at preserving municipalities.

As the guidance from the legislature at the start of the redistricting process stated that one "may consider municipality preservation" (in contrast to other directives which were not optional), all four of these ensembles meet the guidance given by the legislature. As already mentioned, we also provide a third ensemble for both the NC house and NC Senate which is derived from the primary ensemble, but considers the double-bunking of incumbents.

In all cases using the Metropolis-Hasting Markov Chain Monte Carlo Algorithm, we can produce a mathematically representative sample of the redistricting plans that comply with the criteria described.

4.3 County Clusters for State Legislature

In Stephenson v. Bartlett, 562 S.E.2d 377 (N.C. 2002), the North Carolina Supreme Court ruled that North Carolina's state legislative districts should be clustered into groups of counties and that no district should cross between two of the "county clusters." As part of our non-partisan work concerning redistricting, we implemented the algorithmic part of the Stephenson Ruling in a publicly available open-source piece of software [4]. We used this computer software to produce the county clusterings used in this report. The resulting clusterings were described in our publicly released report which can be found here [5]. We understand that the NC Legislature also used this report to determine the possible clusterings. In any case, the clusterings we found coincide with those discussed by the legislature.

There is not a unique choice of statewide clustering. Rather there are parts of the state which can only be clustered in one way, while there are two ways to cluster the counties in other regions. In the state Senate, there are 17 clusters containing 36 of the 50 districts that are fixed based on determining optimal county clusters. These are represented by the color county groupings in Figure 4.3.1. The white numbers annotating each county clustering give the number of districts that the county cluster should contain. Ten of these clusters contain one district, meaning that ten of the 50 senate districts are fixed by the county clusters. The remaining county clusters must be further subdivided into legislative districts. The remaining 14 counties, shown in gray on the map in Figure 4.3.1 are distributed among four groups, each containing two clustering options. Following the nomenclature in [5], we will label the cluster groups by the letters A, B, C, and D . Each group consists of two different possible clusterings which we will label with the numbers 1 and 2. Thus, the first choice in cluster A is labeled A1, and the second choice A2. A complete choice of county clusters then consists of one choice from the A group, the B group, the C group, and the D group.

Similarly, in the NC State House, there are 33 clusters containing 107 of the 120 districts that are fixed based on determining optimal county clusters. These are represented by the color county groupings in Figure 4.3.2. Again, the white numbers annotating each county clustering give the number of districts that the county cluster should contain. Eleven of these clusters contain one district, meaning that eleven of the 120 house districts are fixed by the clustering process. The remaining clusters (shown in gray) are separated into three groups each containing two clustering options. As before, the groups will be demoted by the letters A, B, and C with each of the two options in each group labeled with the numbers 1 or 2.

More details can be found in [5] and [4]. It should be noted that the algorithm used to produce these clusterings only implements the algorithmic portion of the Stephenson v. Bartlett. In particular, it does not address any compliance with the Voting Rights Act.

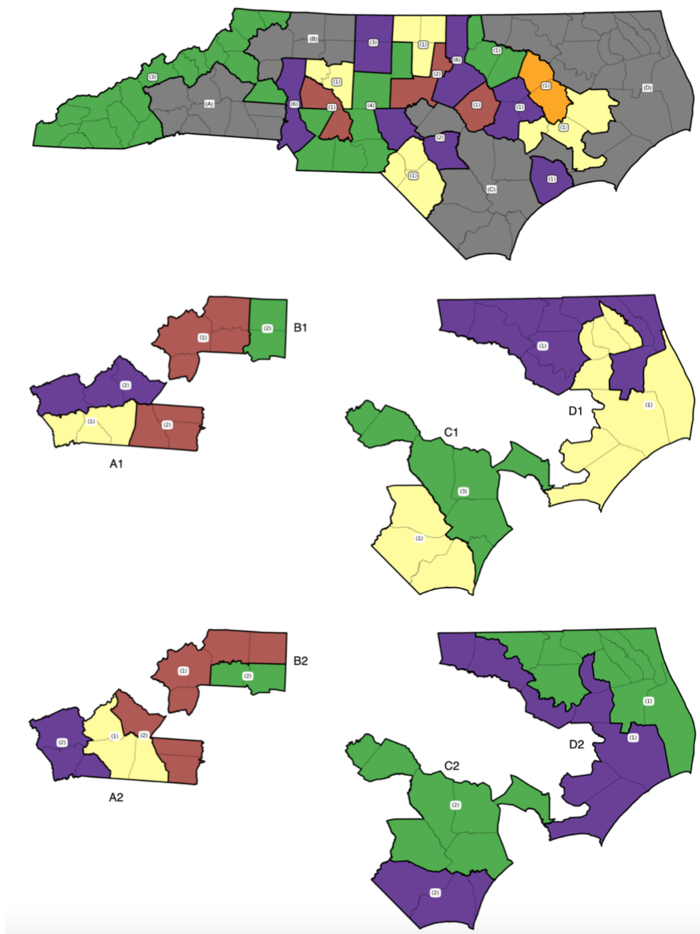


Figure 4.3.1: Senate

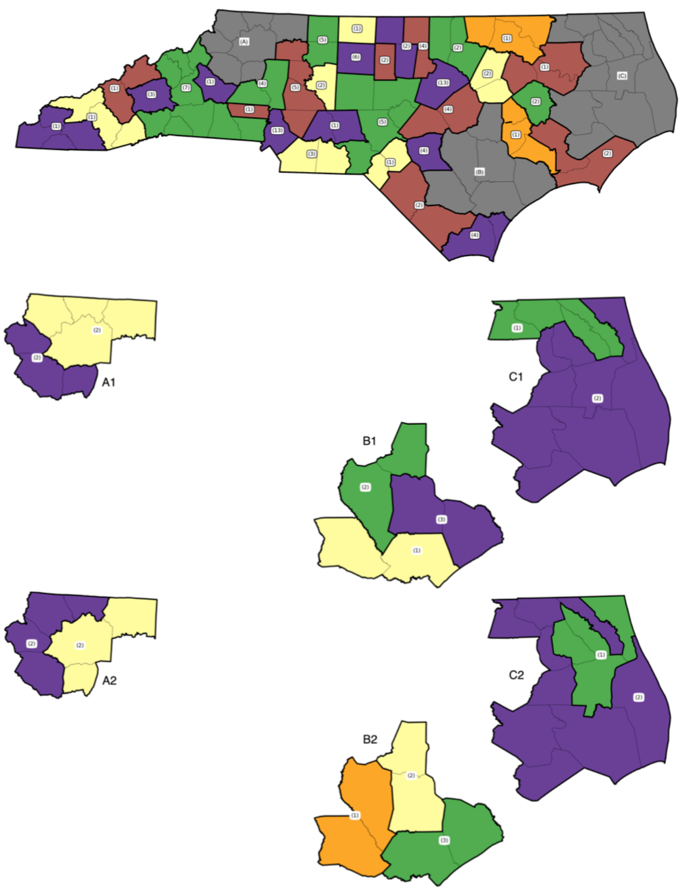


Figure 4.3.2: House

4.4 State Legislature: Ensemble Overview

We now give more details on the different distributions already sketched in Section 4.2. They represent different distributions that emphasize different policies consistent with the Legislature's guidance and historical presidents. All the distributions from which we build our ensembles respect the county clusters we derived in [6] by algorithmically implementing the ruling Stephenson v. Bartlett, 562 S.E.2d 377 (N.C. 2002). That is to say in both the State House and State Senate, the state is segmented into groups of counties referred to as *county clusters* so that the population of each county cluster can be divided into a number of districts each with a population within 5% of the ideal district population. The county clusters are different for the State House and State Senate as the number of districts, and hence the ideal district populations, are different. Each district is constrained to lay entirely within one county cluster.

Beyond the county cluster requirement all of our primary and secondary ensembles for both chambers also satisfy the following constraints:

- The maps minimize the number of split counties. The 2021 redistricting criteria state that "Within county groupings, county lines shall not be traversed except as authorized by Stephenson I, Stephenson II, Dickson I, and Dickson II."

- Districts traverse counties as few times as possible.

- All districts are required to consist of one contiguous region.

- Except for two exceptions, the deviation of the total population in any district is within 5% of the ideal district population. The two special cases are explained in Section 7.2.

- Voting tabulation districts (i.e. VTDs or precincts) are not split (see again the two exceptions with population deviation in Section 7.2)

- Compactness: The distributions on redistricting plans are constructed so that a plan with a larger total isoperimetric ratio is less likely than those with a lower total isoperimetric ratio. (See Section 7.2 and 8.1 for a definition of the isoperimetric ratio.) The total isoperimetric ratio of a redistricting plan is simply the sum of the isoperimetric ratios over each district. The isoperimetric ratio is the reciprocal of the Polsby-Poper score; hence, smaller isoperimetric ratio corresponds to larger Polsby-Poper scores. The General Assembly stated in its guidance that the plans should be compact according to the Polsby-Popper score or the Reock score [9]. We have found that while the Reock is useful when comparing two districts. However, the Polsby-Popper/isoperimetric score is a better measure when generating district computationally. In our previous work, we have seen that this choice did not qualitatively change our conclusions (see [7] and the expert report in Common Cause v. Rucho).

  We tuned our primary ensemble so that compactness scores of the ensemble were comparable to those of the enacted plan. See Section 7, for plots showing the compactness scores.

Municipality Preservation: We now come to the property which distinguishes the Primary and Secondary ensembles. In both chambers of the NC Legislature, we tune the primary ensemble to match the level of municipalities preservation to those seen in the enacted plan. Since municipality preservation is concerned with keeping the voters of a particular municipality together as a block, we concentrate on the number of *ousted voters*. Ousted voters are those who have been removed from the districts which primarily contain the other members of the municipalities. We construct the ensemble to control the total number of ousted voters across the entire state. More details are given in Section 7.2. As already mentioned, we tune the Secondary ensembles differently for the two chambers. Since the Enacted Senate plan was at the lowest end of municipality splitting we observed, we have included a secondary ensemble in the Senate which did not explicitly consider municipality reservation. In the NC House, since the enacted plan did not preserve municipalities to the level we found possible, we included a secondary ensemble which better preserved municipalities.

Incumbency: The effect of incumbency are addressed in a subsequent section of this report.

4.5 Construction of Statewide Ensembles for State Legislature

Statewide ensembles are created by drawing samples from a number of "sub-ensembles." Because of the county cluster structure, we can sample each county cluster independently of the other county clusters. In the house, we sample the Wake and Mecklenburg county cluster groups separately from the rest of the state as they have many more precincts and districts. In the Senate, we sample the Wake county cluster independently since it must split precincts to achieve the 5% population balance. There are several regions of the state that have multiple options for county clusters and we sample each of the county clustering options separately. We then sample the remainder of the state together.

We combine these sub-ensembles by first choosing which of the county clustering options will be used, treating all options equally. With these fixed, we then choose a map from each of the other sub-ensembles and combine them to produce a statewide map. We used this procedure to create an ensemble of 100,000 maps. These ensembles of statewide maps were used to generate the various figures. This number was chosen as it proved to be sufficient for the statistics of the quantities of interest to have converged. That is to say that adding additional maps to the ensemble did not change the results. See Section 7.1 for more details on the sampling method.

4.6 Election Data Used in Analysis

The historic elections we consider are from the year 2016 and 2020. We only consider statewide elections. We will use the following abbreviations: AG for Attorney General, USS for United States Senate, CI for Commissioner of Insurance, LG for Lieutenant Governor, GV for Governor, TR for State Treasure, SST for Secretary of State, AD for State Auditor, CA for Commissioner of Agriculture, and PR for United States President. We add to these abbreviations the last two digits of the year of the election. Hence CI16 is the vote data from the Commissioner of Insurance election in 2016.

5 State Legislature: Main Statewide Analysis

Our analysis shows that the enacted plan for the NC State House is an extreme gerrymander over a wide range of voter behavior seen historically in NC. The effect of this extreme gerrymander is to prevent the Democrats from winning as many seat as they would have had the maps been drawn in a neutral way without political considerations. This gerrymander is achieved by packing Democrats in a number of the most Democratic districts while depleting them from those districts which typically change hands when the public changes its expressed political opinon through the vote. The effect is particularly strong in situations where the Democrats would typically reduce a Republican supermajority to a a simple majority. The enacted map often denies this transition. Similarly the enacted map again behaves in an anomalous fashion by under electing democrats when the typical maps would almost always give the Democrats the majority in the House. This extreme outlier behavior is reflected in the behavior we see at the individual cluster level.

The effect in the Senate is less pronounced. At the cluster level there are a number of strong and extreme outliers signaling extreme partisan gerrymandering. At the statewide level, the structure of the map shows it to be an extreme outlier in the fashion in which Democrats are packed in certain districts and depleted from others. The effect at the statewide level is mostly seen when the Republicans are in danger of losing the supermajority in the Senate. Over this range the anomalous packing and cracking of Democrats leads to a number of extreme outlier behaviors which result in the Republicans maintaining the supermajority when they typically would have lost it under a non-partisan map from the ensemble.

Additionally we see that the reason that the Senate map is typical in many situations stems from the choice to highly conserve municipalities. The municipality preservation is at the extreme end of what we have observed. In contrast, the municipality preservation in the house is less extreme as we can easily create an ensemble which preserves municipalities to a higher degree. For the Senate plan, relaxing the requirement to preserve municipalities leads to an ensemble that is more favorable to the Democrats, meaning that the enacted plan would be an extreme outlier in more situations. Put differently, prioritizing municipality preservation in the Senate plan appears to enable more maps that favor Republicans. By contrast, for the House plan, where the enacted map does not prioritize preserving municipalities, my analysis finds that such a prioritization would not have favored the Republican party.

5.1 NC State House

Figure 5.1.1 shows the distribution of Democratic seats elected under a number of historical elections which capture plausible voting patterns in North Carolina elections. The elections are arranged vertically by the statewide Democratic vote share, from most Republican at the bottom to the most Democratic at the top. The Democratic seats elected under each election by the enacted plan is marked with a yellow dot.

It is important to remember that the single number of statewide vote fraction is not sufficient to categorize an election. Elections with similar statewide vote fractions can have dramatically different seat counts since the votes can be concentrated differently geographically. An example of this is shown in Figure 5.1.8 which shows the Collected Seat Histograms for an ensemble that places more weight on preserving municipalities that the enacted plan or the primary ensemble. Notice that the AG20 votes produce more democratic seats typically than either AG16 or GV16 even though the statewide vote fraction of AG20 is sandwiched between AG16 and GV16. (Recall the definitions of these abbreviations given in Section 4.6.)

Returning to Figure 5.1.1, we see that the enacted map is atypical in its favoring of the Republican party in every one of the elections considered and an outlier or extreme outlier in the vast majority of the elections. Additionally, the enacted plan is an extreme outlier when the Republicans are likely to lose either the Super-majority or control of the chamber. Observe that in the vast majority of plans in the primary ensemble (Figure 5.1.1) the votes in PR16, LG20 and CL20 produce a simple majority for the Republican party in the NC State House (and not a supermajority). Yet under the enacted plan, the Republican Party maintains the supermajority in all three cases.

Similarly, in a large number of the ensemble plans the Democrats hold the majority in the chamber under the voting patterns given by AD20, SST20, and GV20. (Under GV20 the Democrats have the majority most of the time, under AD20 roughly half the time and under SST roughly 75% of the time.) Yet, under the enacted plan the results are extreme outliers, giving the Republicans the majority with a safety margin of a few seats in all cases.

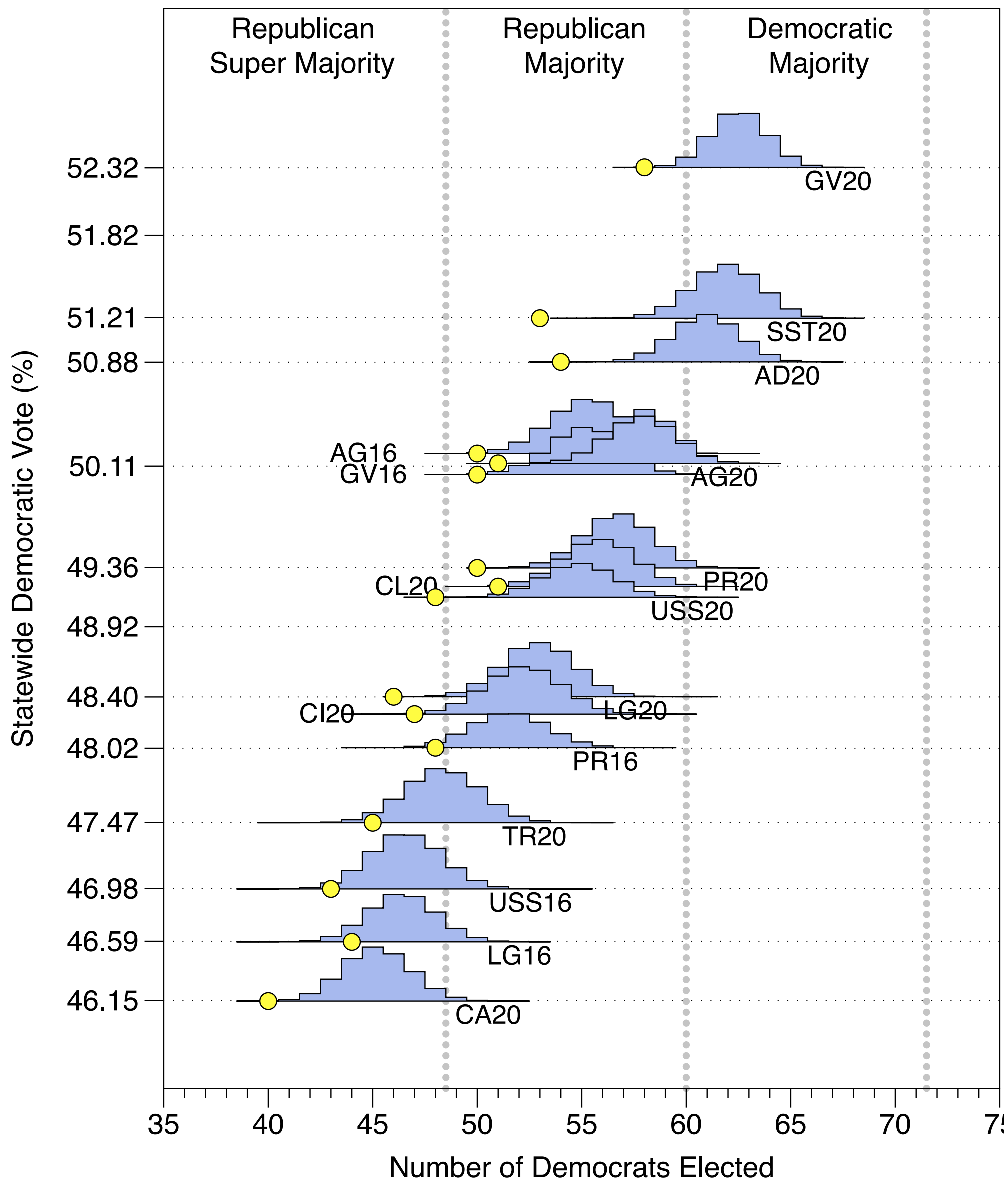


Figure 5.1.1: The Collected Seat Histogram for the Primary Ensemble on the NC House. The individual histograms give the frequency of the Democratic seat count for each of the statewide elections considered from the years 2016 and 2020. The histograms are organized vertically based on the statewide partisan vote fraction for each election. The more Republican elections are placed lower on the plot while more Democratic elections are placed higher. Three dotted lines denote the boundary between where the supermajorities and simple majorities are in force. The yellow dot represents the enacted plan.

As already observed, Figure 5.1.1 helps to identify the properties of the Enacted Map under different electoral environments. There is a clear trend as one moves to more Democratic elections, the atypical results (already tilted to toward

% Dem	Election	% Outlier	# Outlier	# Samples
52.32%	GV20	0.118%	118	100000
51.21%	SST20	0.000%	0	100000
50.88%	AD20	0.007%	7	100000
50.20%	AG16	0.451%	451	100000
50.13%	AG20	0.005%	5	100000
50.05%	GV16	0.399%	399	100000
49.36%	PR20	0.007%	7	100000
49.22%	CL20	0.759%	759	100000
49.14%	USS20	0.012%	12	100000
48.40%	LG20	0.009%	9	100000
48.27%	CI20	0.461%	461	100000
47.47%	TR20	5.569%	5569	100000
46.98%	USS16	3.066%	3066	100000
46.59%	LG16	11.778%	11778	100000
46.15%	CA20	0.094%	94	100000

Table 1: NC House Collected Seat Histogram Outlier Data. Starting from the left, the first column gives the statewide partisan makeup of the of the election under consideration whose abbreviation is given in the second column from the left. The right most column gives the total number of plans in the ensemble considered which is 100,000. The second column from the right gives the number of those 100,000 plans which elect the same or less Democrats under the given election. These are the plans which are as much or more of an outlier than the enacted map. The middle column is the percentage of plans which are more or equal of an outlier. (It is calculated by dividing the 2nd column from the right by 100,000 and multiplying by 100 to make a percentage.) The extremely low percentages in the middle column shows that the enacted plan is an extreme outlier across many different electoral settings.

the Republican party) in the more Republican elections in Figure 5.1.1 trend into extreme outliers as we shift to the more Democratic leaning elections.

To make the above table more quantitative, in Table 1 we tabulated the number of maps which produced the same or fewer seats for the Democrats in each of the elections we consider. We see that the enacted map is an extreme outlier. Across the vast majority of elections, the house map behaves as an extreme outlier in favor of the Republican party.

In the three elections where the results are not an extreme outlier (TR20, USS16, and LG16), the enacted plan is still atypically tilted to favor the Republican party. These three elections have a strong statewide Republican vote fraction. Hence, there is no need for a gerrymander as the Republicans have the needed votes to often keep a supermajority under even a typical map.

We will see in Figure 5.1.2 and 5.1.3 below that when these three elections are shifted (using the uniform swing hypothesis) to produce plausible voting fractions at a larger statewide Democratic vote fraction, then the results are also extreme outliers.

It is also worth noting that the bias in the enacted plan from what non-partisan map would produce systematically is the favor of the Republican party. Not once is the tilt even mildly in the favor of the Democrats.

To better control for other variation, we now include a number of Collected Seat Histograms built from a single election which has been shifted to create a sequence of elections with different statewide partisan vote fractions but the same spatial voting patern.

In Figures 5.1.2 and 5.1.3, we see that the same phenomena from Figure 5.1.1 is repeated again and again. As the vote share increases to the point where the primary ensemble for the NC House would typically break the Republicans supermajority, the enacted plan under elects Democrats to an extent which makes it an extreme outlier. This exceptional under-electing of Democrats persists past the point where almost all of the ensemble maps would have given the majority to the Democrats. In many cases the enacted map fails to respond to the shifting will of the electorate, leaving the control in the Republican hands. In addition to presenting these figures, we have also animated this affect with movies that have been submitted.

To better understand the structures responsible at the district level for the extreme outlier behavior seen in Table 2 and Figures5.2.1 to 5.2.2, we now turn to the rank-order-boxplots as described in Section 3.4. It is easy to see the abnormal structures of the enacted plan which are responsible for its extreme outlier behavior. The pattern revealed is one often seen in gerrymandered maps; namely *packing and cracking*. This refers to the depleting of one party from districts which typically would be competitive but often elect a representative from their party and instead place them in districts which were already overwhelmingly safe for either party. In Figures 5.1.4, 5.1.5, and 5.1.6, a version of this pattern is repeated. The number of Democrats seen in the districts which usually would be moderate in their partisan makeup has been decreased with a corresponding increase in the number of Democrats in the more Democratic districts where their presence has little effect on the election outcome. We give the specifics in the captions of each figure. We will see that this type of structure will be repeated in many of the individual clusters which are analyzed in Section 6.1. In addition to presenting these figures, we have also animated this affect with movies that have been submitted.

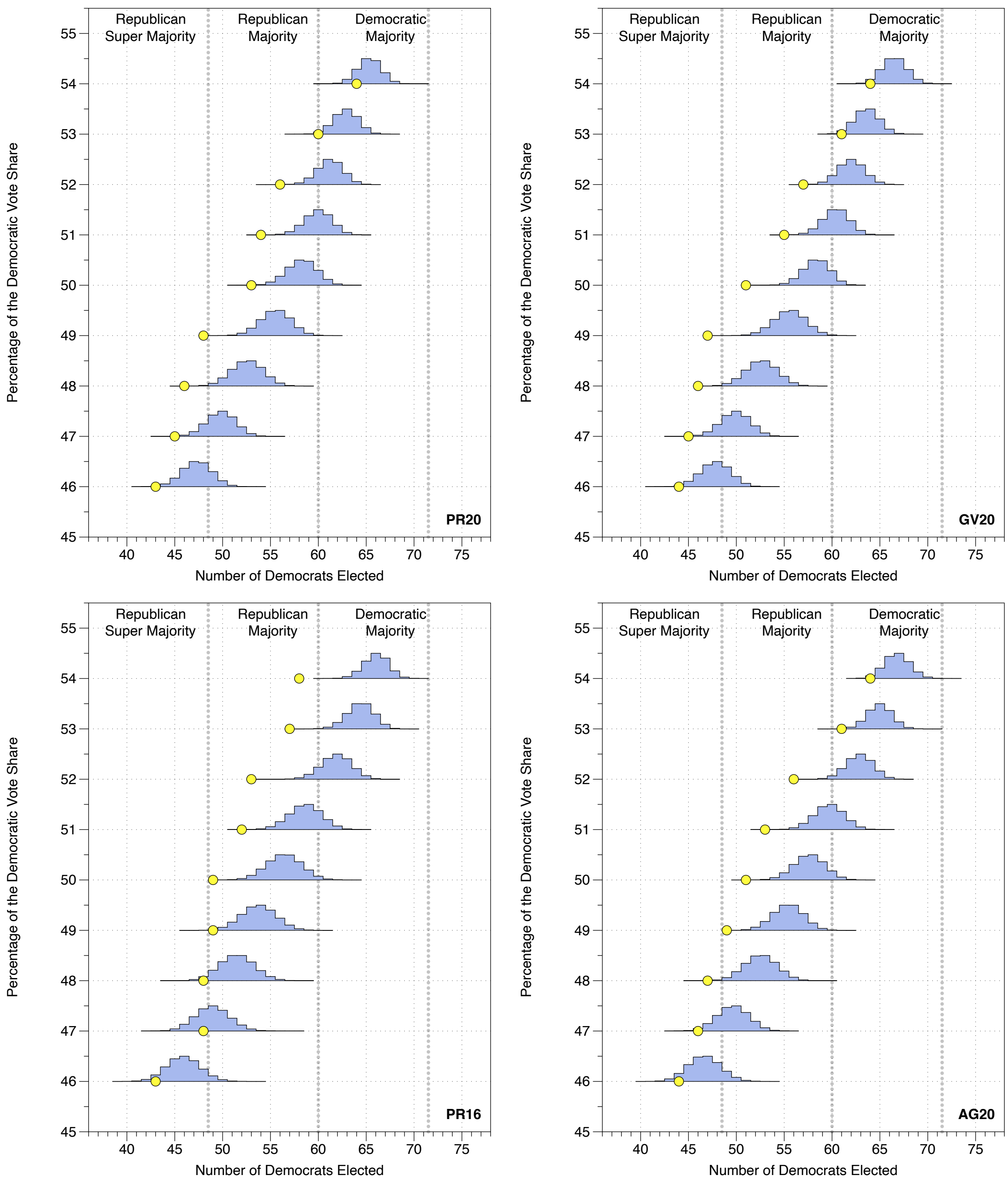


Figure 5.1.2: The individual histograms give the frequency of the Democratic seat count in the ensemble for each of the shown statewide elections, with a uniform swing. The histograms are organized vertically based on the statewide partisan vote fraction. The more Republican swings are placed lower on the plot while more Democratic swings are placed higher. Three dotted lines denote the boundary between where the supermajorities and simple majorities are in force. The yellow dot is the enacted plan.

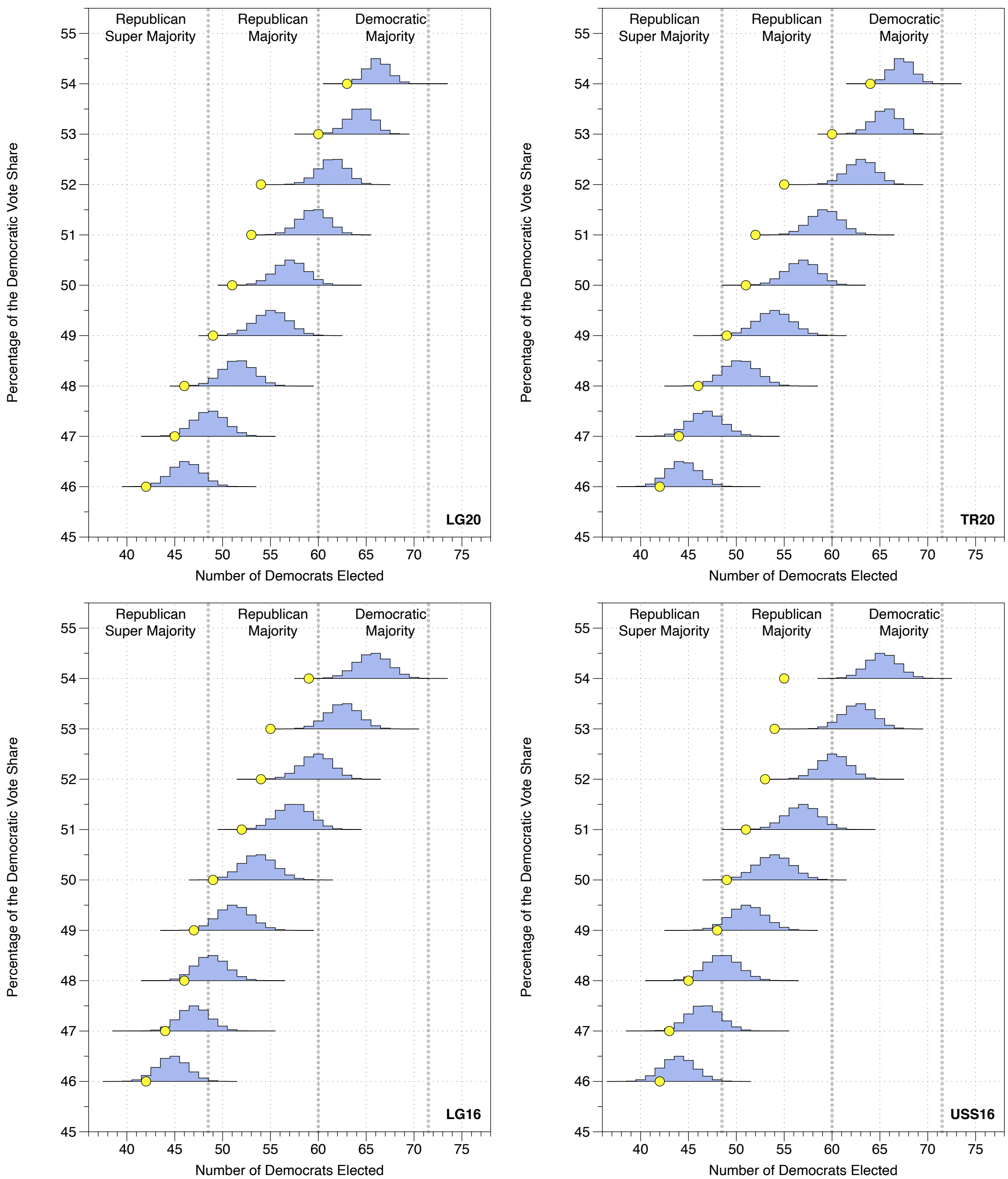


Figure 5.1.3: The individual histograms give the frequency of the Democratic seat count in the ensemble for each of the shown statewide elections, with a uniform swing. The histograms are organized vertically based on the statewide partisan vote fraction. The more Republican swings are placed lower on the plot while more Democratic swings are placed higher. Three dotted lines denote the boundary between where the supermajorities and simple majorities are in force. The yellow dot is the enacted plan.

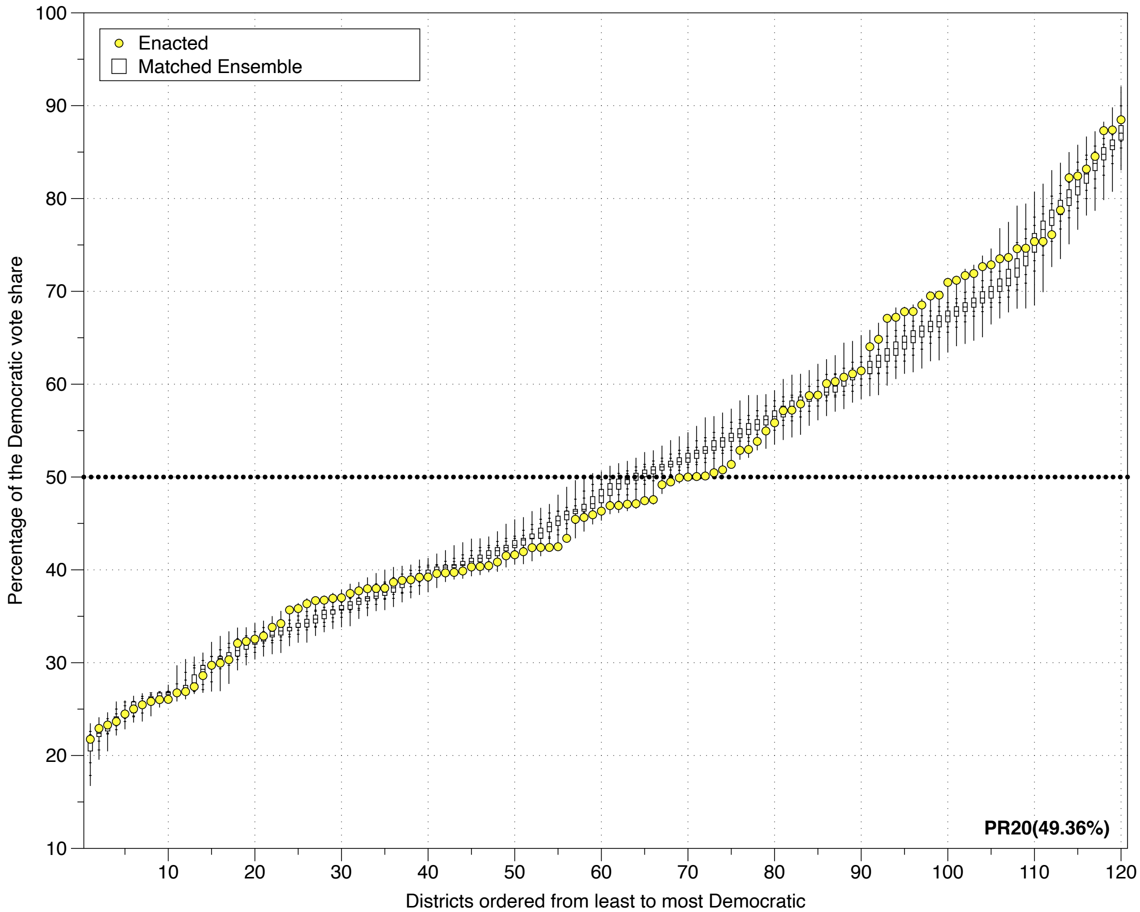


Figure 5.1.4: The yellow dots represent the democratic vote fraction of the enacted map under the PR20 vote count when the district are ordered from most Republican on the left to most Democratic in vote share on the right. The box-plots show the range of the same statistic plotted over the primary ensemble. From around the 60th to 80th district the yellow dots all well below the boxplots of the ensemble. This result is that many dots fall well below the dotted 50% line than usually would; and hence more Republicans are elected than typical. To achieve this effect, the fraction of Democrats is increased in the already strongly democratic districts ranging from the 90th to 105th most Democratic districts. This structure does not exist in the non-partisan ensemble and is responsible for the map's extreme outlier behavior.

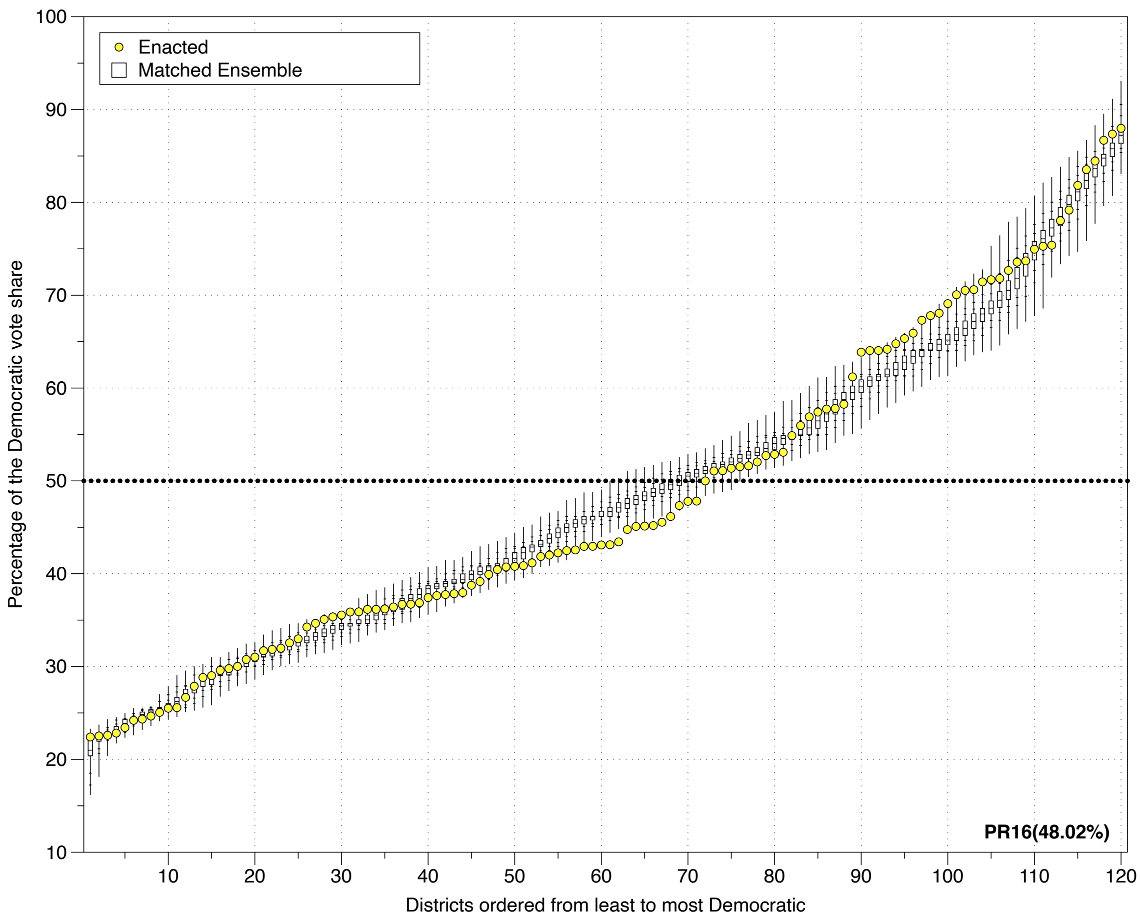


Figure 5.1.5: A similar structure to that seen in Figure 5.1.4 is repeated here. The low 50s to the high 70s have had the number of democrats depleted while the districts from the high80s to around 105 have an excess of Democrats.

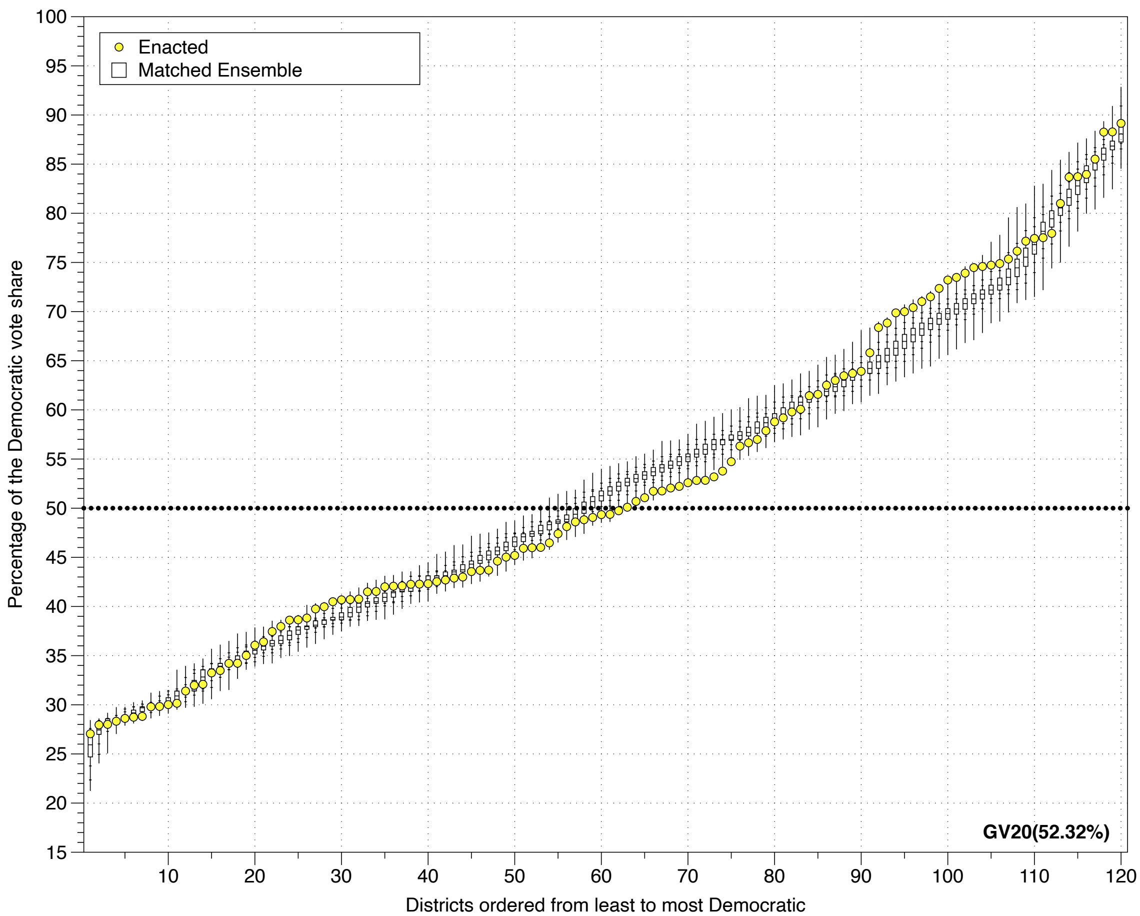


Figure 5.1.6: Mirroring what was seen in Figure 5.1.4 and Figure 5.1.5, we have abnormally few Democrats from around the 60th to the 80th most Republican and abnormally many Democrats packed in the districts in the low 90s to the just below 110.

NC House: Primary Ensemble considering Incumbency.

Figure 5.1.7 shows the Collected Seat Histogram analogous to Figure 5.1.1, but for an ensemble which pairs the same or fewer incumbents than the enacted plan. The other considerations are left unchanged from the Primary ensemble. Comparing the two figures, we see no qualitative change in the behavior of the ensemble. Hence the previous conclusions continue to hold. In particular, a desire to prevent the pairing of incumbents cannot explain the extreme outlier behavior of the enacted plan.

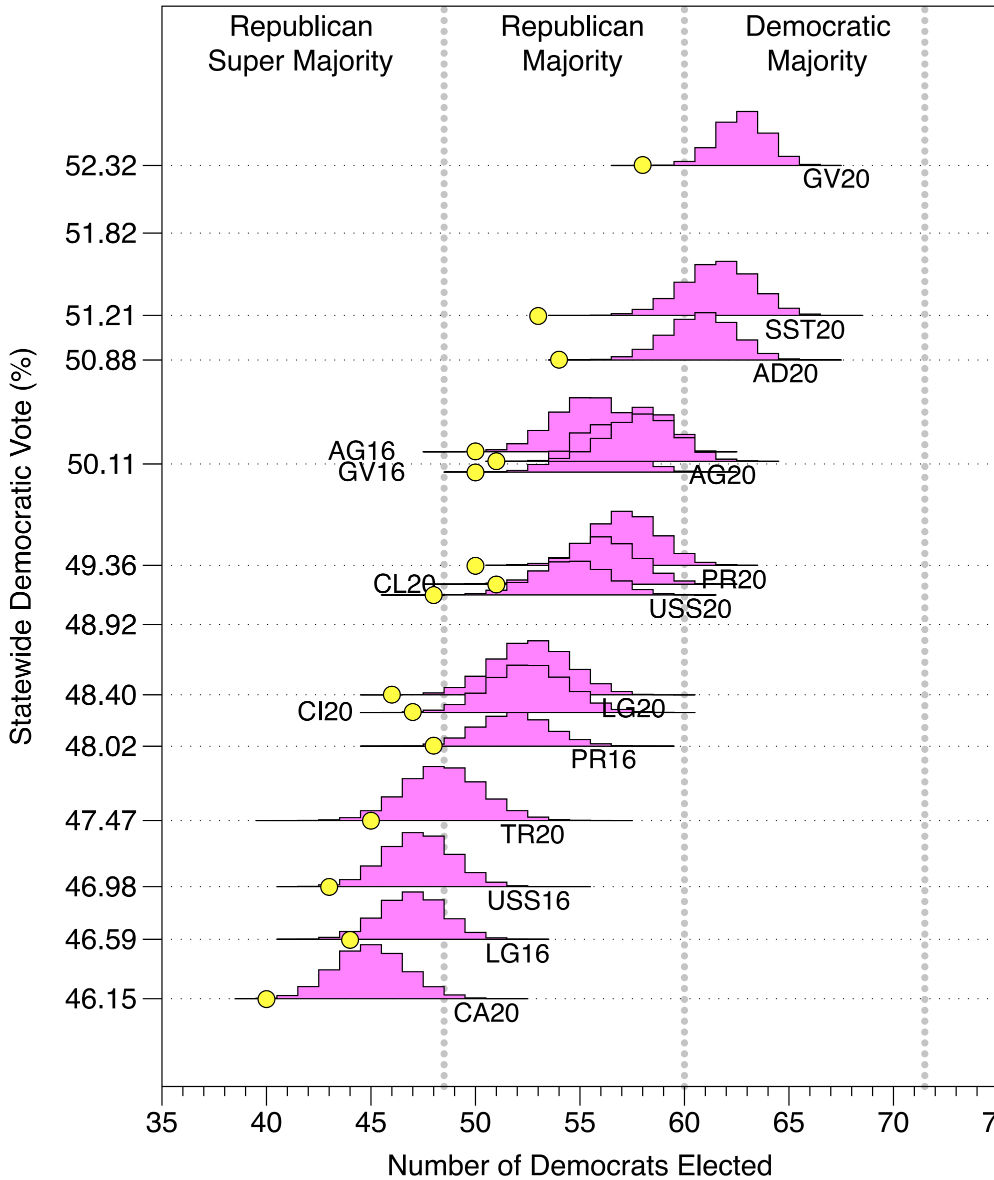


Figure 5.1.7: The Collected Seat Histogram for the Primary Ensemble on the NC House with incumbency considerations added. See Figure 5.1.1 for full description.

NC House: Secondary Distribution

The ensemble used to produce Figure 5.1.8, put more weight on preserving municipalities than either the enacted plan or the Primary Ensemble, which is tuned to match the enacted plan. This enacted plan is still an extreme outlier with respect to this secondary ensemble. We still see that the enacted map resists relinquishing the supermajority under PR16, CI20 and LG20 when this secondary ensemble almost always does. Similarly as the elections become more Democratic in AD20, SST20 and GV20 and the ensemble regularly would give the majority to the Democrats the enacted map dramatically under elects Democrats. In other words, we find that if the mapmakers had made an effort to prioritize preservation of municipalities in the House, that effort would not have led to a map that was more likely to favor Republicans.

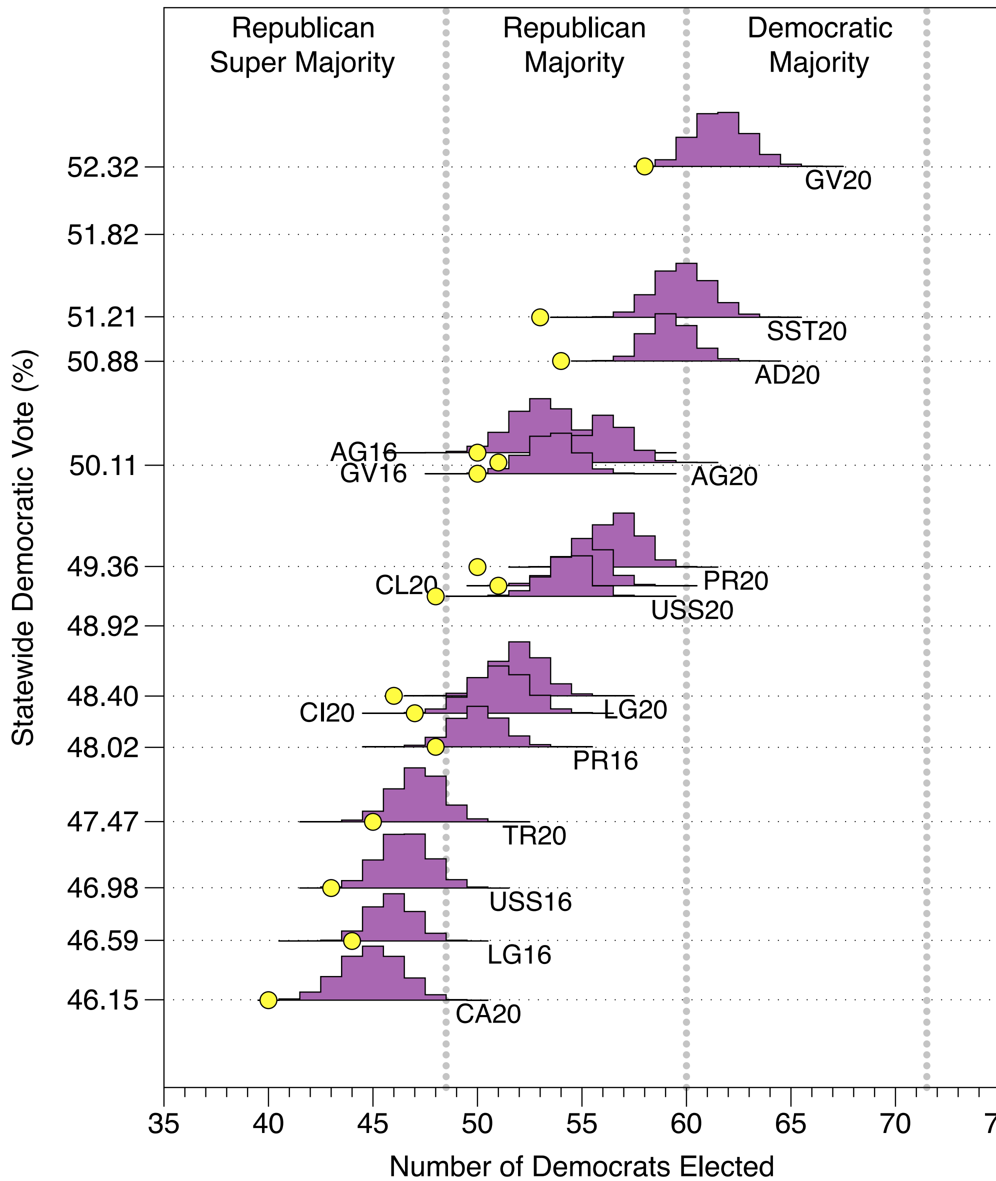


Figure 5.1.8: The Collected Seat Histogram for the Secondary Ensemble on the NC House. The Secondary Ensemble for the NC House is centered on distributions which better preserve municipalities than the enacted plan. See Figure 5.1.1 for full description.

% Dem	Election	% Outlier	# Outlier	# Samples
52.32%	GV20	16.343%	16343	100000
51.21%	SST20	35.184%	35184	100000
50.88%	AD20	42.880%	42880	100000
50.20%	AG16	12.129%	12129	100000
50.13%	AG20	4.332%	4332	100000
50.05%	GV16	0.075%	75	100000
49.36%	PR20	6.220%	6220	100000
49.22%	CL20	5.365%	5365	100000
49.14%	USS20	14.052%	14052	100000
48.40%	LG20	0.000%	0	100000
48.27%	CI20	0.322%	322	100000
47.47%	TR20	5.726%	5726	100000
46.98%	USS16	43.176%	43176	100000
46.59%	LG16	44.943%	44943	100000
46.15%	CA20	1.123%	1123	100000

Table 2: NC Senate Collected Seat Histogram Outlier Data. Starting from the left, the first column gives the statewide partisan makeup of the election under consideration whose abbreviation is given in the second column from the left. The right most column gives the total number of plans in the ensemble considered which is 100,000. The second column from the right gives the number of those 100,000 plans which elect the same or less Democrats under the given election. These are the plans which are as much or more of an outlier than the enacted map. The middle column is the percentage of plans which are more or equal of an outlier. (It is calculated by dividing the 2nd column from the right by 100,000 and multiplying by 100 to make a percentage.) The number of fairly small to extremely small percentage in the middle column between 50.13% (AG20) and 47.47% (TR20) are another signature of the anomalous behavior seen visually in Figure 5.2.1 over the same range of vote percentages.

5.2 NC State Senate

We will see in our cluster-by-cluster analysis that the NC Senate map has a number of clusters that are outliers. Their structures are systematically in favor of the Republican party. As discussed in Section 3.2, we often see maps that express their outlier status under a specific voting climate; often when one party is in danger of losing the majority or super-majority. The enacted map for the NC Senate shows this behavior.

Figure 5.2.1 is the plot for the NC Senate analogous to Figure 5.1.1, which was for the NC House. Most of the outlier behavior at the state level for the enacted NC Senate map is concentrated in the interval between 47.5% statewide Democratic vote share and around 50.5% statewide Democratic vote share. In this range, the enacted map is always an outlier and often an extreme outlier under the votes considered. This range is significant for a number of reasons. First, this is a range of statewide vote fraction where many North Carolina elections occur. Secondly, looking at Figure 5.2.1 we see that over this range the ensemble shows that one should expect the Republican super-majority (less than 21 Democratic Seats) to switch to a simple Republican majority (between 21 and 24 Democratic Seats). Yet the enacted map often resists this switch, breaking the supermajority only when the PR20 and CL20 votes are considered. In both of these elections, the ensemble places the typical number of Democratic seats well away from the supermajority line and centered between it and the simple majority line.

To make Figure 5.2.1 more quantitative, we have included Table 2 which shows the number of maps where the primary ensemble elects less democrates in that election than the enacted map.

Looking at Table 2 we see that a number of the elections in the critical partisan range of around 47.5% to 50% are extreme outliers (GV16, LG20, and CI20) while other (AG20, PR20, and TR20) show atypical behavior all favoring the Republican candidates. It is again important to notice that the enacted plan is never seen to favor the Democratic party relative to what is expected from the Primary non-partisan ensemble. The enacted map ranges between tilted to the Republican party to being an extreme partisan outlier. The importance of the range of statewide Democratic between 47.5% to 50% by looking at Figure 5.2.1. The primary ensemble shows that is within this range that one expects a Republican supermajority to become a simple majority. The effect of the enacted plan is to suppress this by under electing Democrats.

We will in the cluster-by-cluster analysis in Section 6.2 that a number of individual clusters are extreme outliers in their partisan structure.

To better control for other variation we now include a number of Collected Seat Histograms built from a single election which has been shifted to create a sequence of elections with different statewide partisan vote fractions but the same spatial voting pattern.

The large jump that we see in Figures 5.2.3 to 5.2.5 between the 33nd most Republican district and the 35th most

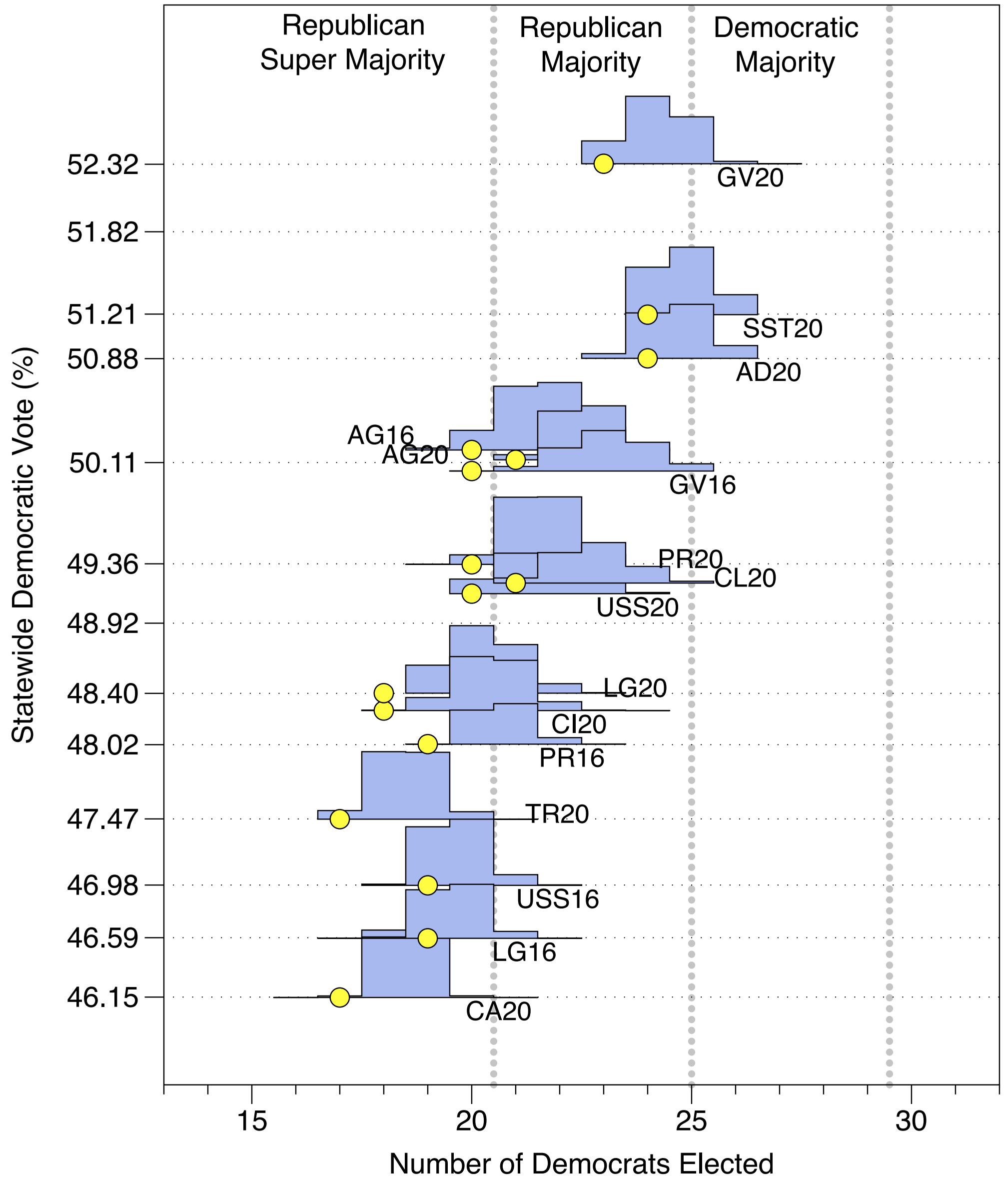


Figure 5.2.1: The Collected Seat Histogram for the Primary Ensemble on the NC Senate. The individual histograms give the frequency of the Democratic seat count for each of the statewide elections considered from the years 2016 and 2020. The histograms are organized vertically based on the statewide partisan vote fraction for each election. The more Republican elections are placed lower on the plot while more Democratic elections are placed higher. Three dotted lines denote the boundary between where the supermajorities and simple majorities are in force.

Republican district means that over a large range of swings in the partisan character of the election the outcome will change at most by one seat.

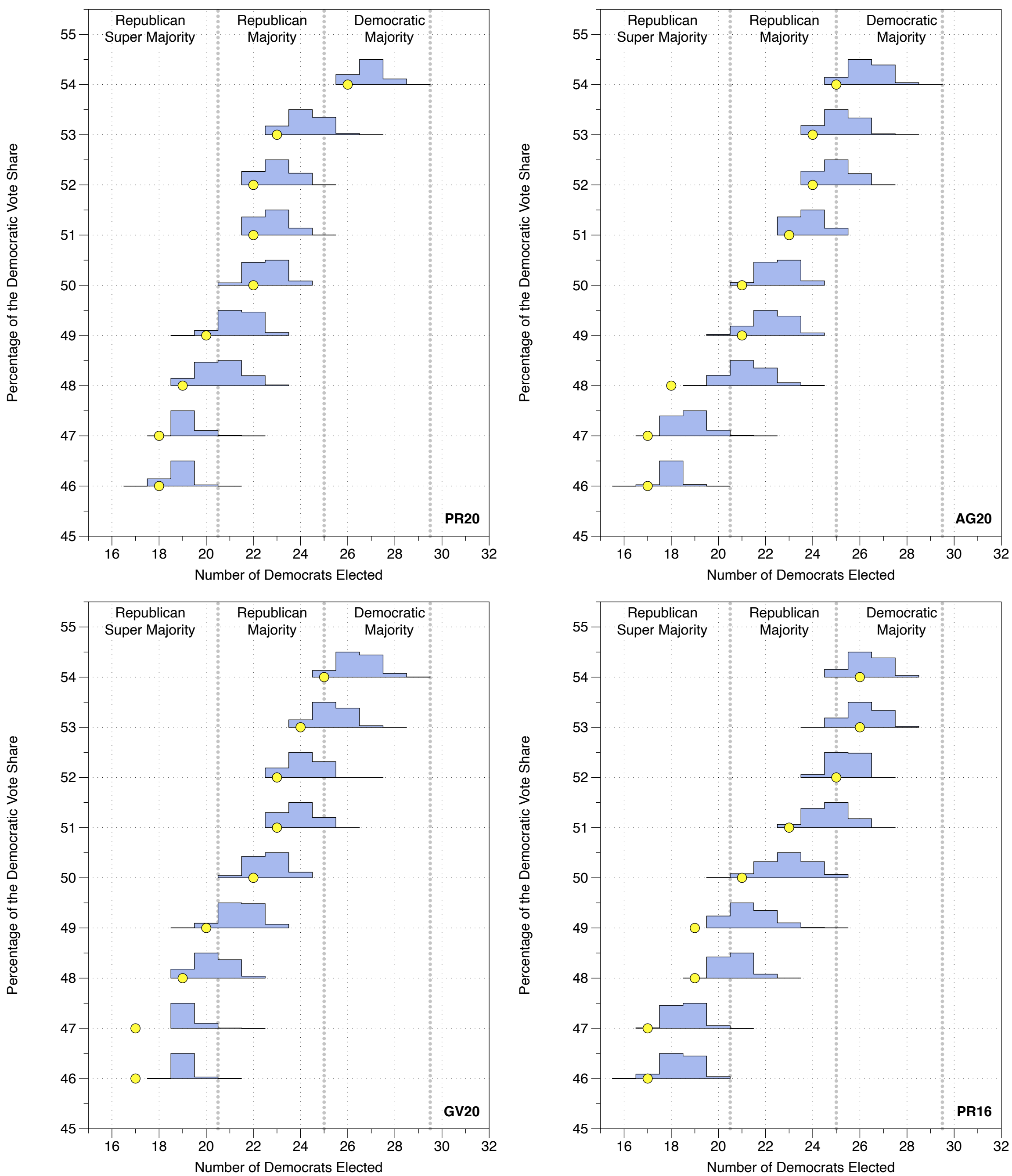


Figure 5.2.2: The Collected Seat Histograms for the Primary Ensemble on the NC House built from a collection of voting data generated via uniform swing.

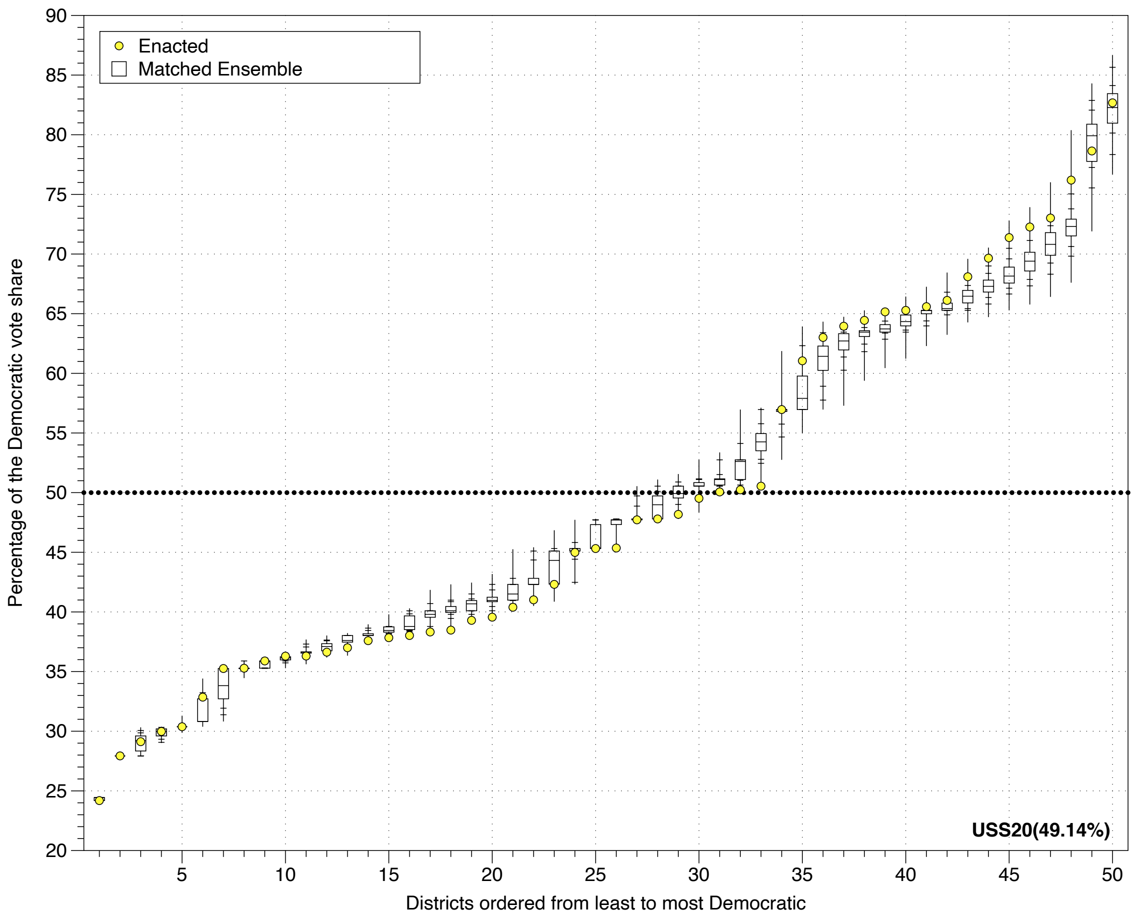


Figure 5.2.3: The yellow dots represent the democratic vote fraction of the enacted map under the USS20 vote count when the district are ordered from most Republican on the left to most Democratic in vote share on the right. The box-plots show the range of the same statistic plotted over the primary ensemble. Essentially all of the districts between the 15th most Republican and the 33rd most Republican have abnormally few Democrats. This is compensated by packing abnormally many Democrats the 35th to the 47th most Republican districts. This structure is an extreme outlier and does not occur in the ensemble.

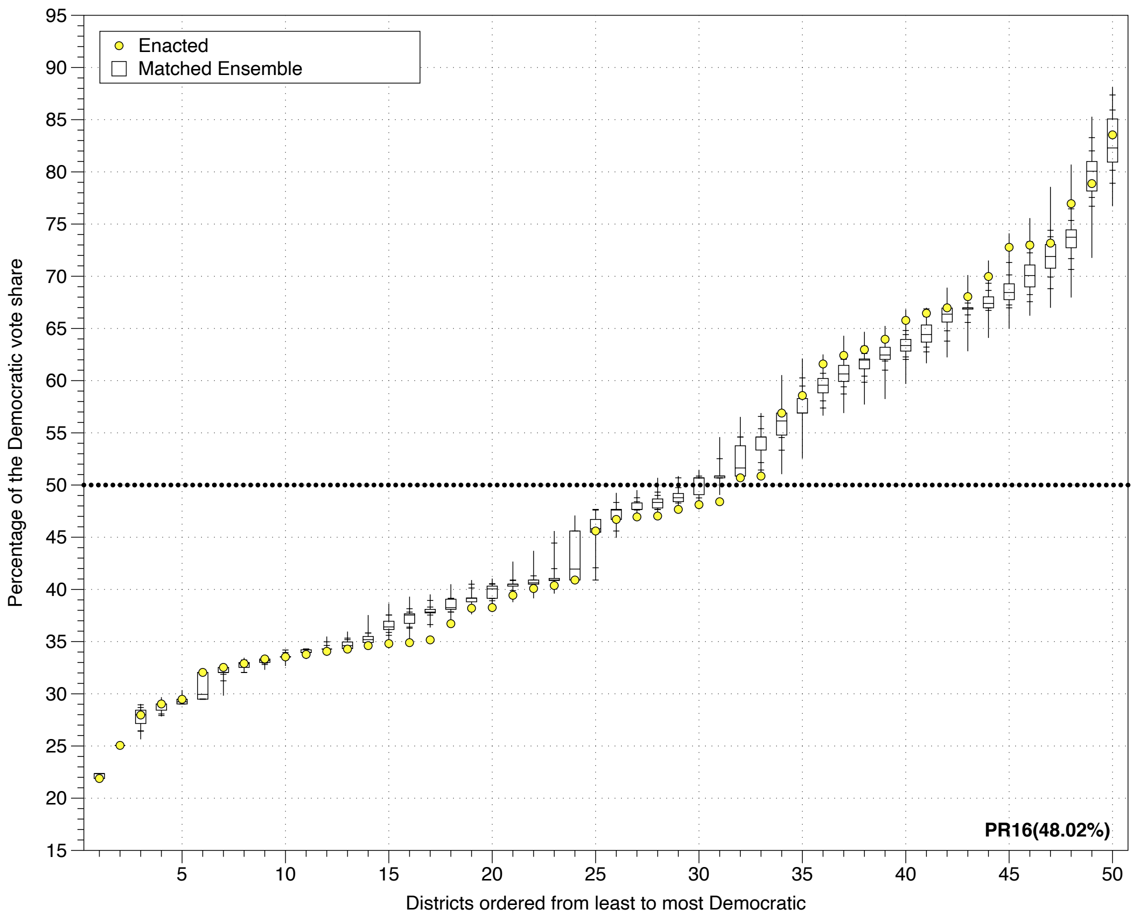


Figure 5.2.4: A similar structure to that seen in Figure 5.2.3 is repeated here over a nearly identical range of districts.

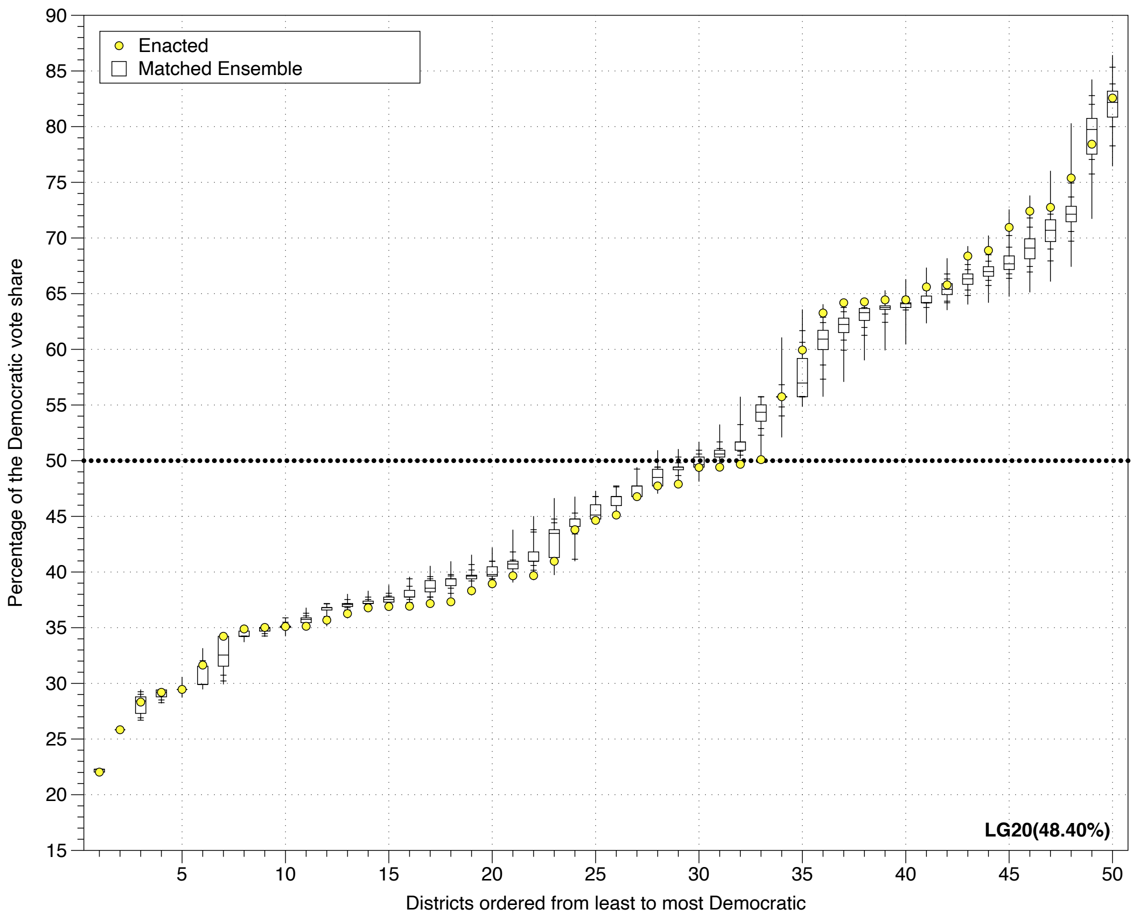


Figure 5.2.5: A similar structure to that seen in Figure 5.2.3 is repeated here.

NC Senate: Primary Ensemble considering Incumbency.

Preserving incumbency has little qualitative effect on the observations we have made. Looking at 5.2.6, we see that the election between and including GV16 and TR20 in the Figure 5.2.6 are all extreme outliers. This is in fact more extreme that the enacted map was under the Primary ensemble. It reinforces that this gerrymander seems to be most efective at the statewide level when the Republican supermajority is possible but in question.

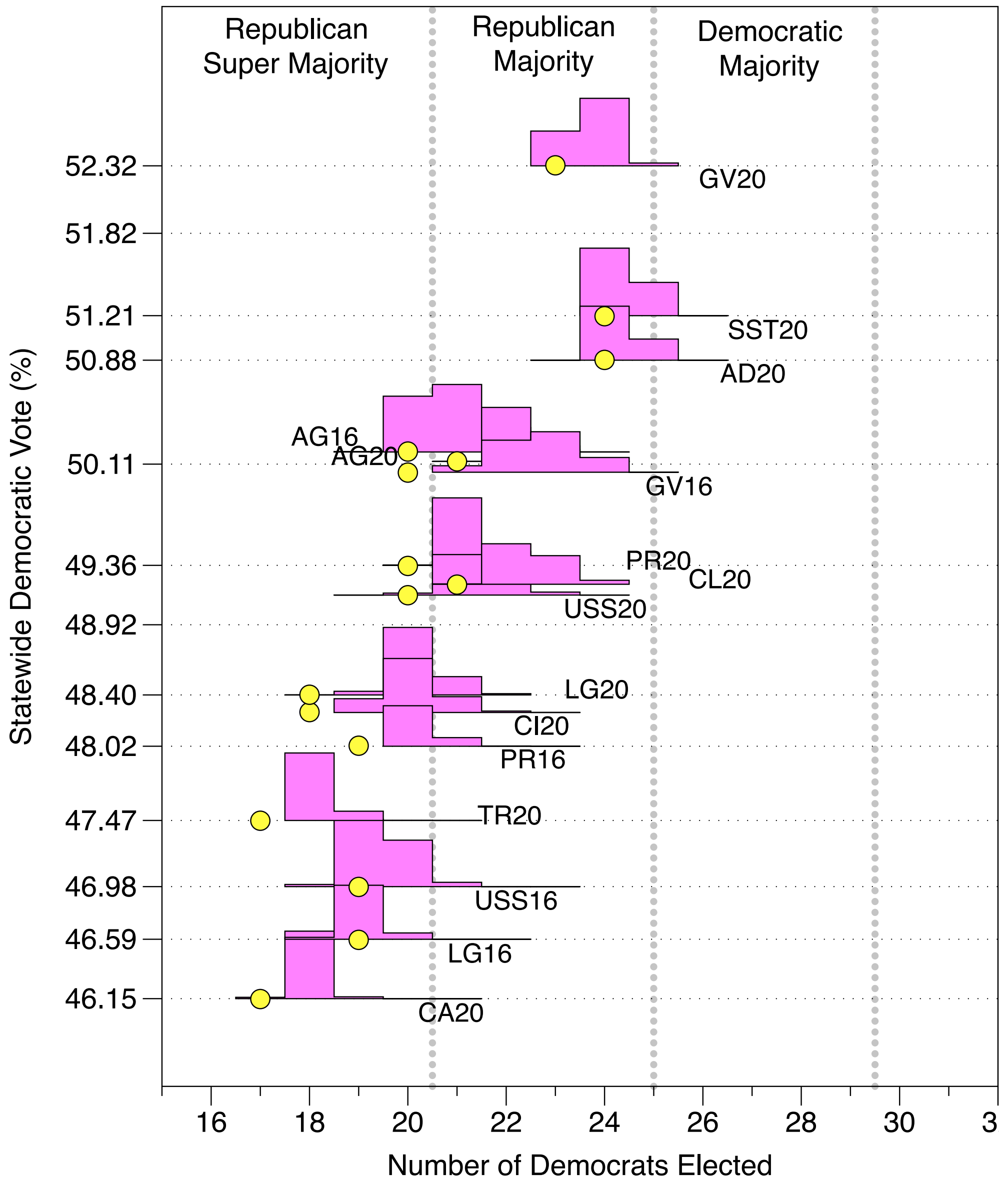


Figure 5.2.6: The Collected Seat Histogram for the Primary Ensemble on the NC Senate with incumbency considerations added. See Figure 5.1.1 for full description.

NC Senate: Secondary Distribution

When municipal preservation is not prioritized, the enacted plan becomes an outlier in all but the two most Republican elections as shown in Figure 5.2.7. Additionally, in most cases it was an extreme outlier when municipal preservation is not considered.

In other words, when municipal preservation is not prioritized, the ensemble produced is more favorable to the Democrats, meaning that the enacted plan appears as an extreme outlier in more situations than in the ensemble that matched the enacted map in prioritizing municipality. Put differently, the decision to prioritize municipality preservation in the Senate plan appears to have enabled more maps that favor Republicans.

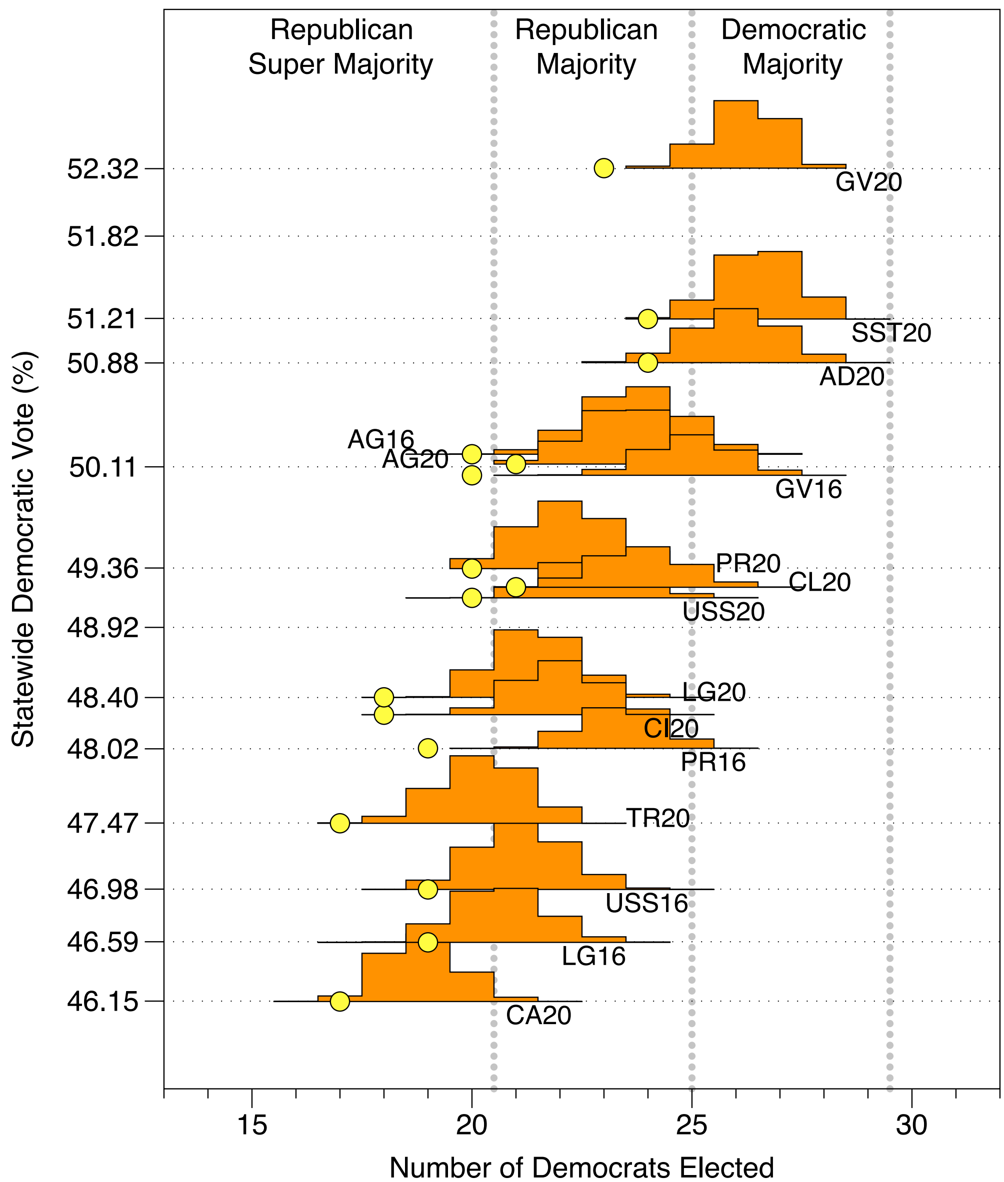


Figure 5.2.7: The Collected Seat Histogram for the Secondary Ensemble on the NC Senate. The Secondary Ensemble for the NC Senate is centered on distributions which do not explicitly consider municipality preservation. See Figure 5.1.1 for full description.

6 State Legislature: Selected Cluster by Cluster Analysis

Using the same tools, we now turn our analysis to the individual cluster. We find that a number of cluster demonstrate significate cracking and packing. In some cases this leads to changes in the partisan make of the representative typically elected from the region. In other cases, it makes the districts insensitive to changes in the voters political outlook as expressed in their votes.

6.1 NC State House

6.1.1 Mecklenburg

The ranked ordered histogram for the Mecklenburg cluster using the primary ensemble (which matches the number of people displaced from municipalities) is given in Figure 6.1.1. Across all of the voting patterns considered, we see that the two most Republican Districts (districts 98 and 103) have exceptionally few Democrats. This has the effect of making them more likely to elect a Republican when many (and often almost all) ensemble plans elect a Democrat in those districts. Specifically, that is the case under LG20, AG20, USS20, CL20, AD20 and SST20. Under GV20 and PR20, the two most Republican districts barely elect Democrats even though the majority of the ensemble plans safely elect Democrats. Under CA20 and TR20, the enacted plan safely elects two Republicans while under the ensemble the races are much closer, swinging in both directions under different plans. In these two elections, the enacted map elects a third Republican (in District 104) when the ensemble of maps typically would not. All of this is achieved by packing exceptionally many Democrats into the 6th through 9th most Democrat district, as shown in Figure 6.1.1 where the enacted plan is consistently at the extreme top of the range seen in the ensemble. All of these facts make the plan an extreme outlier in this cluster.

In fact, ranging over all of the elections considered, the Democratic fraction in the four most republican districts in the ensemble is greater than that in the enacted plan in less than 1.7% of the plans with it dipping as low as around 0.5% in a few cases. More dramatically, the percentage of plans in the ensemble where the fraction of Democrats, in the four most Democratic districts, is always less than 0.11% with it often dipping as low as 0.02% or lower.

As already discussed, it was possible to oust many less people from municipalities than the enacted plan does. Figure 6.1.2 shows the secondary ensemble which constrains municipalities much more strongly. We seen that structures highlighted above persist in this ensemble; again making the enacted map an extreme outlier.

Municipal Splits and Ousted Population: In Figure 6.1.3, we see that the enacted plan ousts people from municipalities at a number that is comparable to the primary ensemble but typically more than the Secondary House ensemble.

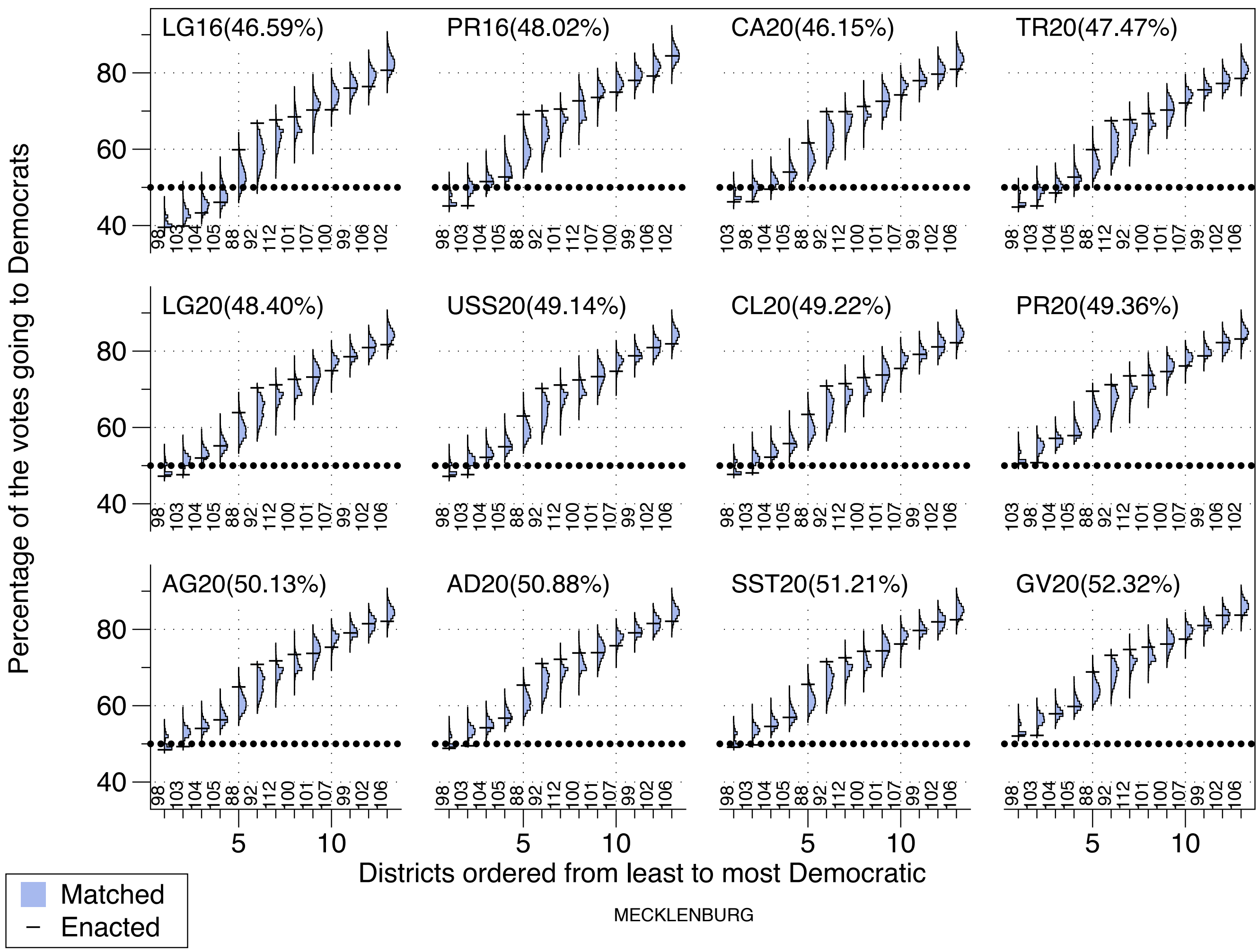


Figure 6.1.1: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

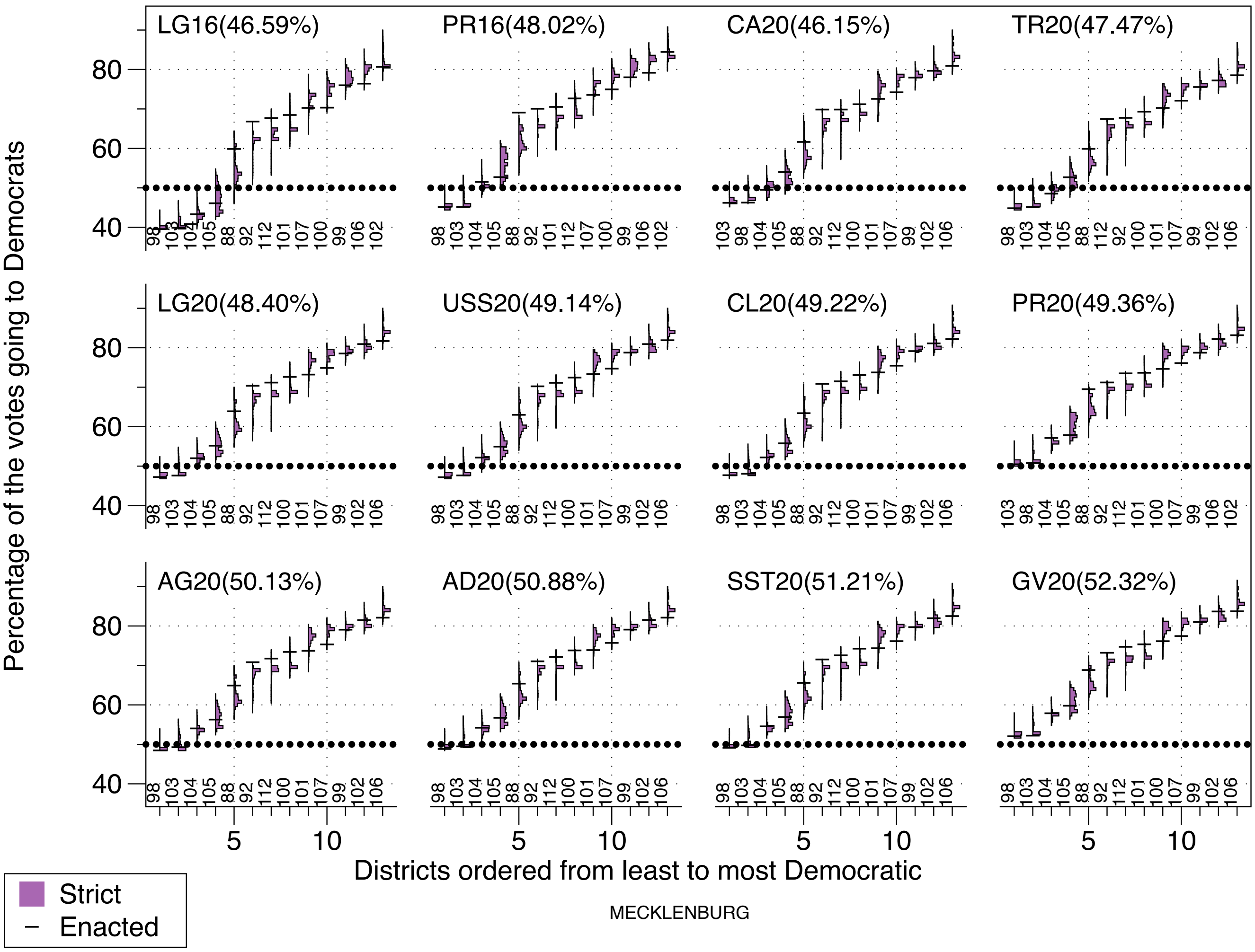


Figure 6.1.2: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

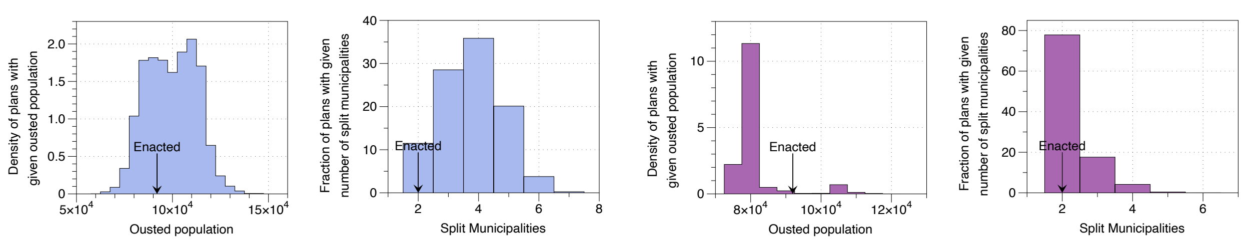


Figure 6.1.3: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.2 Wake

In the Wake cluster, we again see the depleting of Democrats from the two most Republican districts (Districts 37 and 35) while packing Democrats into the next several districts, as in the Mecklenburg cluster. The effect is to swing the two most Republican districts into play in elections where they would not be under the ensemble. Furthermore, the enacted plan makes them safer for Republicans in situations when the ensemble maps would typically have it as a toss-up.

Across all of the elections considered, the number of maps in the ensemble which have a lower Democratic vote fraction in the two most Republican districts than in the enacted plan is less than 0.42% except for the CA20 election where it is 1.2%.

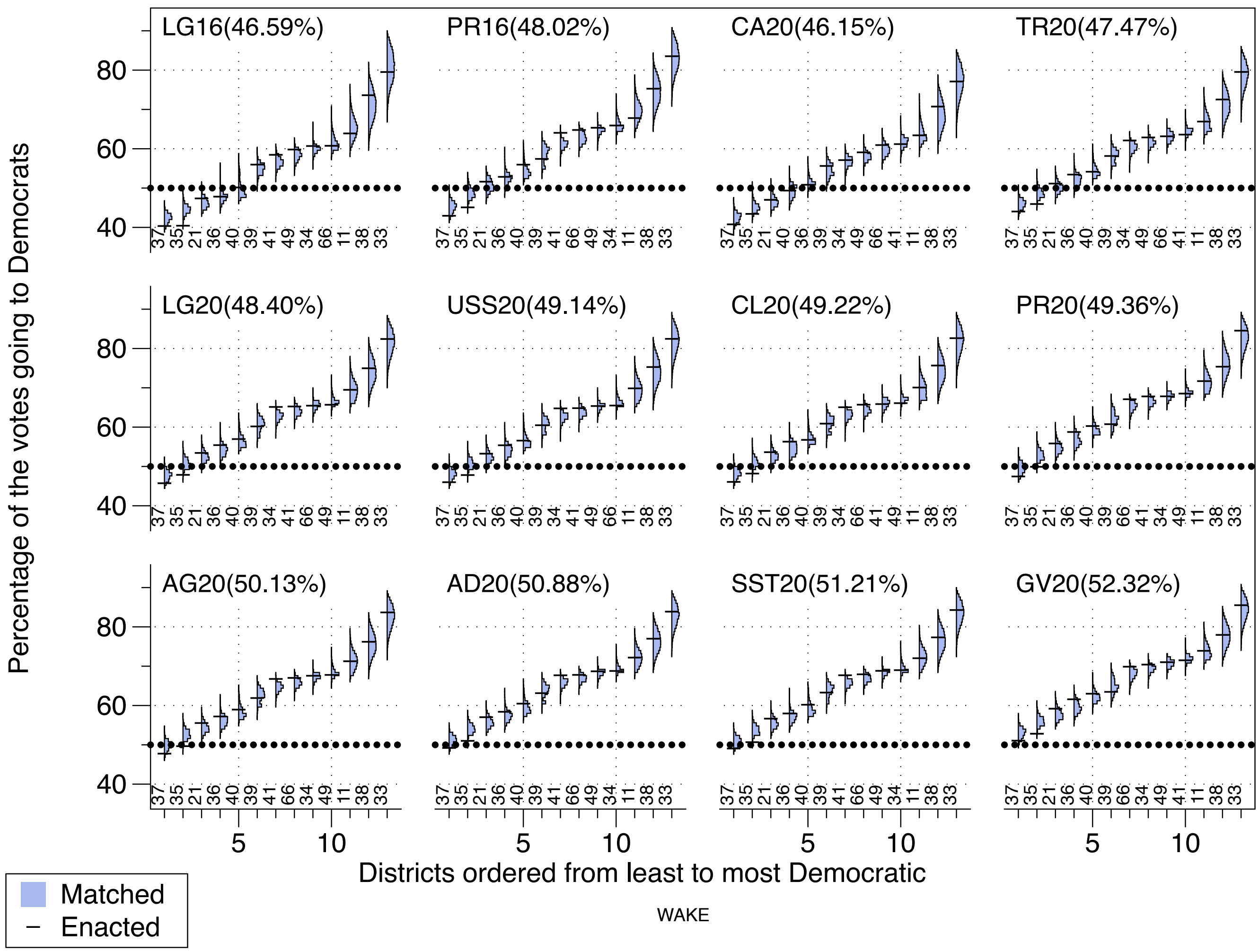


Figure 6.1.4: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

As shown in Figure 6.1.5, the trend continues under the secondary ensemble which better preserves municipalities.

Municipal Splits and Ousted Population:

In Wake we see from Figure 6.1.6 that the enacted plan consistently ousts more people than the primary ensemble and significantly more than the secondary ensemble.

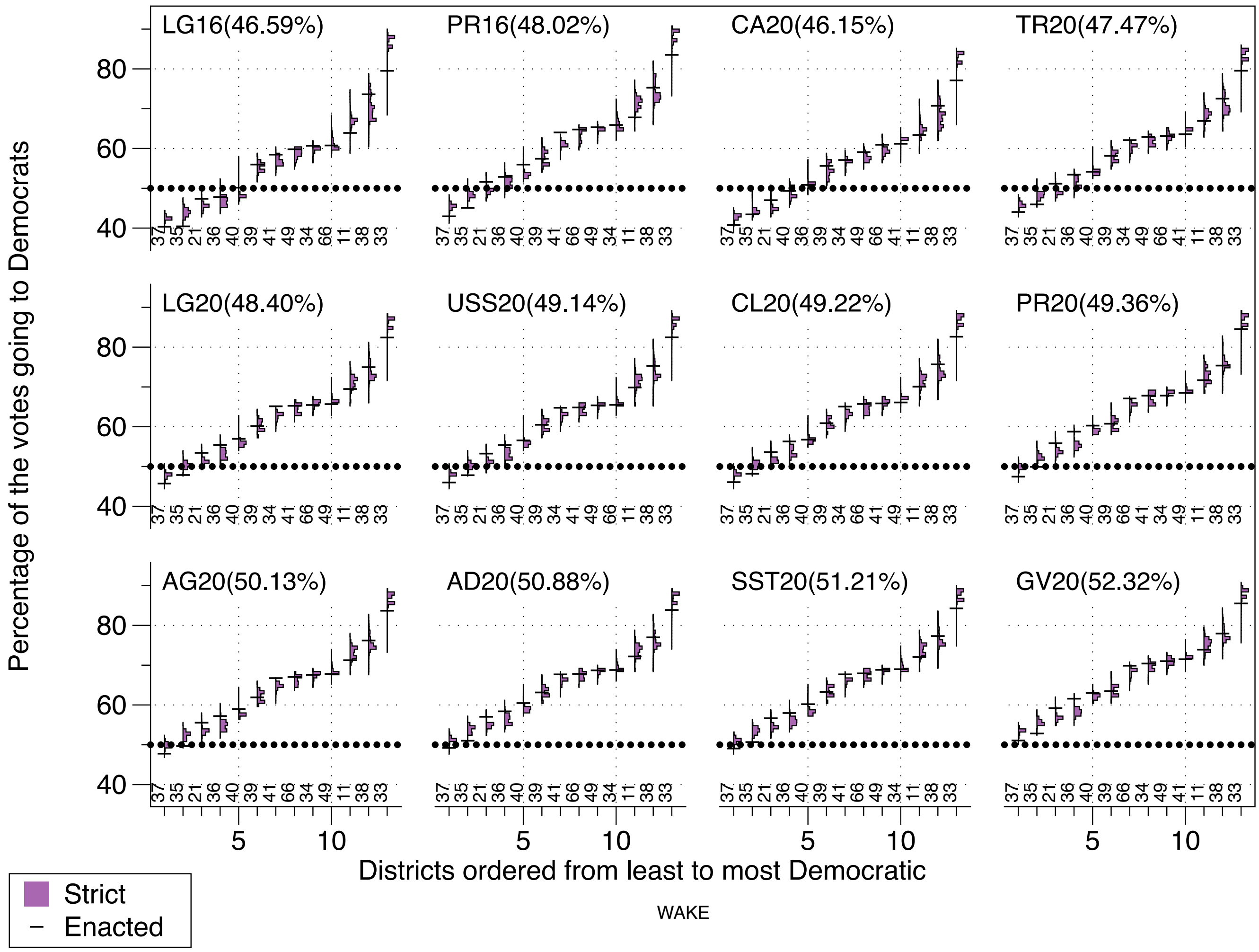


Figure 6.1.5: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

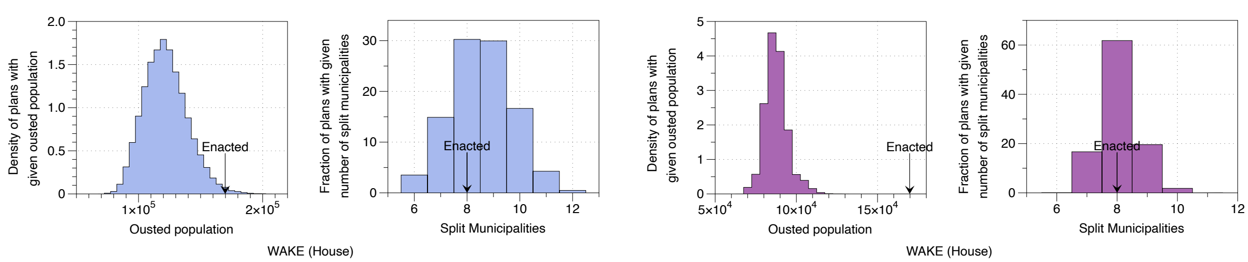


Figure 6.1.6: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.3 Forsyth-Stokes

Again in Figure 6.1.7, showing the primary ensemble in the Forsyth-Stokes cluster, we see the most Republican districts depleted of Democrats while excess Democrats are packed in safe democratic districts and in the safest Republican district are moved to competitive districts. The effect is apparent in all of the elections, but varies slightly across different voting patterns. In all cases, we see the Democratic makeup of the 3rd most Republican district pulled below the range typically seen in the ensemble often resulting in this district electing a Republican when it would not typically. In the three elections where the 3rd-most Republican district still elects a Democrat (GV20), the map's depletion of Democrats from the second most Republican district is enough to reliably elect a Republican in that district when typically the election would vary between being close and strongly favoring the Democrats.

Ranging over all of the elections considered, less than 0.02% of the plans in the ensemble have a lower Democratic fraction in the three most Republican districts than the enacted plan signaling extreme cracking. Additionally, less than 1.3% of the plans in the ensemble have a larger Democratic in the two most Democratic districts than the enacted plan.

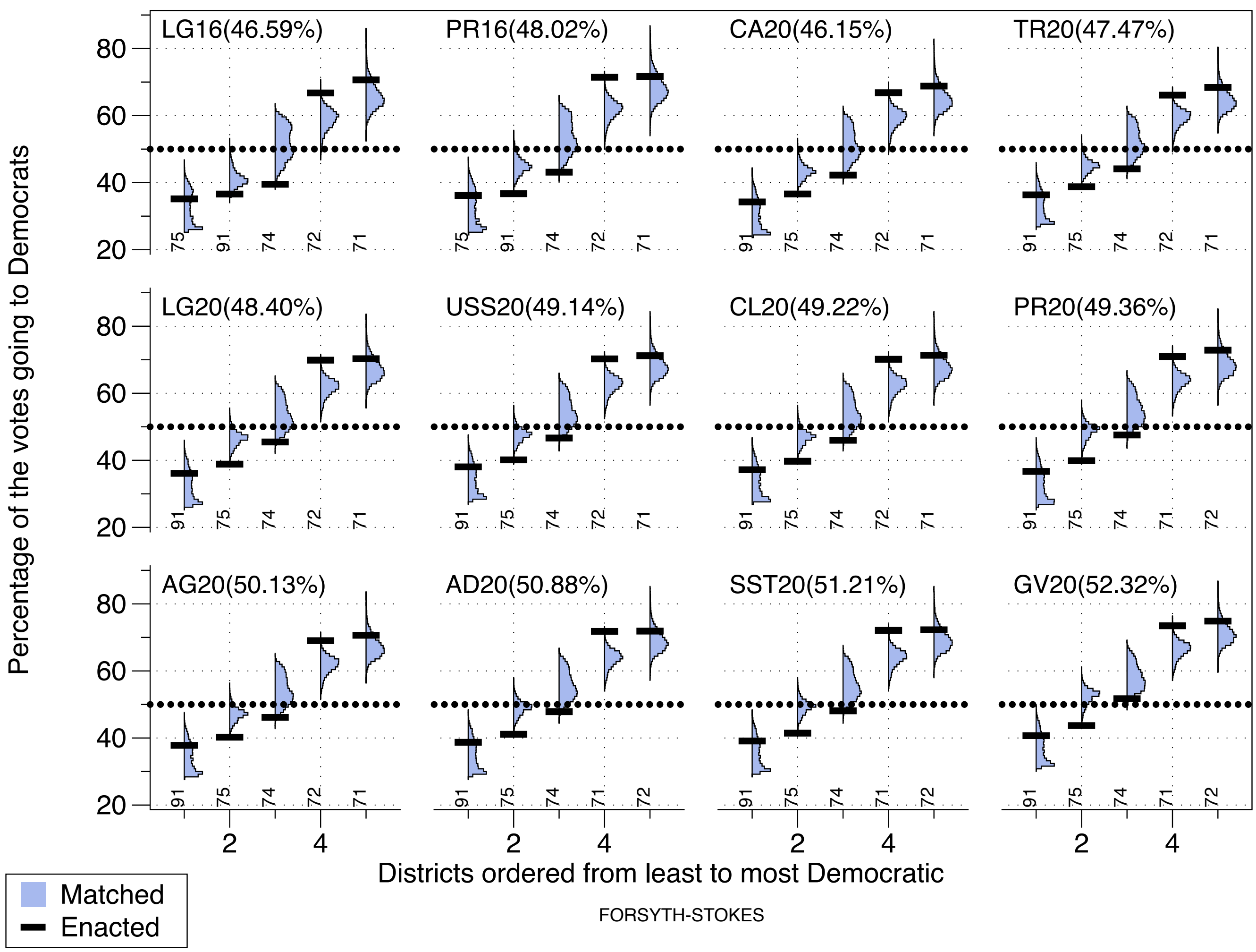


Figure 6.1.7: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

As shown in Figure 6.1.8, the trend continues under the secondary ensemble which better preserves municipalities. Some of the effects are more extreme and in this cluster, this ensemble leads to more partisan districts. Nonetheless, the enacted map still regularly elects a Republican in the third most Republican district even thought it is typically more firmly Democratic under this ensemble.

Municipal Splits and Ousted Population:

From Figure 6.1.9, we see that in Forsyth-Stokes the enacted plan ousts a number of people comparable to the primary ensemble but consistently more than the secondary ensemble.

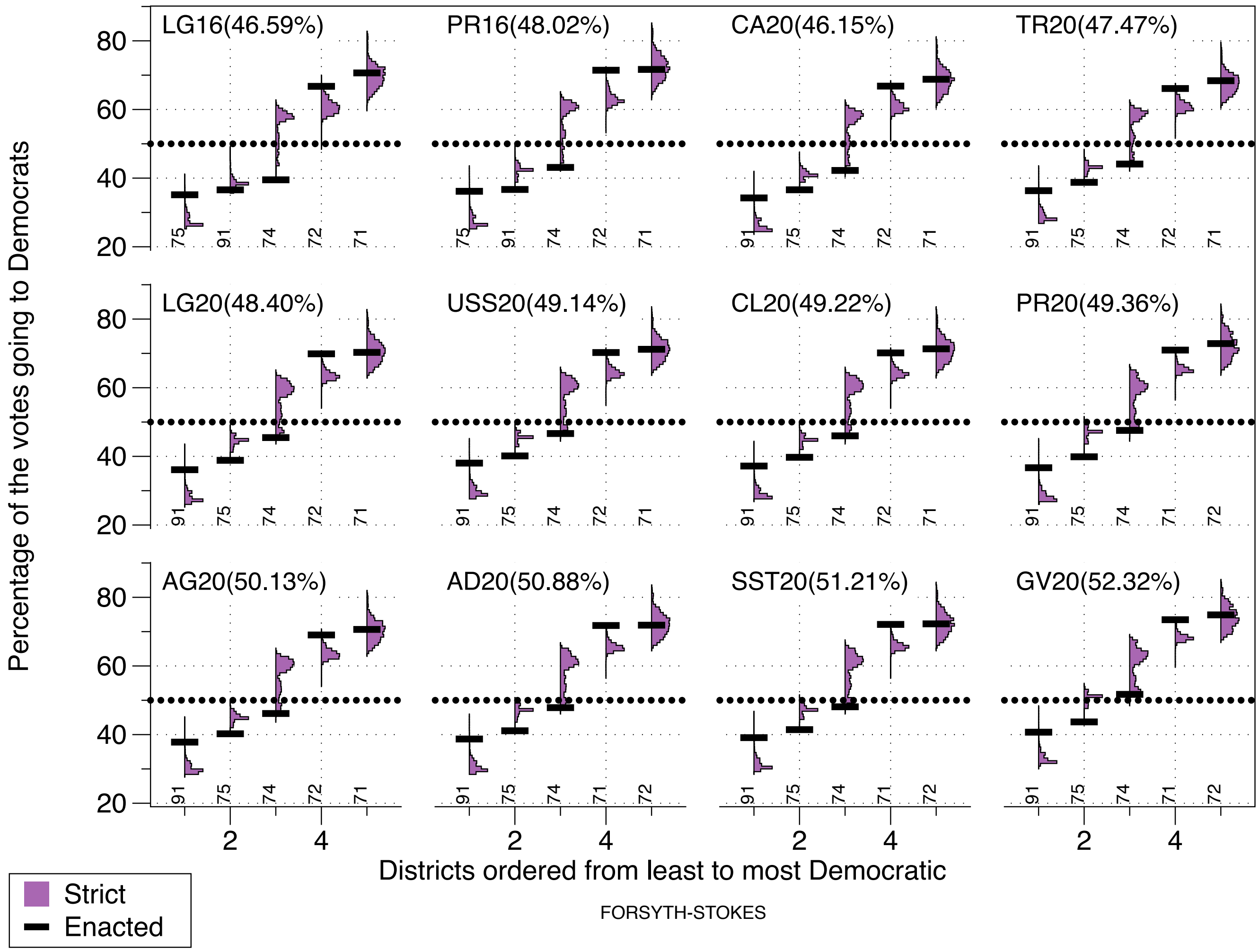


Figure 6.1.8: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

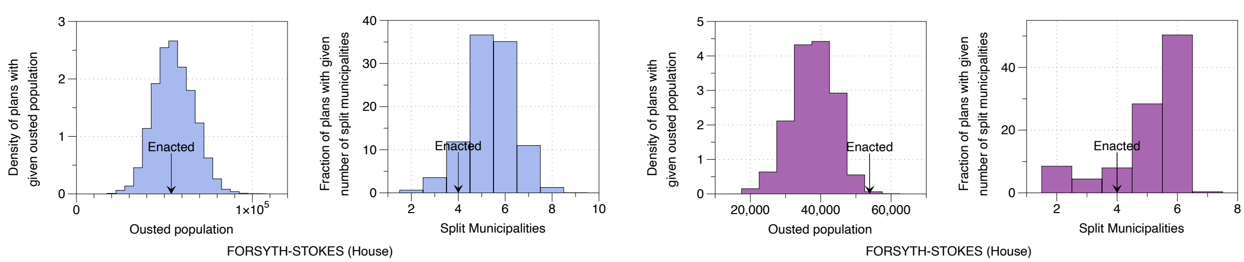


Figure 6.1.9: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.4 Guilford

The pattern seen previously is again repeated in an extreme fashion in the Guilford County. The two most Republican Districts (districts 59 and 62) have abnormally few Democrats when compared to what is seen in the primary ensemble and the more Democratic districts (numbered 57, 58, 60, and 61) have exceptionally many Democrats packed into them. The effect is that the enacted plan regularly (and often safely) elects two Republicans under election climates which would rarely or never do so.

Over all of the elections considered and all of the around 80,000 plans in the ensemble, none of the plans have a higher Democratic fraction in the four most Democratic districts or a lower Democratic fraction in the two most Republican districts, in comparison to the enacted plan. . In other words, this cluster shows more cracking and packing of Democrats than every single plan in the nonpartisan ensemble.

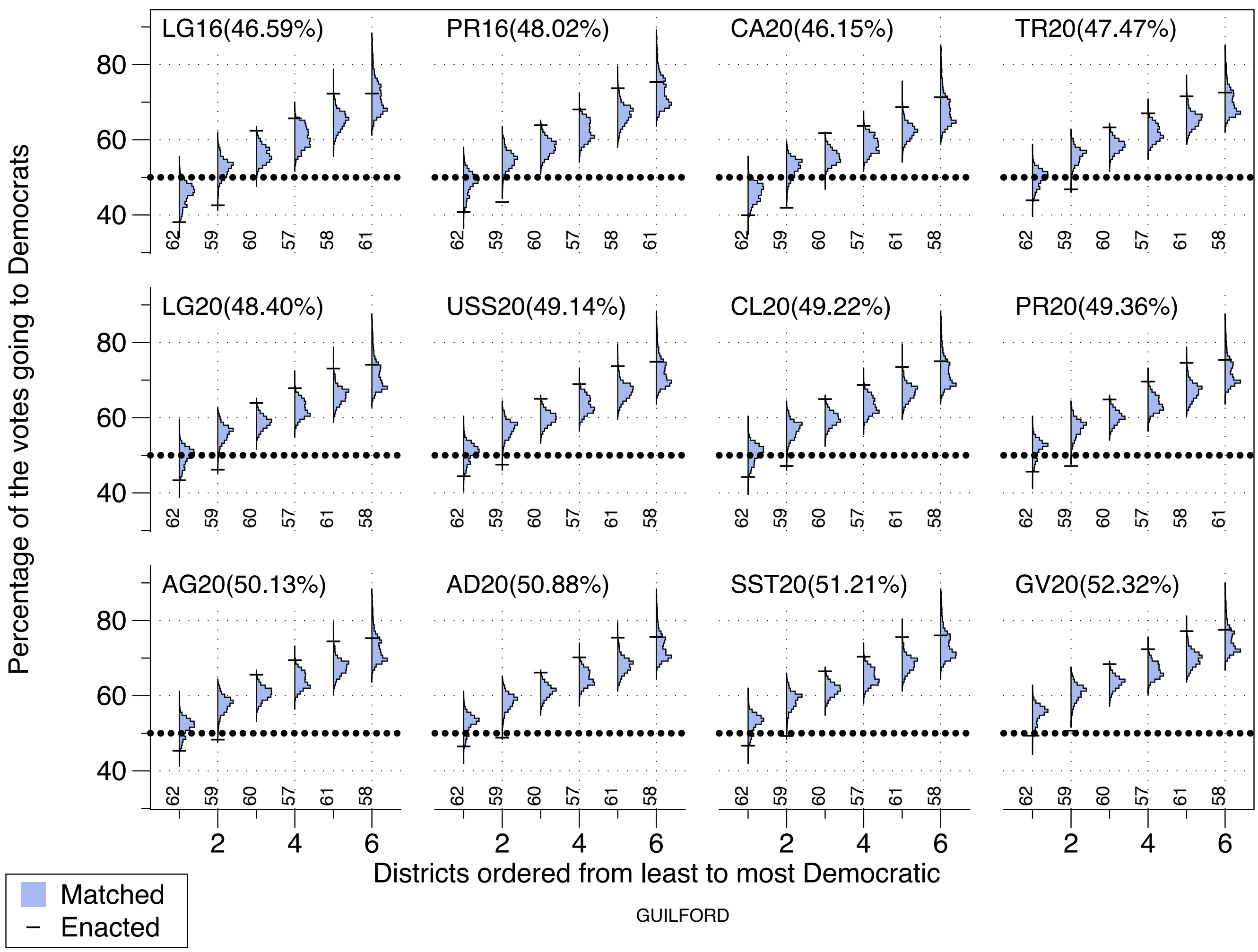


Figure 6.1.10: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

In Figure 6.1.11, we see the effect of considering the the ensemble that more strongly preserves municipalities than the enacted plan. The ensemble reliably has four democratic districts and a 5th which typically leans Republican but sometimes is competitive. Yet, the enacted plan gives one clearly Republican district and one which is often safely Republican and at times competitive.

Municipal Splits and Ousted Population: From Figure 6.1.12, we see that in Guilford the enacted plan ousts a number of people comparable to the primary ensemble but constantly more than the secondary ensemble.

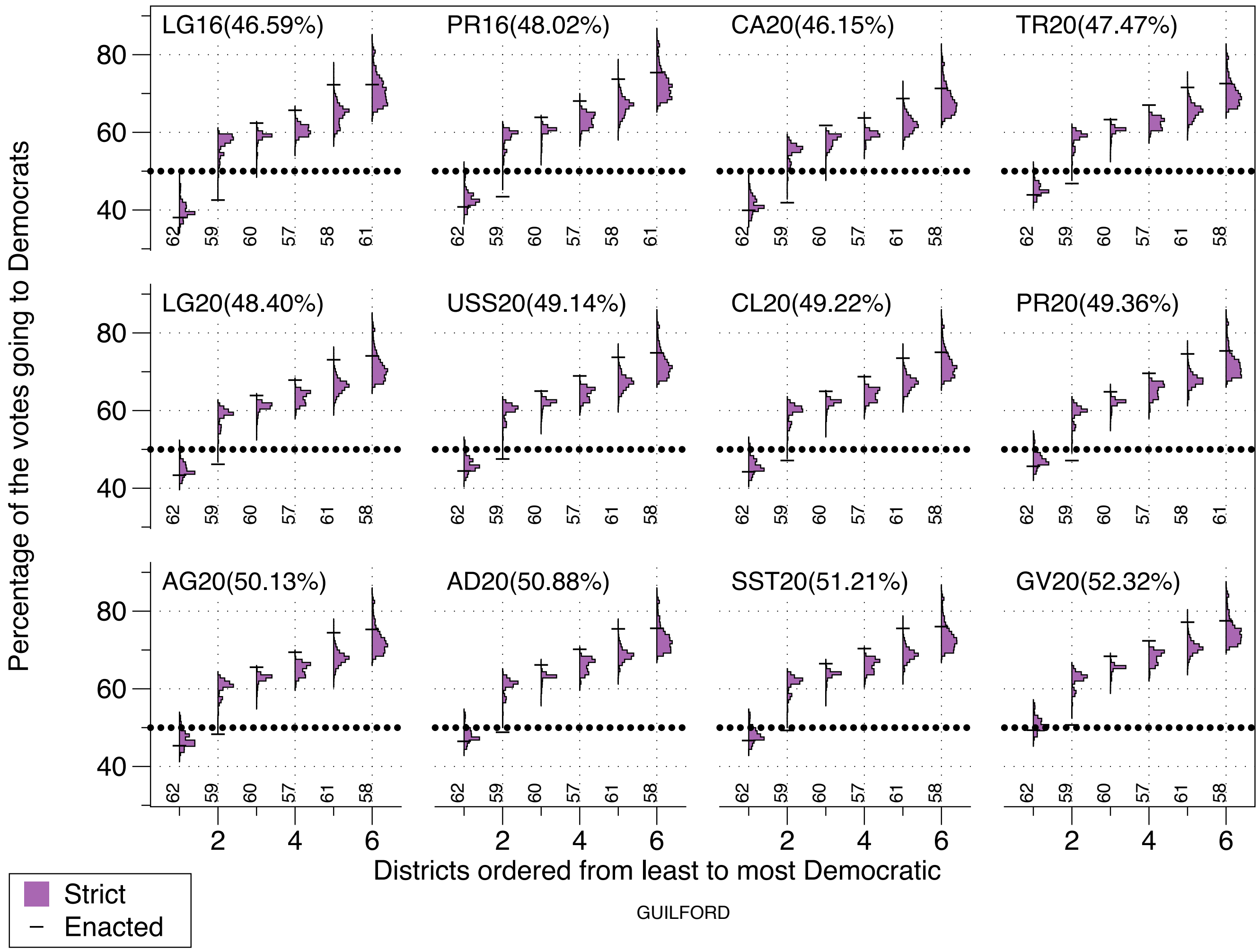


Figure 6.1.11: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The “ – ” on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the “ – ”. This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

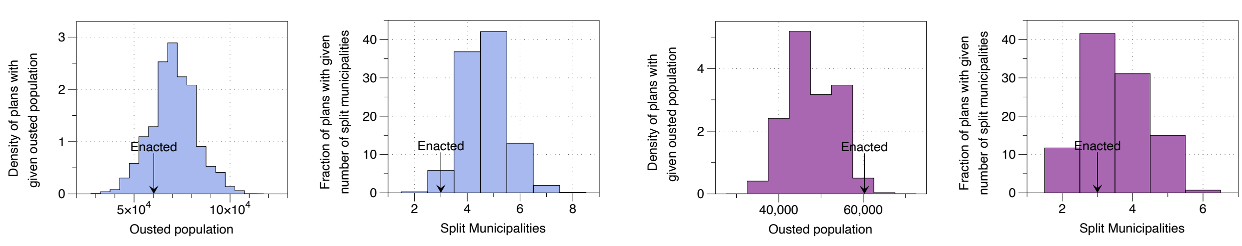


Figure 6.1.12: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.5 Buncombe

As seen in Figure 6.1.13, the primary ensemble shows two Democratic districts with a third typically leaning Democratic but sometimes in play. However, the enacted map produces one district which is typically Republican. This is achieved by packing unusually many Democratic in the most Democratic district (district 114) leaving abnormally few Democrats for the most Republican district (district 116).

Ranging over the elections considered, at most 1.2% of the plans in the ensemble have a lower democratic fraction in the most Republican district in the ensemble than the enacted plan does. The percentage of plans with a larger Democratic fraction in the most Democratic district in the ensemble fluctuates around 5%.

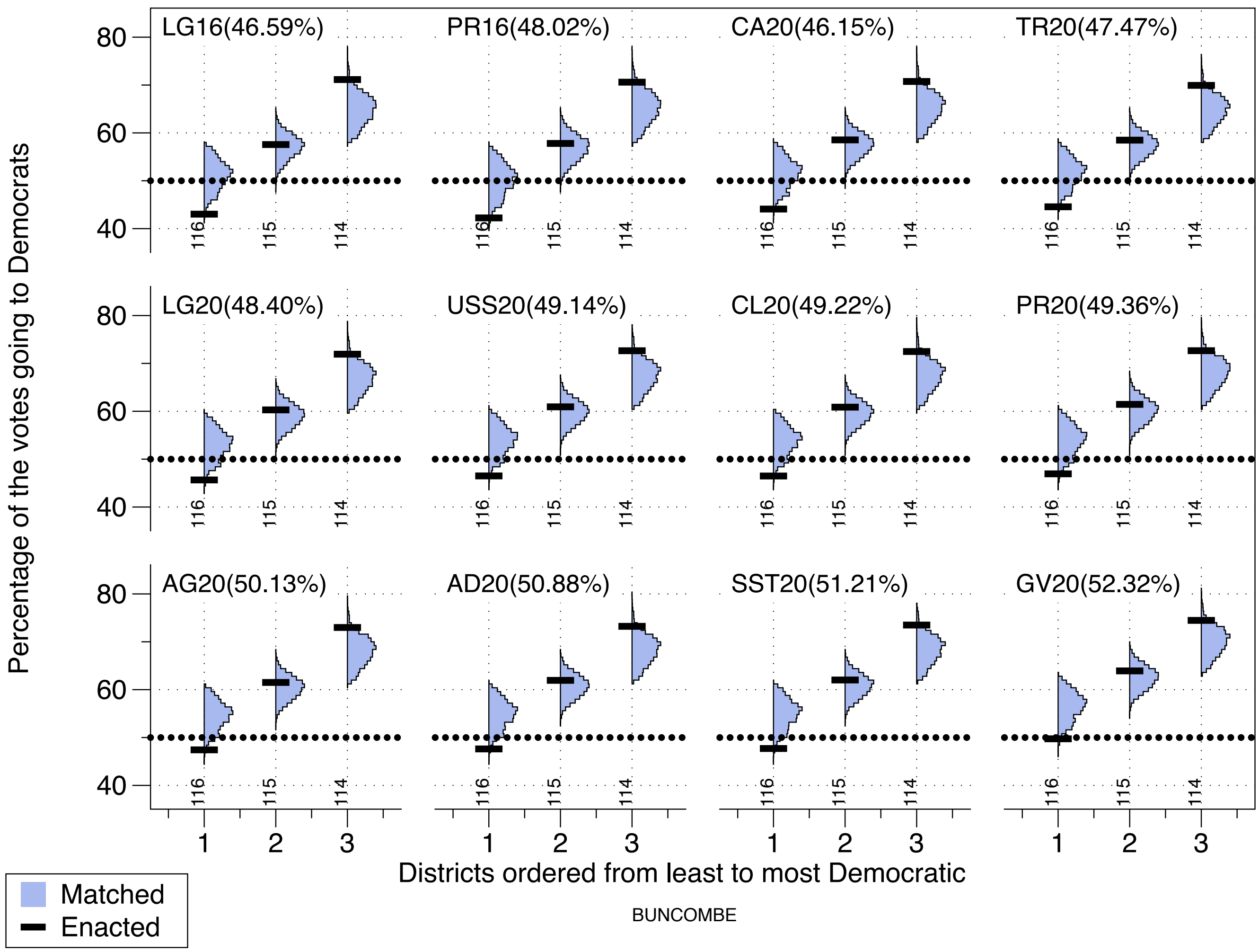


Figure 6.1.13: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

The same pattern of depleting Democrats from the most republican district so that it often elects a Republican when it typically would not under the ensemble is again seen in Figure 6.1.14 which shows the results under the secondary ensemble.

Municipal Splits and Ousted Population: From Figure 6.1.15, we see that there is not a lot of difference between the two ensembles in the number of ousted people. Both are comparable to the enacted map.

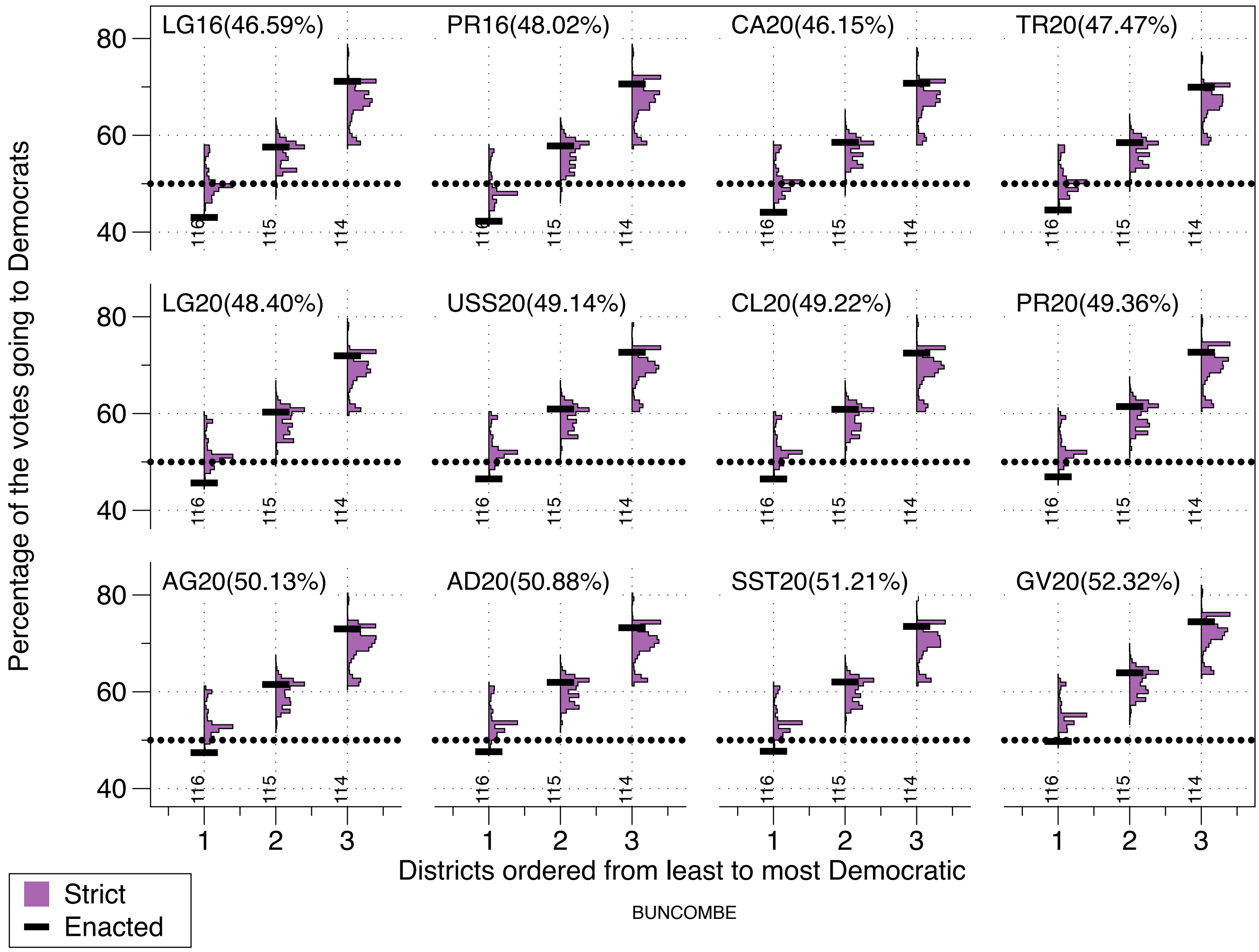


Figure 6.1.14: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

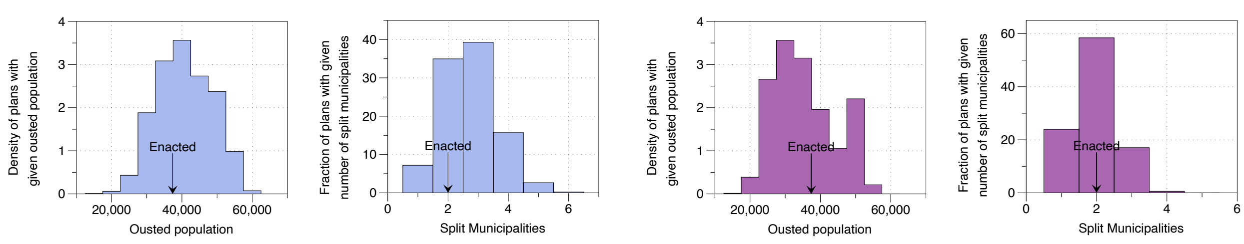


Figure 6.1.15: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.6 Pitt

Pitt County only has two districts. The enacted places atypically many Democrats in the most Democratic district (district 8) while placing atypically few in the most Republican district (district 9). This maximizes the chance that the second district will elect a republican. In many cases, it does when many of the ensemble maps would not. By maximizing the difference in the partisan makeup of the two districts, the enacted map minimized the degree to which the enacted map responds to the shifting opinions of the electorate.

Across the elections considered, the percentage of plans in the ensemble which have a higher fraction of Democrats in the most Democratic district than the enacted plan fluctuates between 1.1% and 5.3%.

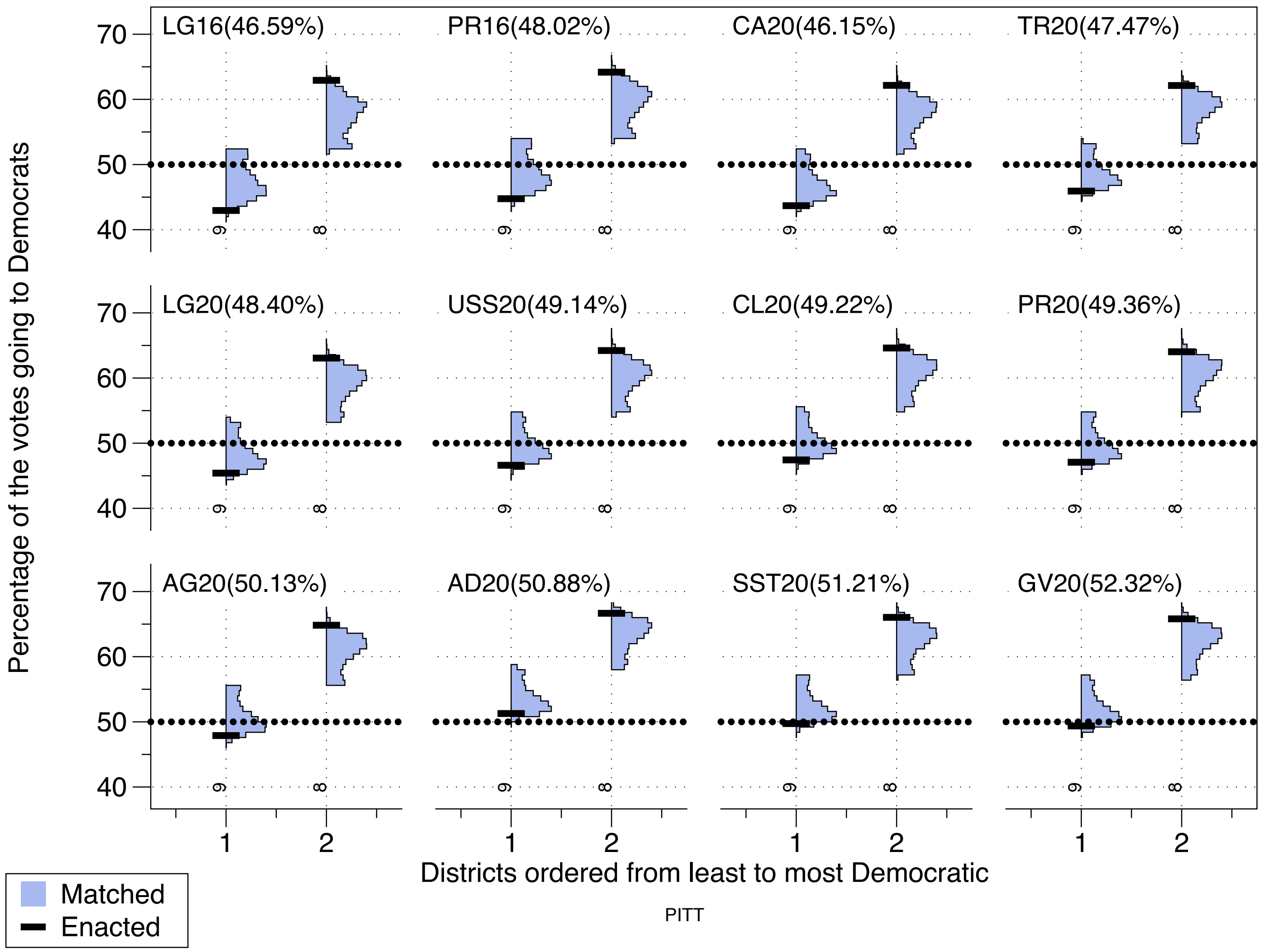


Figure 6.1.16: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

The same pattern is repeated in Figure 6.1.17 which uses the secondary ensemble which better preserves municipalities than the enacted map.

Municipal Splits and Ousted Population: From Figure 6.1.18, we the number of ousted people in the primary ensemble is comparable to the enacted plan but more than the secondary ensemble.

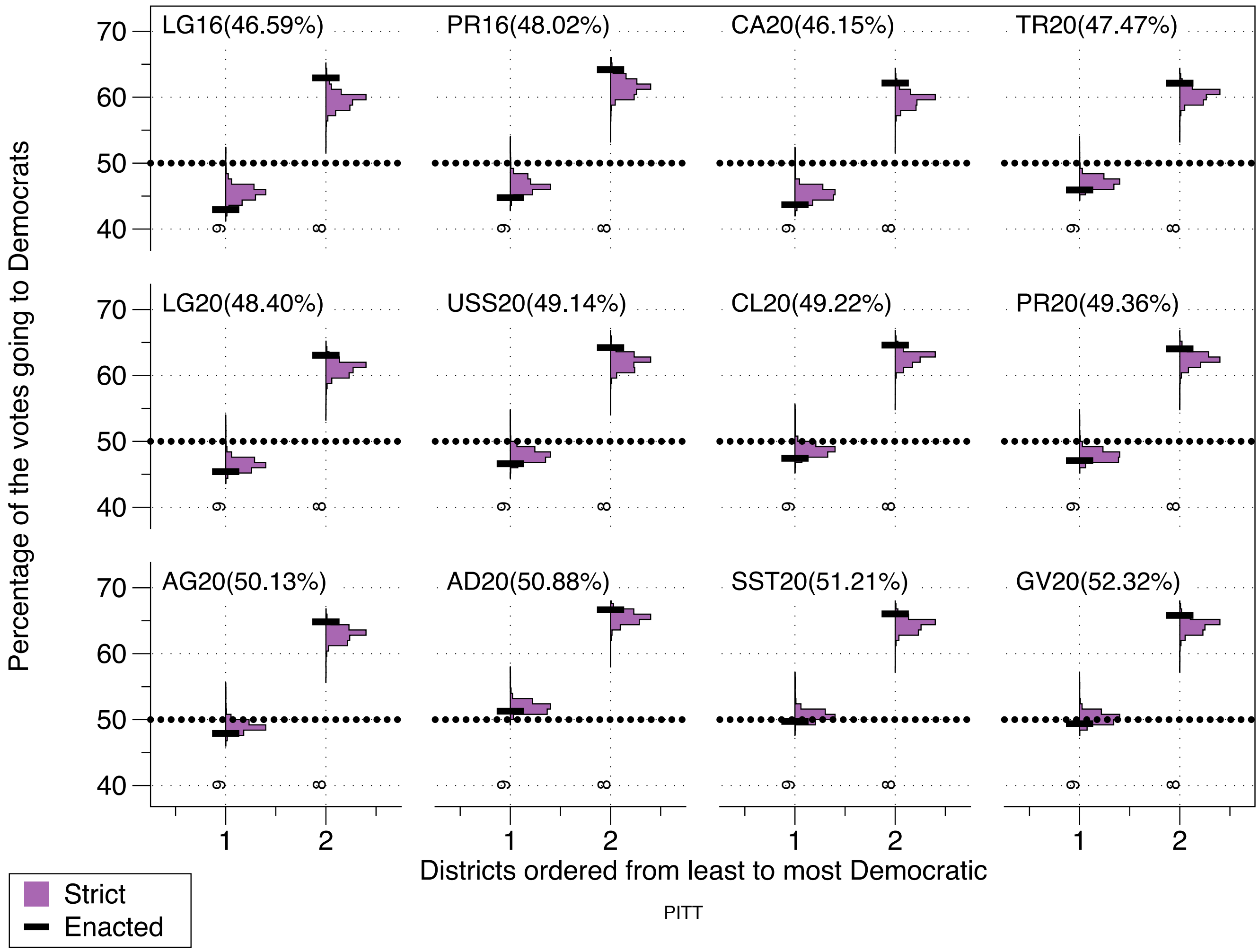


Figure 6.1.17: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

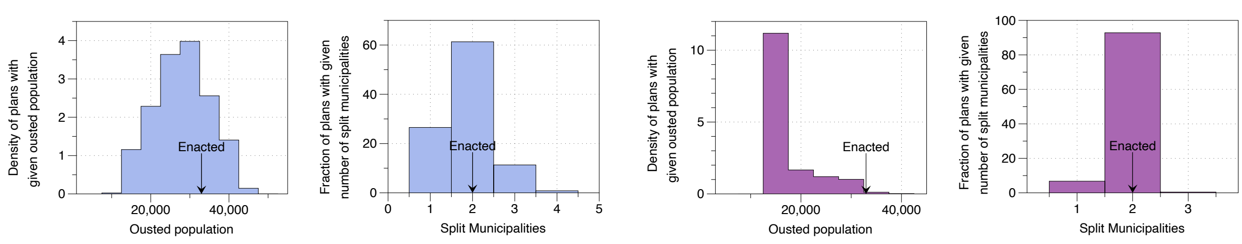


Figure 6.1.18: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.7 Duplin-Wayne

In the Duplin-Wayne county cluster the two districts are safely Republican under the elections considered. The enacted map is typical, falling in the middle of the observed democratic fraction on the Histograms.

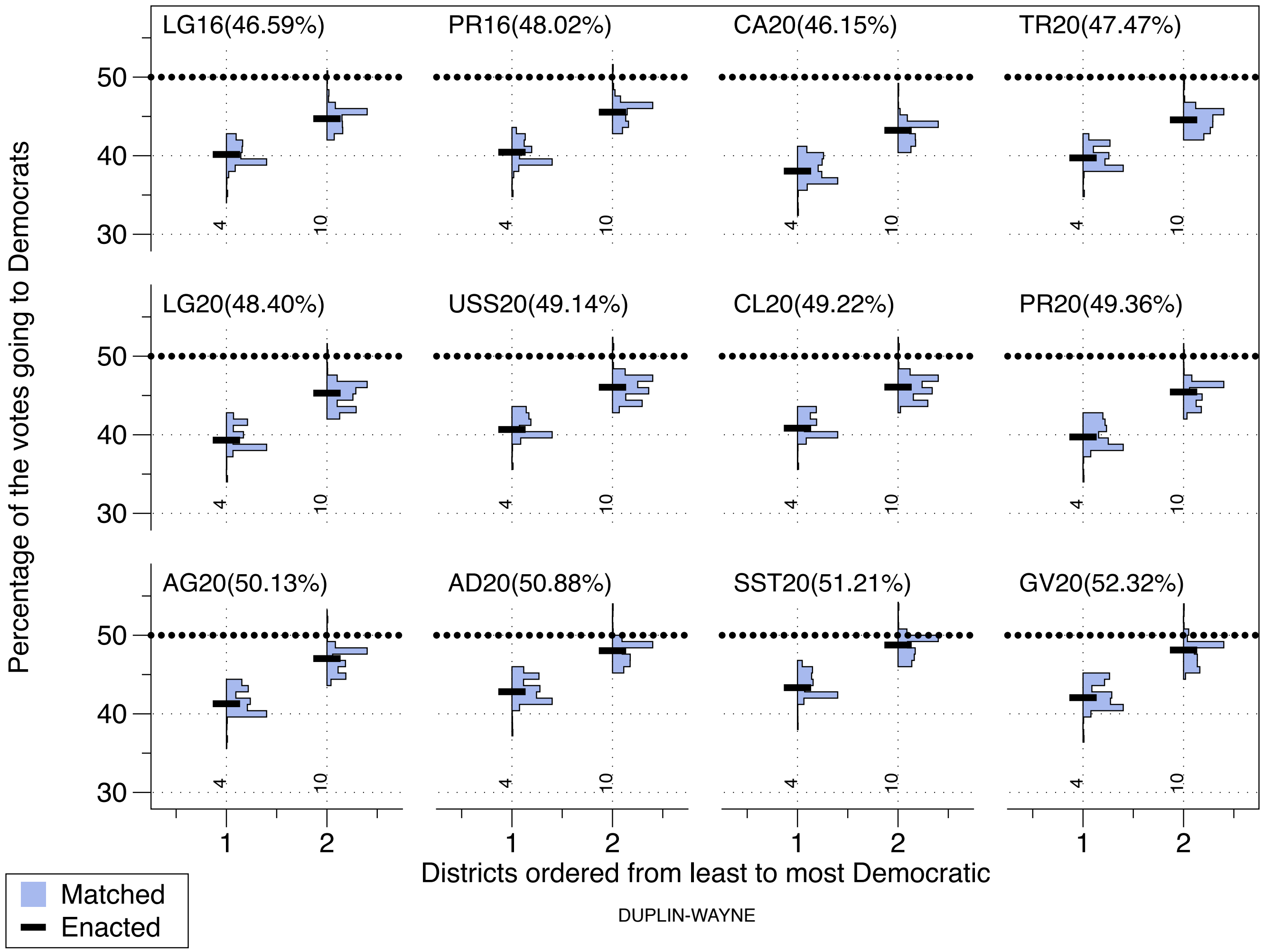


Figure 6.1.19: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

As seen in Figure 6.1.20, the distribution has extremely small variance when municipalities are better preserved. Here there seem to be a little less Democrats in the most Democratic district than typical, but this has little effect as the two districts are firmly Republican and the distribution is highly concentrated.

Municipal Splits and Ousted Population: From Figure 6.1.21, we seen that the number of people ousted by the enacted plan is at the lower end of the typical amounts seen in the Primary ensemble or the secondary ensemble.

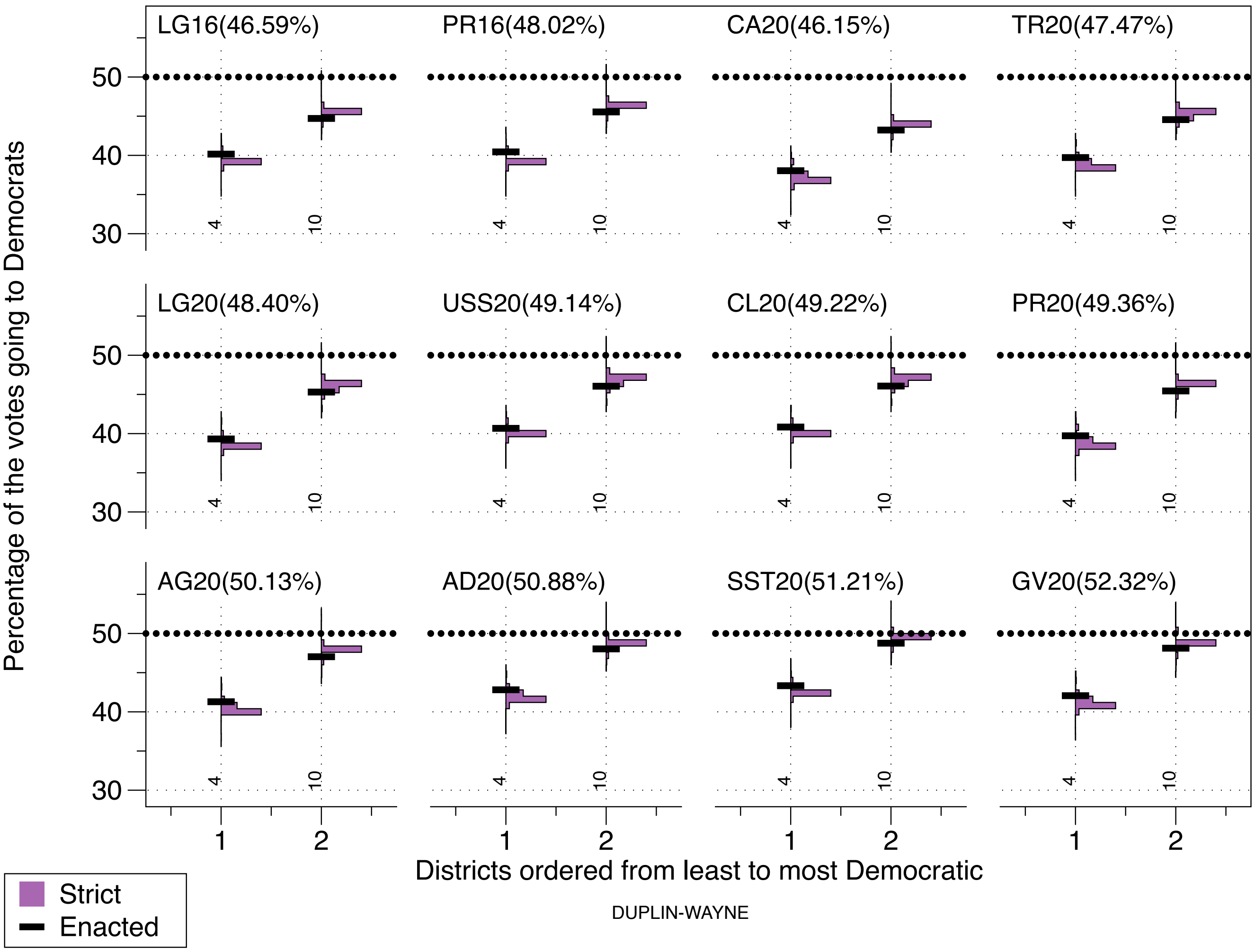


Figure 6.1.20: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

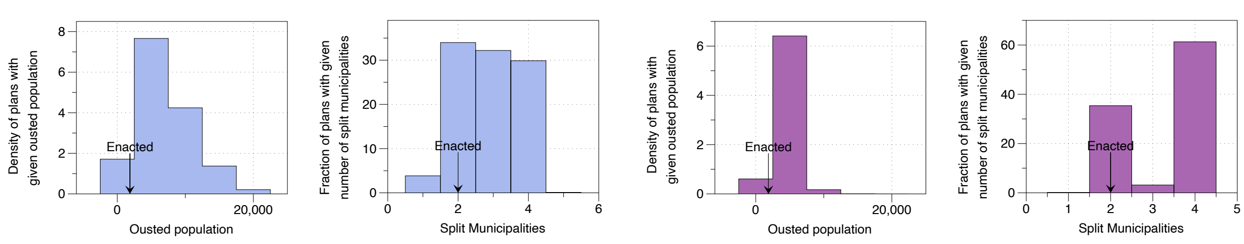


Figure 6.1.21: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.8 Durham-Person

As seen in Figure 6.1.22, under the primary ensemble Durham-Person cluster typically has three exceedingly Democratic districts and one more moderately Democratic district. The enacted plan places abnormally few Democrats in the most Republican district (district 2). This is accomplished by packing more Democrats in the most Democratic districts (districts 29 and 30). The effect is sufficient to pick up a Republican seat in a few elections where the seat typically would have remained democratic according to the non-partisan primary ensemble.

Not a single map in the non-partisan ensemble across any of the elections considered has a smaller fraction of Democrats in the most Republican district than the enacted plan does. This signals extreme cracking. In all but two elections the fraction of plans which have a higher Democratic vote fraction than the enacted plan is less than 0.62%. The two exceptions are LG16 (3.5%) and CA20 (1.2%).

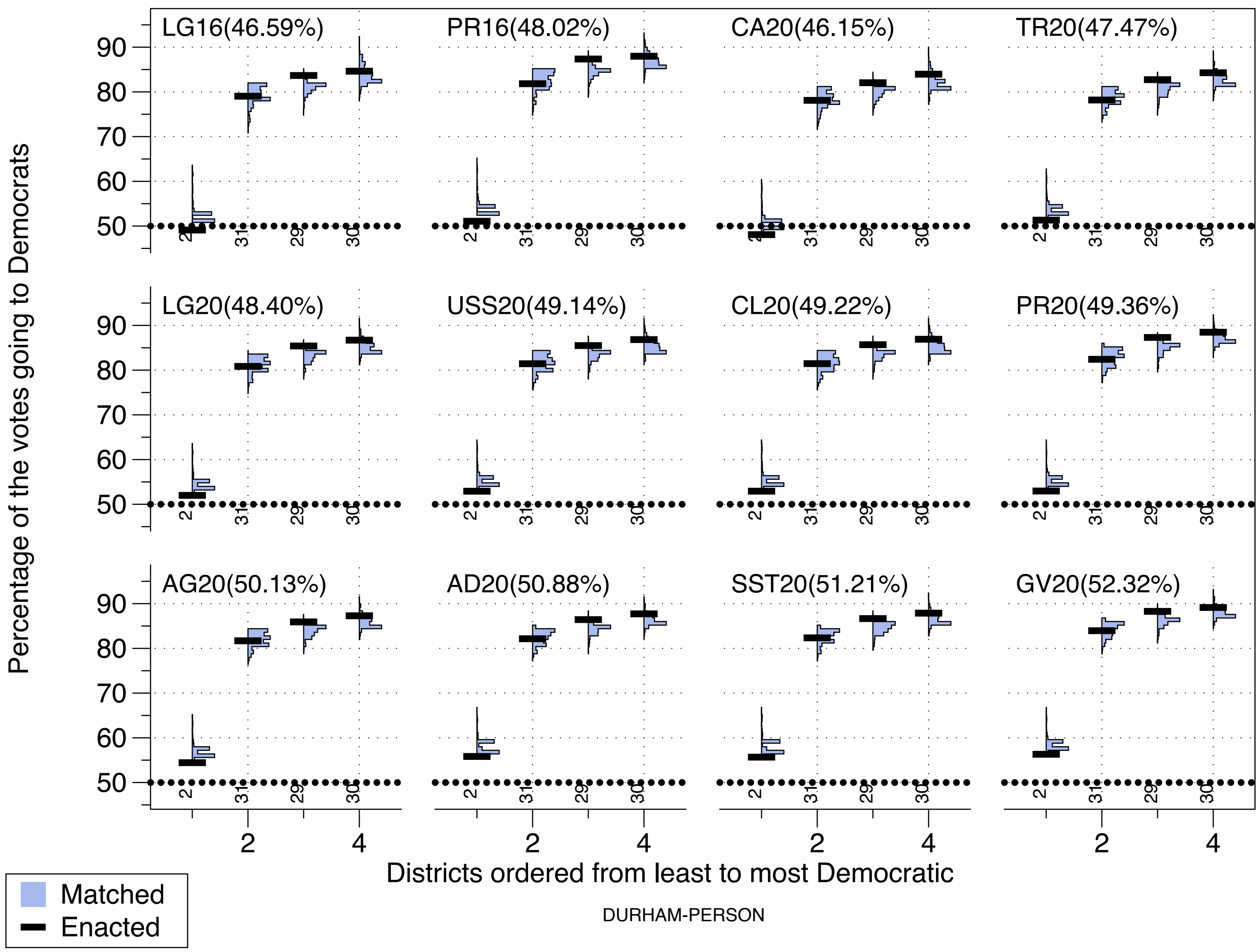


Figure 6.1.22: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

A similar effect is seen in 6.1.23, for the ensemble which better preserves municipalities.

Municipal Splits and Ousted Population:

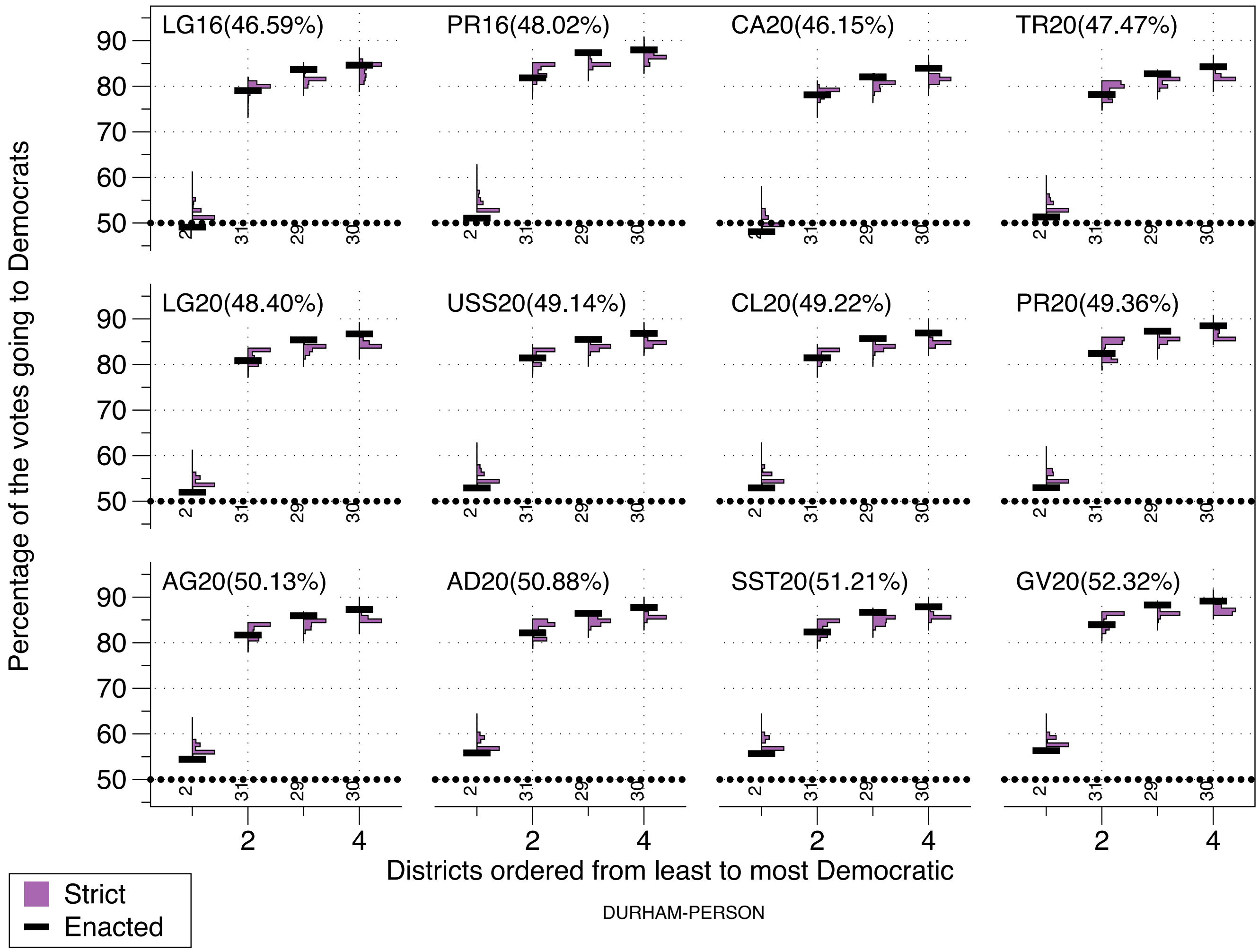


Figure 6.1.23: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

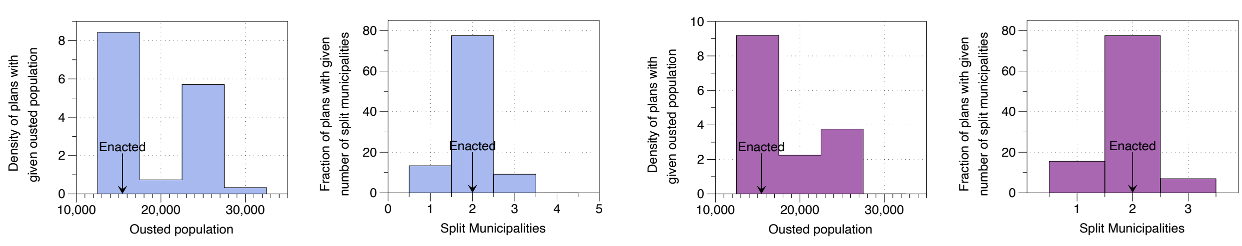


Figure 6.1.24: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.9 Alamance

From Figure 6.1.25, we see that though the enacted map tends have more Democrats in the more Democratic district and less in the less democratic district it not an outlier on its own.

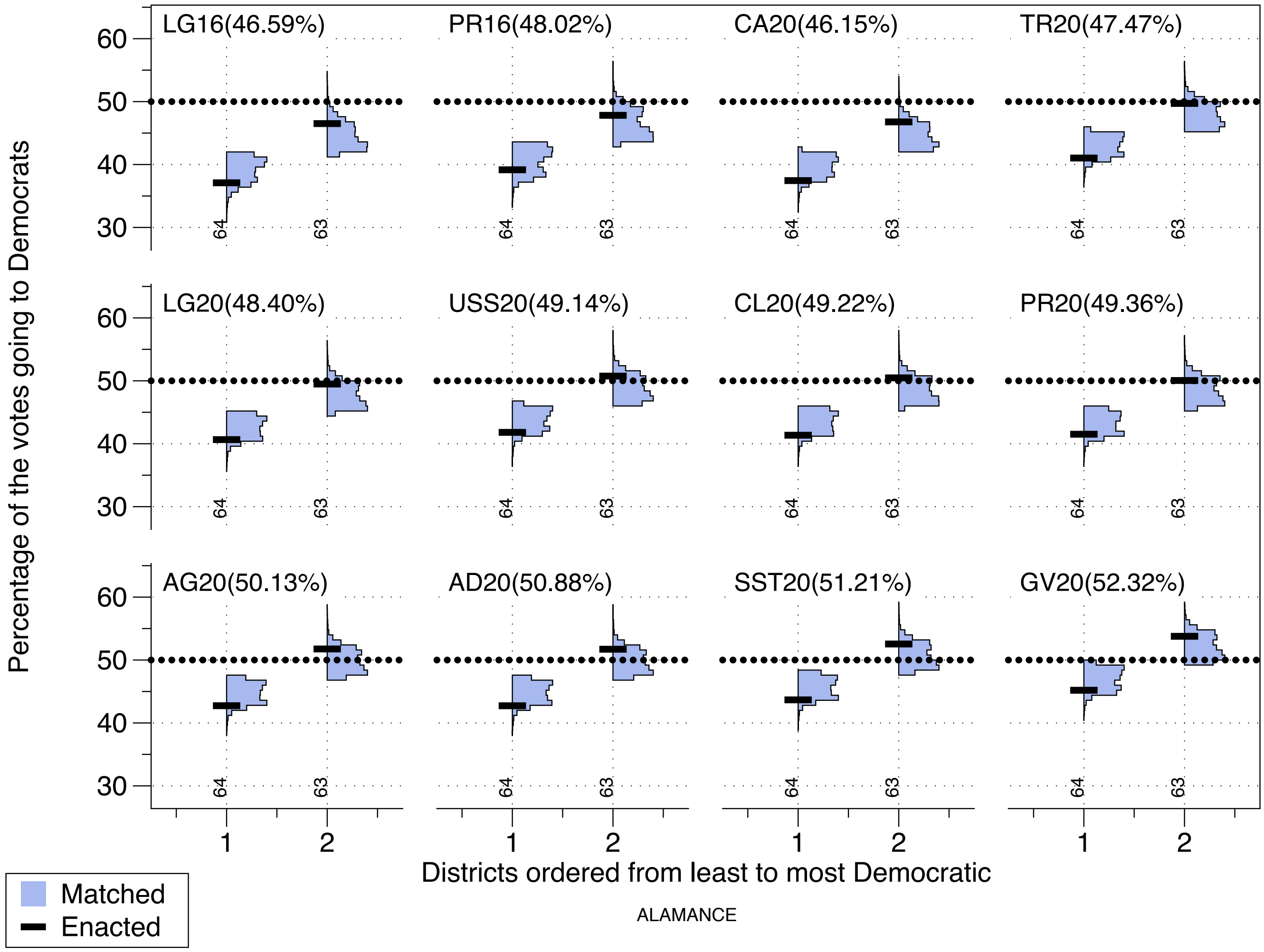


Figure 6.1.25: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

Figure 6.1.26 tells a similar story to Figure 6.1.25,

Municipal Splits and Ousted Population:

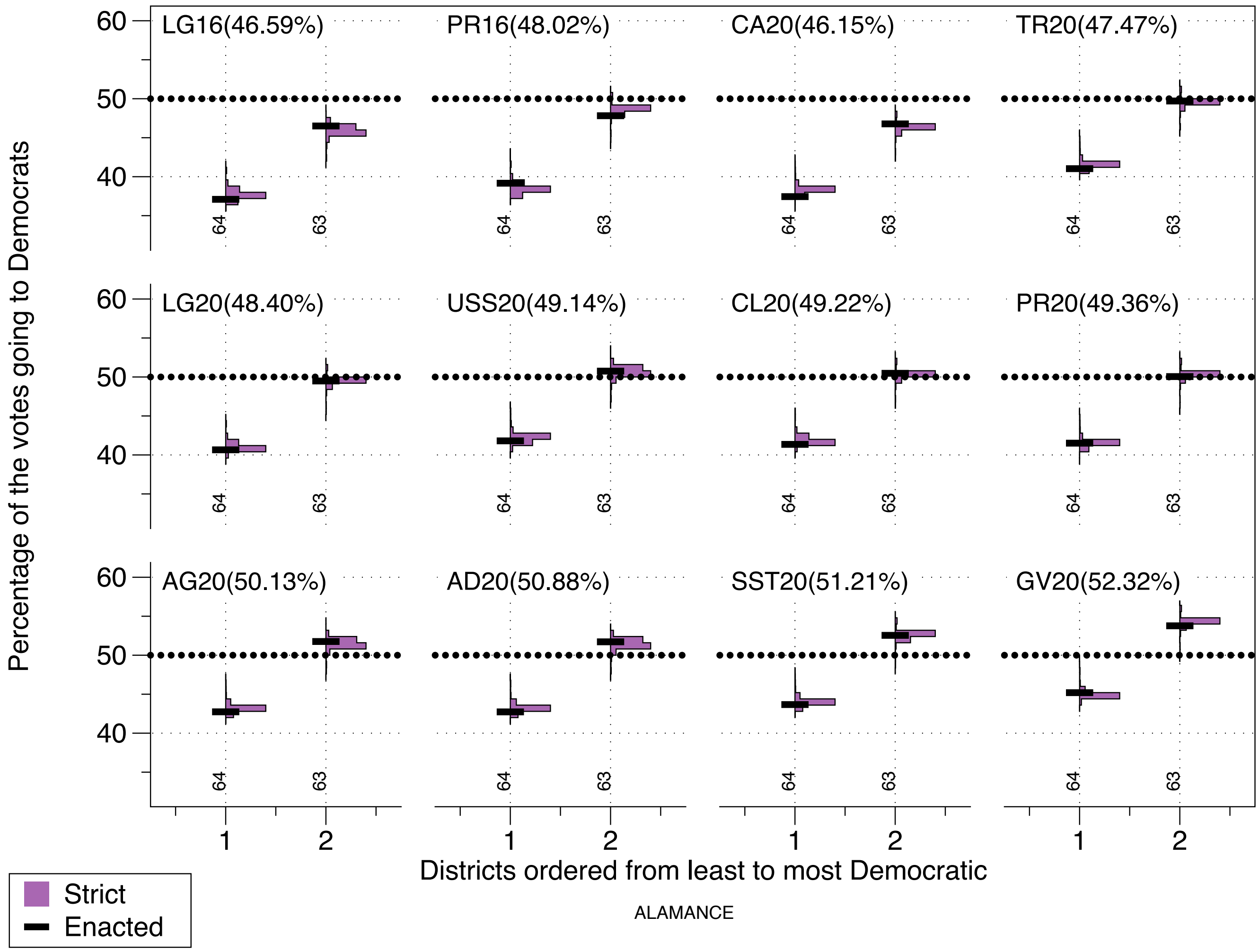


Figure 6.1.26: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The “ – ” on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the “ – ”. This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

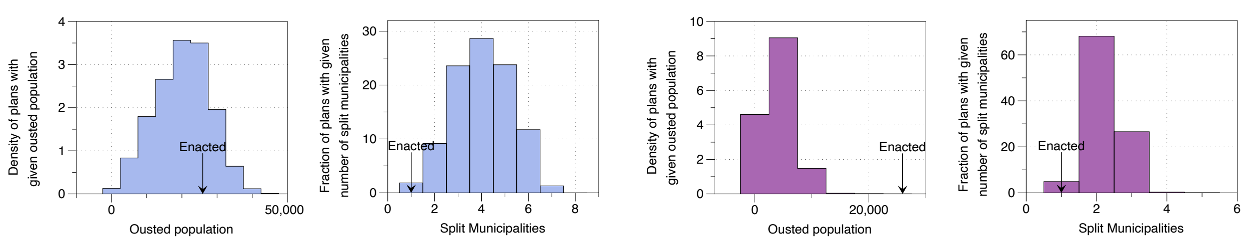


Figure 6.1.27: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.10 Cumberland

Looking at Figure 6.1.28, we again see outlier behavior in Cumberland County. We see that the districts in the enacted plan have been constructed so that the two most Republican districts (district 43 and 45) have a similar partisan makeup. Typically, one is more Democratic and one is more Republican. This is achieved by removing republicans from the most republican district and Democrats from the most democratic two districts. While the effect on the most Republican district individually is within the typical range, the combined effect creates an enacted cluster which is an strong outlier.

For each of the elections considered, the number of plans in the ensemble with smaller fraction of democrats in the second most republican district is typically around 1% with, for a few elections, the percentage reaching as high as 7% or as low as 0.4%.

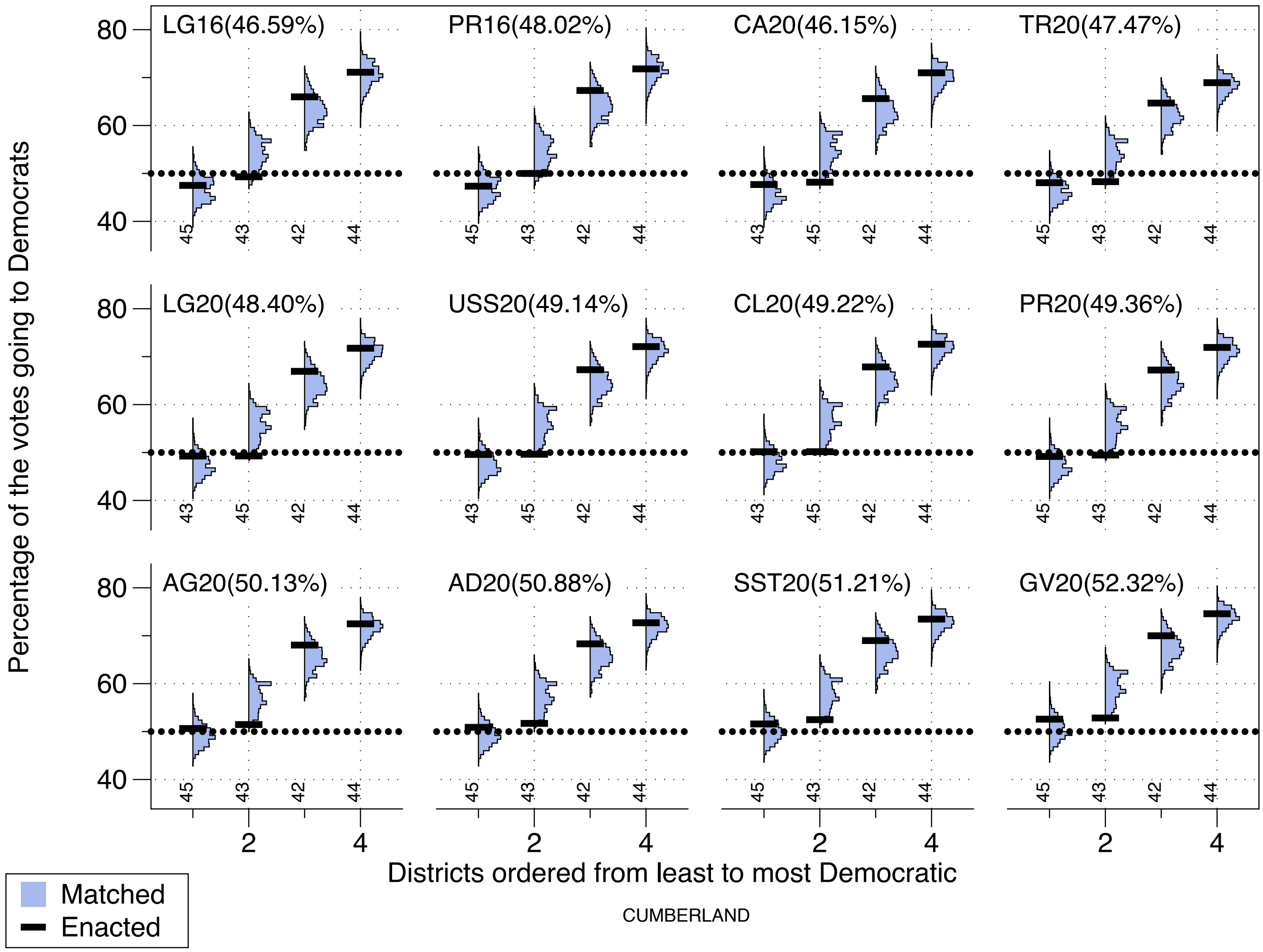


Figure 6.1.28: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

Looking at Figure 6.1.29, we see that the structure of the enacted map is a more extreme outlier for the secondary ensemble which better preserves municipalities. In an ensemble that better preserves municipalities, the most Republican district is typically more republican and the second most Republican district more Democratic. This makes the enacted plan which squeezes the two together with an large outlier.

Municipal Splits and Ousted Population:

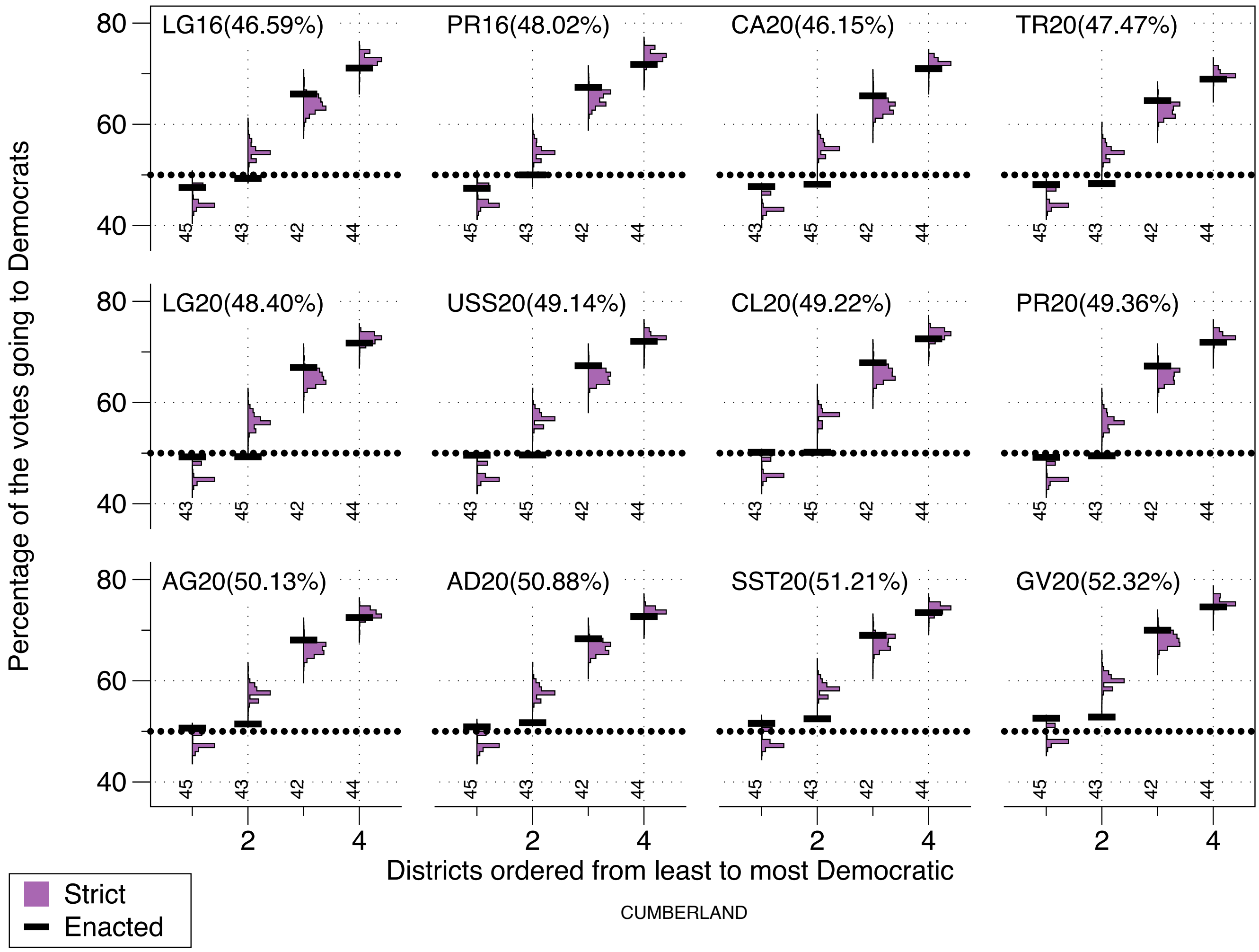


Figure 6.1.29: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

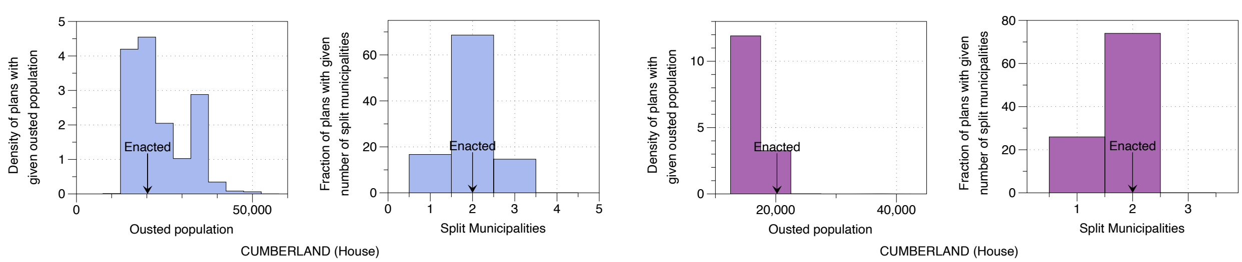


Figure 6.1.30: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.11 Cabarrus-Davie-Rowan-Yadkin

In the Cabarrus-Davie-Rowan-Yadkin county cluster, there are abnormally few Democrats in the most Democratic district (district 82). This is accomplished by placing abnormally many Democrats in the next three most democratic districts (districts 73, 76, and 83 – all of which are safe Republican districts). The effect is to make the most Democratic district a relatively reliable Republican seat (being won by the Republicans in all of the elections considered). Under the ensemble, it would switch parties in a number of the elections and regularly be a close contest.

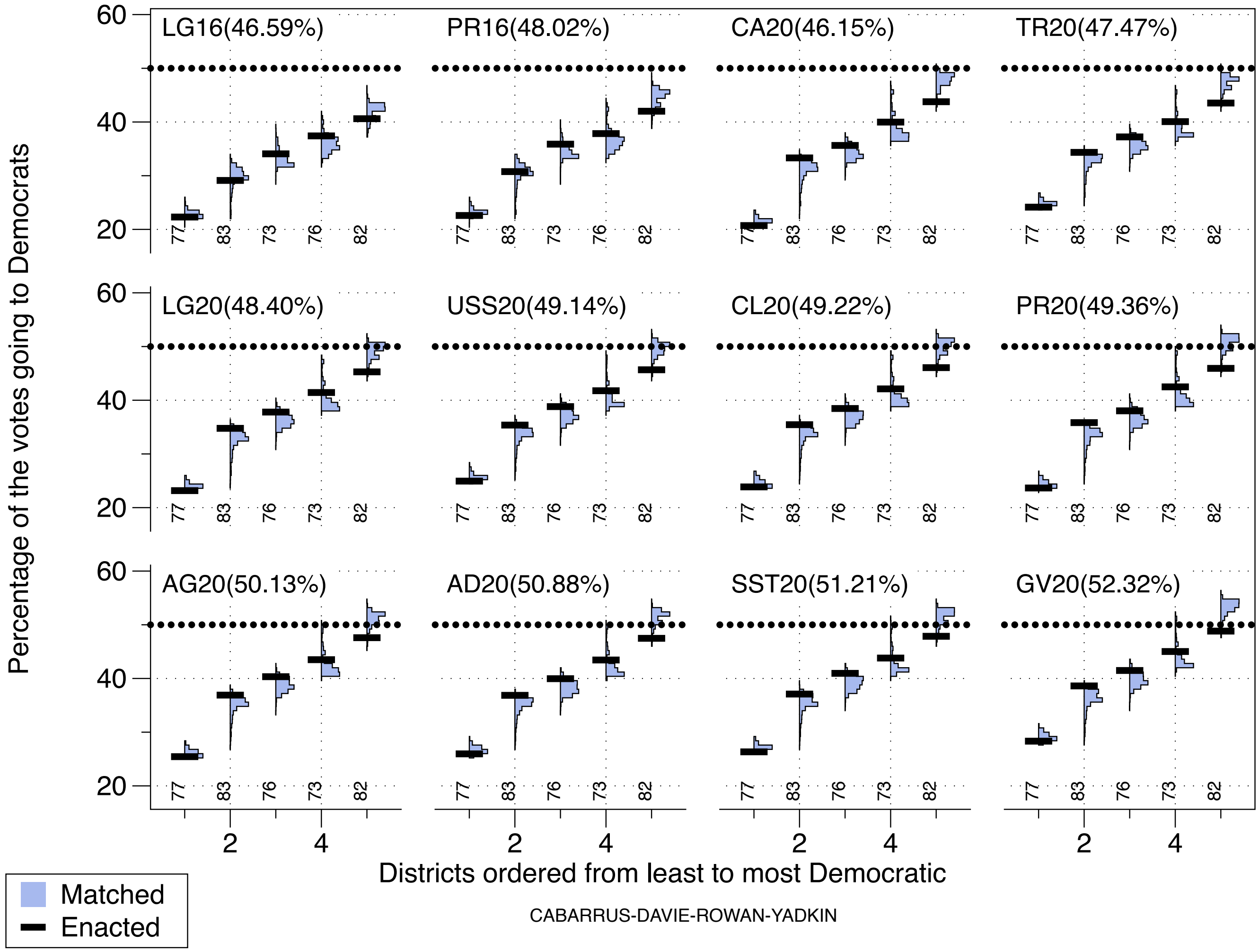


Figure 6.1.31: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

Looking at Figure 6.1.32, we see that the same pattern persists under the secondary ensemble which better preserves municipalities.

Municipal Splits and Ousted Population:

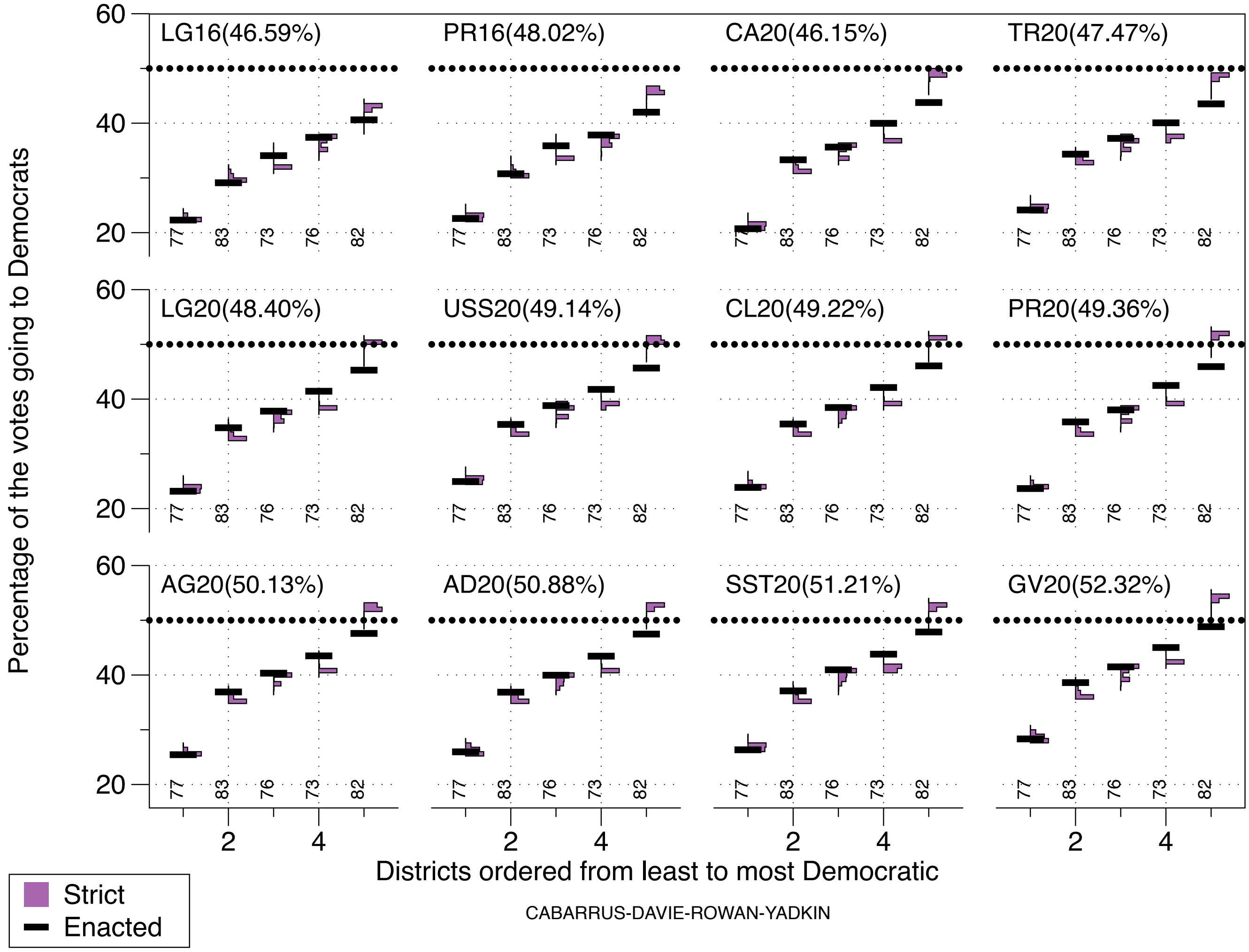


Figure 6.1.32: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

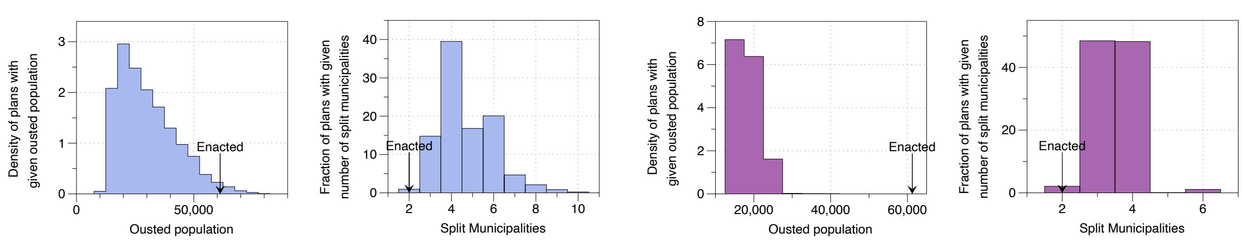


Figure 6.1.33: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.1.12 Brunswick-New Hanover

In the Brunswick-New Hanover county cluster, Figure 6.1.34 shows that the most Democratic district (district 18) has had abnormally many Democrats packed into it and the most Republican has had abnormally few Republicans placed in it, while the second-most Democratic district (district 20) has been depleted of Democrats. This makes the enacted plan much less responsive to changes in the the enacted plan preferences of the voters. The Republican party typically wins the second most democratic district in the enacted plan even though it would go to the Democrats under a number of elections when the neutral maps in the primary ensemble are used. Over each of the elections considered, the fraction of plans in the ensemble

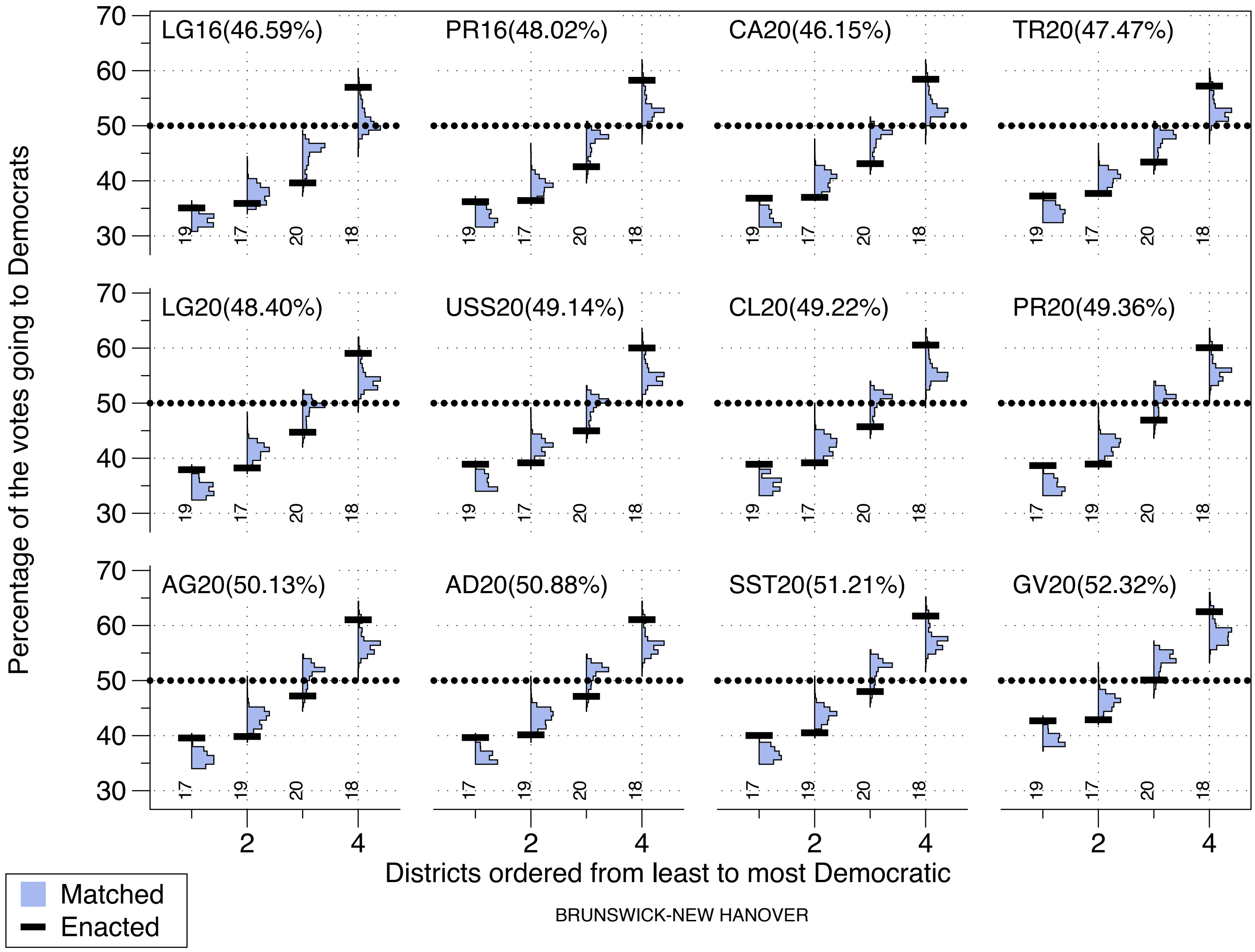


Figure 6.1.34: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

when a lower Democratic vote fraction in the second and third most Republican districts in the ensemble compared to the enacted plan map is always less than 0.5% and often much smaller.

Under the secondary ensemble which better preserves municipalities shown in Figure 6.1.35, we see that the same structure persists. The enacted map becomes a more extreme outlier since this ensemble reduced the variance of the marginals and aligns the outcome gradual progression which ensures the map is fairly responsive to changes in the voter's preference, a property not shared by the enacted map.

Municipal Splits and Ousted Population:

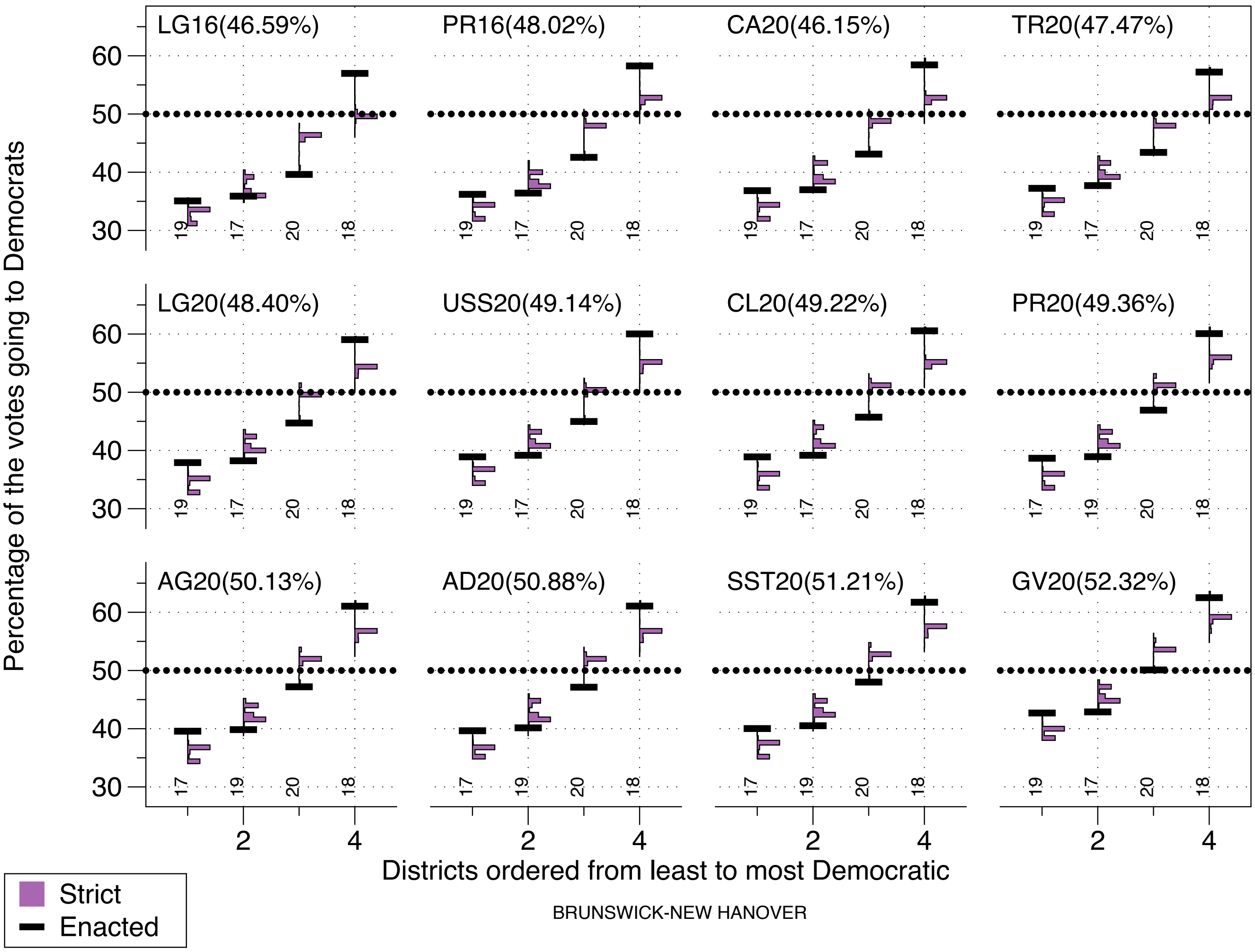


Figure 6.1.35: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Secondary ensemble which better preserves municipalities than the enacted plan.

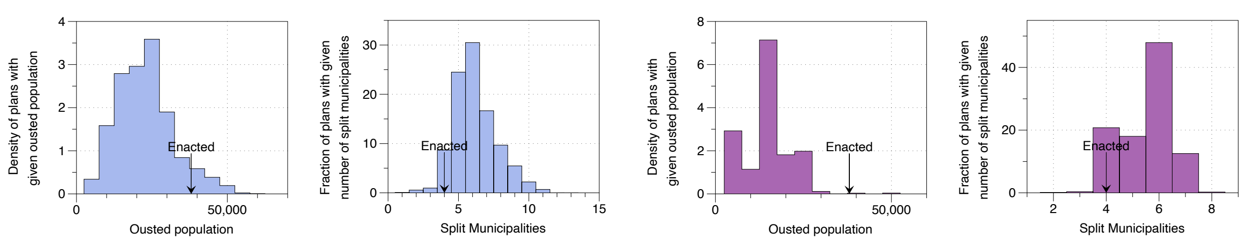


Figure 6.1.36: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2 NC State Senate

Though the principal Senate ensemble, which prioritizes municipality preservation in line with the enacted plan, does not have as dramatic a shift towards the Republicans at the statewide level in comparison to the House, we still see a number of cases of extreme packing and cracking at the individual cluster level. Without exceptions, the effect is to minimize the effect of the Democratic votes and make the outcome of the election insensitive to a wide range of swings in the partisan vote fraction.

In the NC Senate, we again see the effect of prioritizing municipal preservation in our ensemble. When municipal preservation was not prioritized, there are two major effects. First, the enacted maps become extreme outliers, as the typical results swings are much less tilted to the Republican Party. Second, the two parties are much less separated. Requiring a high level of municipal preservation often leads the separation of the two political parties between disjoint districts. This in turn produces maps that are much less responsive to swinging public opinion. In other words, the results of the elections do not change over a wider range of statewide vote ranges.

6.2.1 Iredell-Mecklenburg

In this cluster, the second most Republican district (District 41 in the enacted plan) is the principal district whose outcome varies from election to election. In the enacted plan, unusually few democrats have been placed in this district to maximize the chance that the district elects a Republican. See Figure 6.2.1. In many elections, this means that the Republican wins this district under the enacted plan, whereas a Democrat would win the district under the a majority of ensemble plans.

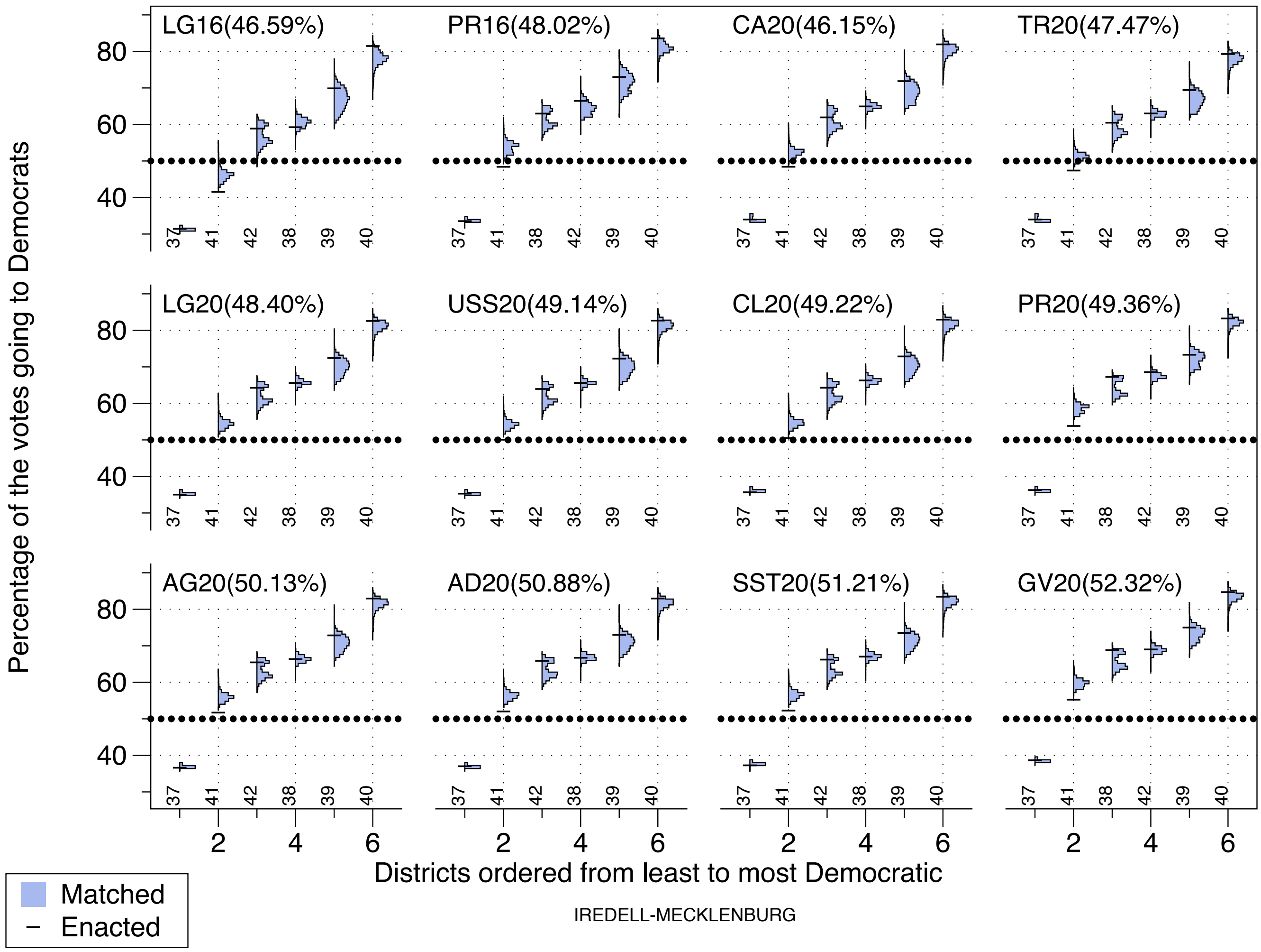


Figure 6.2.1: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

For each of the 2020 and 2016 elections we have consider, we found that none of approximately 80,000 plans in our ensemble had as low a fraction of Democrats in the two most Republican districts in the Iredell-Mecklenburg cluster as the enacted plan. Similarly, in the vast majority of the elections the ensemble had no plans with a higher fraction of democrats packed in the four most Democratic districts. In two elections 0.01% of the plans had a higher fraction of Democrats packed in the four most Democratic districts.

The effect discussed above is essentially the same when the municipality preservation is not prioritized. See Figure 6.2.2.

Municipal Splits and Ousted Population:

We see that in the Iredell-Mecklenburg cluster, the number of ousted people in the enacted plan is comparable the number of ousted people in the ensemble prioritizing municipalities. The enacted plan splits two municipalities which coincides with the most typical number split by the ensemble prioritizing municipalities. Though this ensemble sometimes splits a number more municipalities, it typically displaces a comparable number of people to the enacted plan.

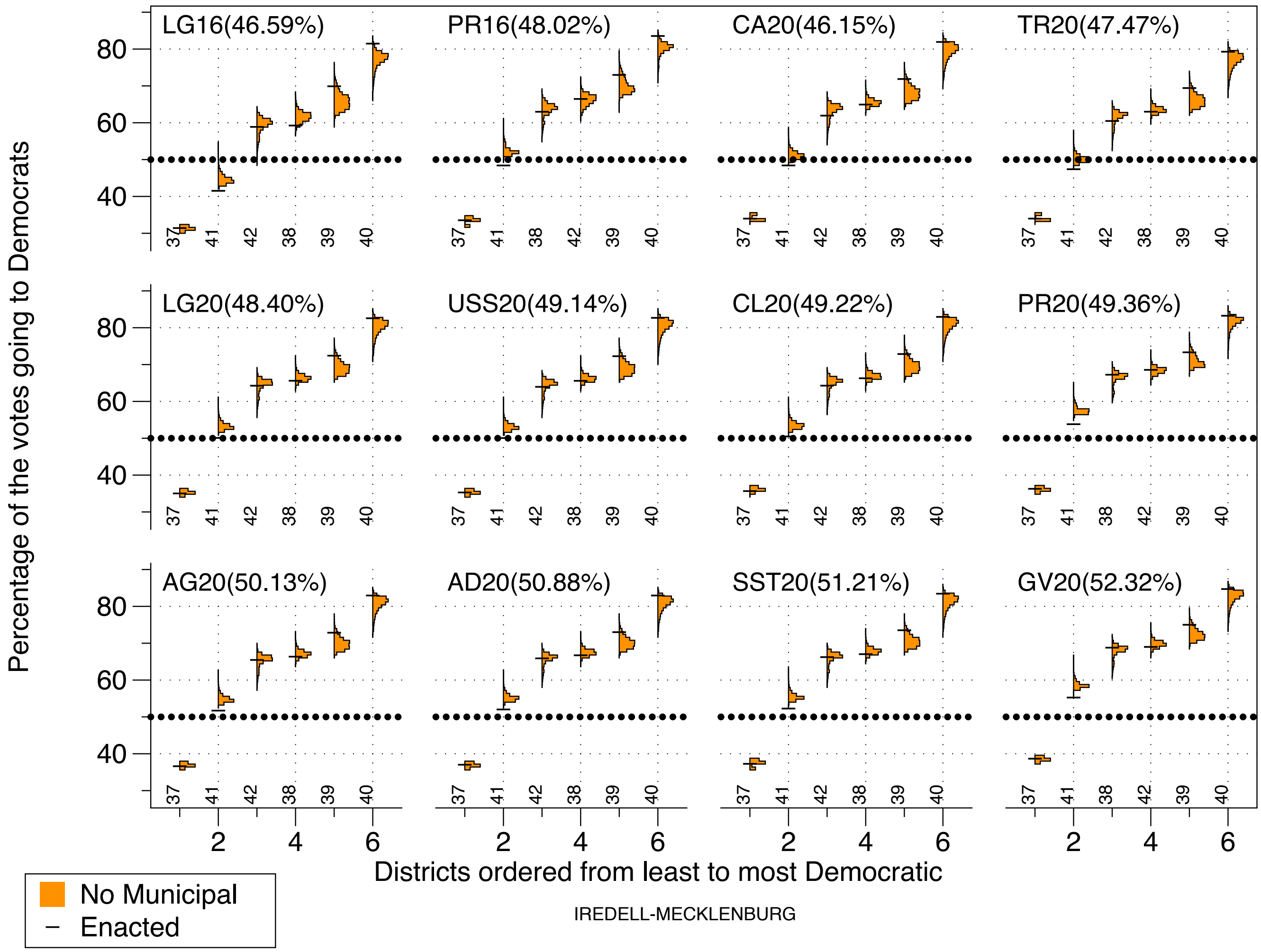


Figure 6.2.2: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the NC Senate Secondary ensemble which does not explicitly preserves municipalities.

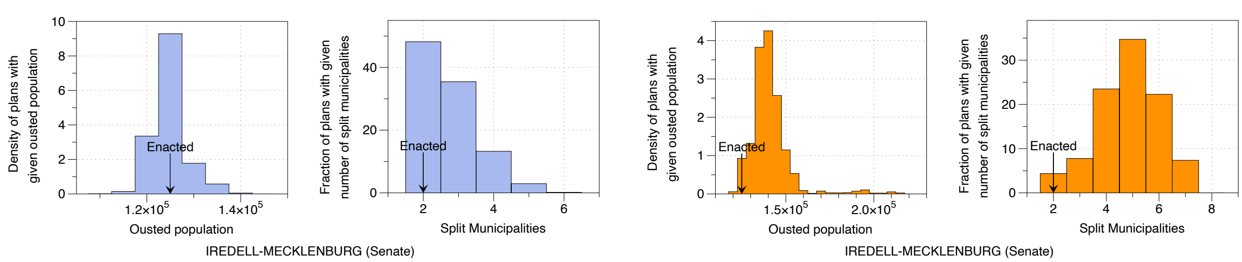


Figure 6.2.3: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2.2 Granville-Wake

The enacted plan is chosen to be at the extreme edge of the ensemble. It maximizes the chance of the Republicans winning Districts 17 and 18 by packing a larger than typical number of Democrats in districts 14, 15, 16, and 18. The effect is shown in Figure 6.2.4 across the 12 elections. For each of the 2020 and 2016 elections we have consider, we found that none of

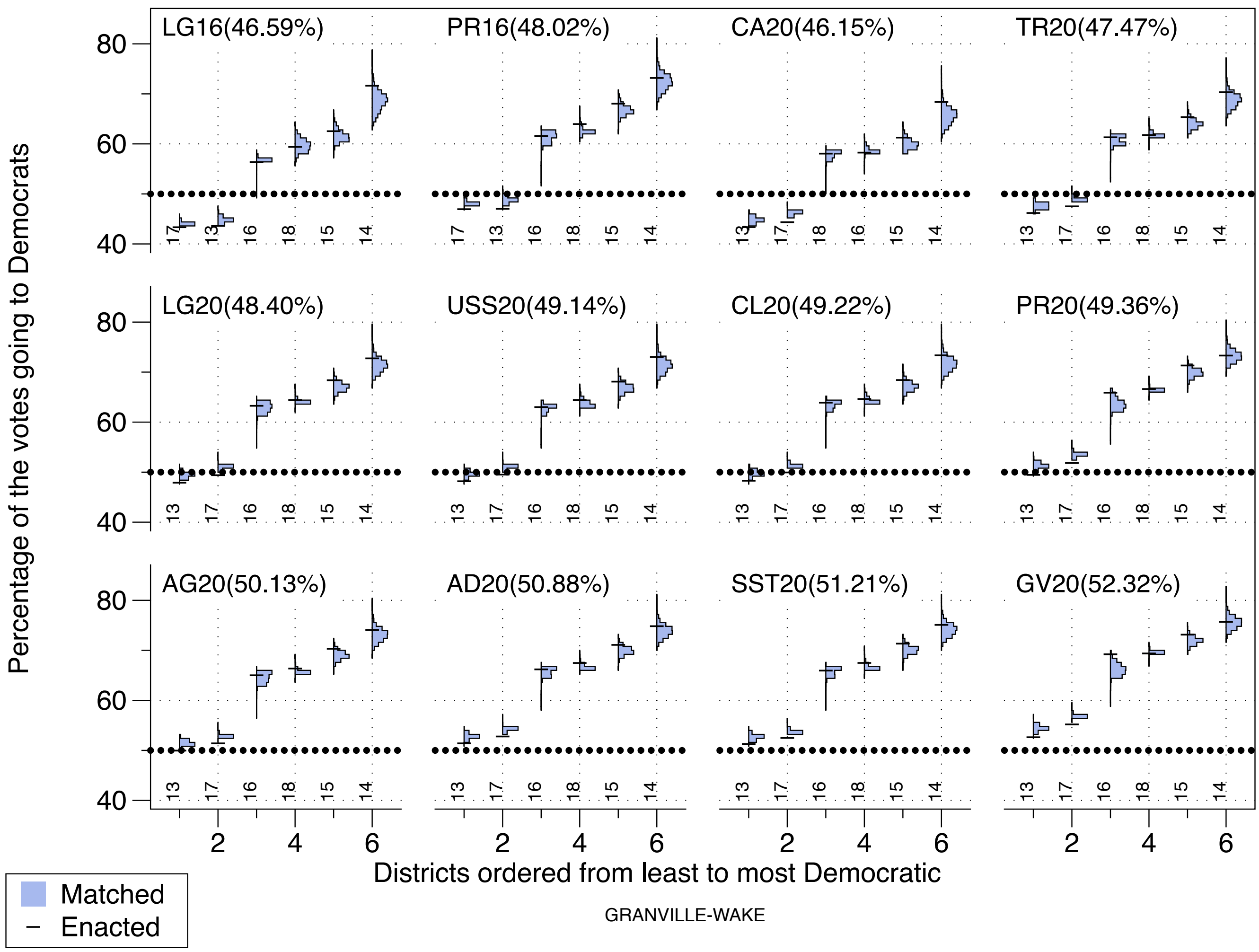


Figure 6.2.4: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

approximately 40,000 plans in our ensemble had as low a fraction of Democrats in the two most Republican districts in the Granville-Wake cluster as the enacted plan. Similarly, in six of the elections, the ensemble has no plans with more democrats packed in the four most Democratic districts. In six elections at most 0.022% of the plans had a higher fraction of Democrats packed in the four most Democratic districts than the enacted plan.

In this cluster, the prioritization of municipal preservation has a dramatic effect of packing Democrats in four districts and Republicans into two districts. The effect is show in Figure 6.2.5 across the 12 elections.

Municipal Splits and Ousted Population:

We see that in the Granville-Wake cluster, the number of ousted people in the enacted plan is significantly more than the number of ousted people in the ensemble prioritizing municipalities. The enacted plan splits three municipalities which coincides with the most typical number split by the ensemble prioritizing municipalities. Though this ensemble sometimes splits a number more municipalities, it typically displaces significantly fewer people than the enacted plan. From the perspective of the number of people ousted, the enacted plan is situated squarely between our ensemble prioritizing municipal preservation and that which does not.

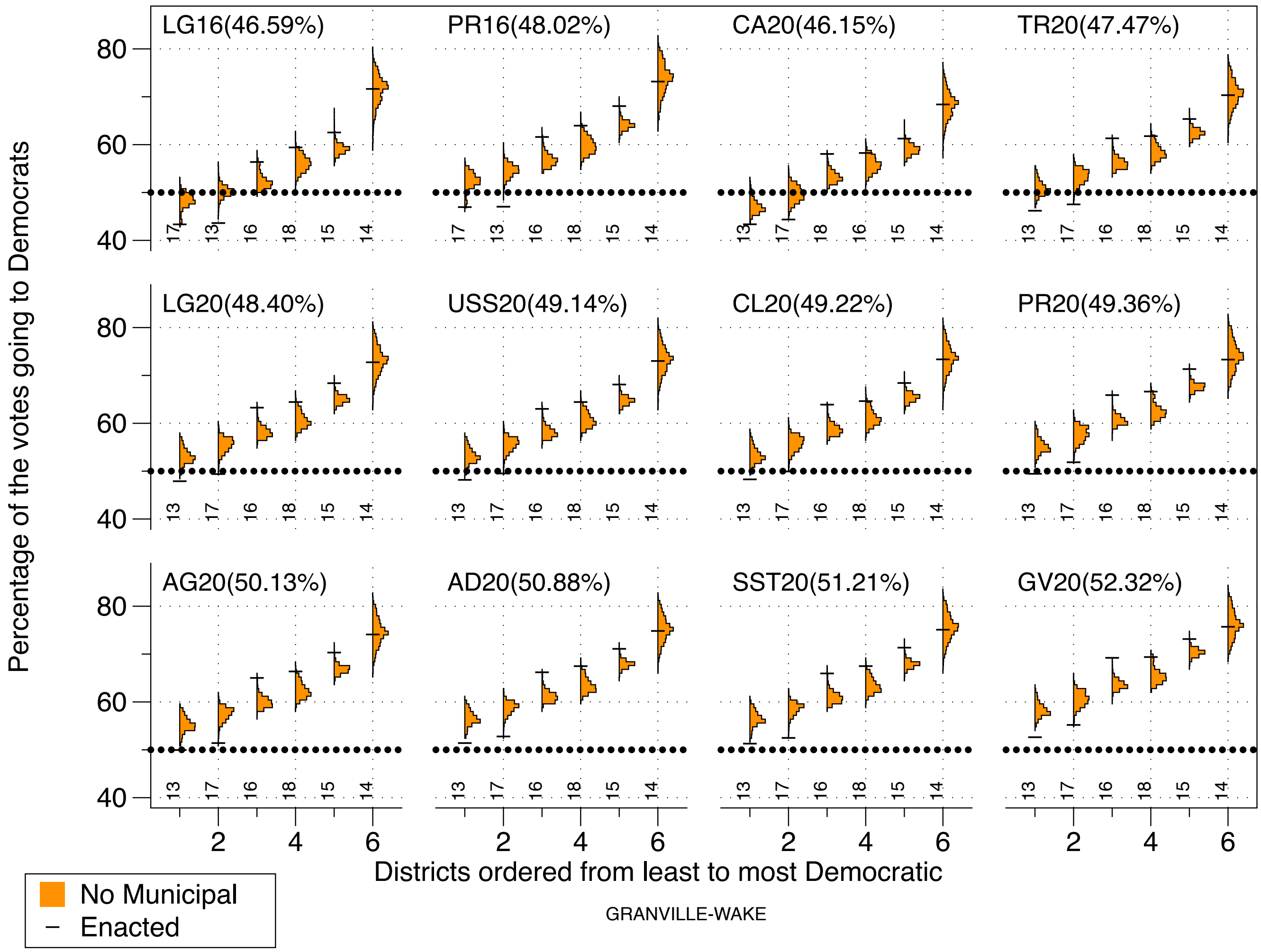


Figure 6.2.5: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the NC Senate Secondary ensemble which does not explicitly preserves municipalities.

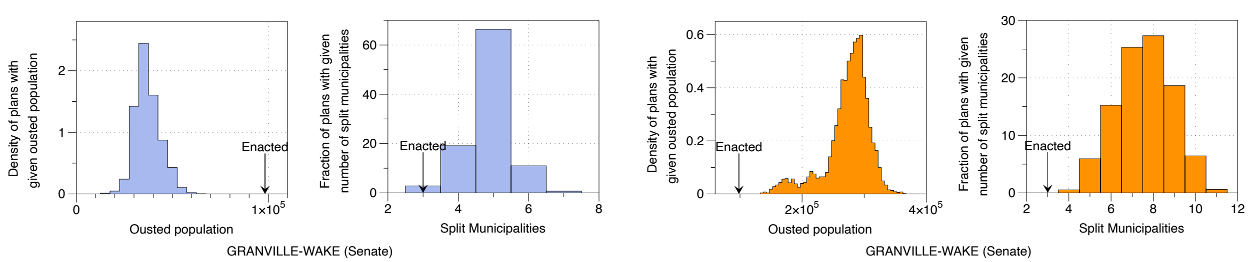


Figure 6.2.6: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2.3 Forsyth-Stokes

There are only two districts in this cluster. The districts in the enacted plan are chosen to maximize the number of Democrats in the more democratic district and the number of republicans in the most Republican district. The map is an extreme outlier in both of these regards. The effect is a maximally non-responsive map. The effect is shown in Figure 6.2.7 across the 12 elections. Of the almost 80,000 maps in the ensemble, less than 1% had as low a fraction of Democrats in the most

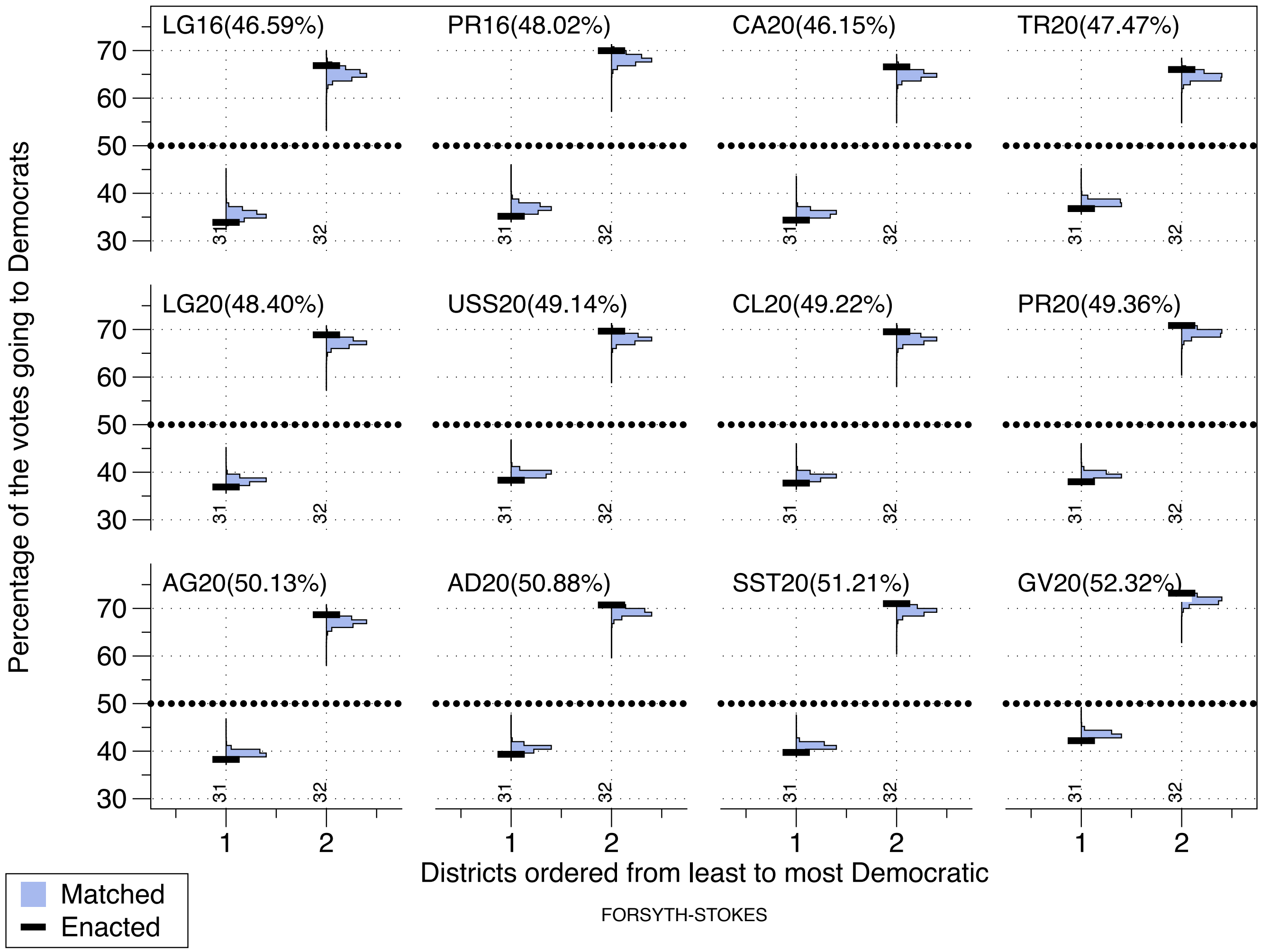


Figure 6.2.7: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

Republican district under the 2020 and 2016 elections considered. And between 1% and 5% of the plans had such a high Democratic fraction in the most Republican District.

When municipal preservation is not prioritized, the enacted map becomes an even more extreme outlier; showing an extreme level of packing of Democrats into one district and Republicans into the other. The effect is shown in Figure 6.2.8 across the 12 elections.

Municipal Splits and Ousted Population: In the Forsyth-Stokes Cluster we see that the number of people ousted from municipalities is comparable between the enacted plan and the municipality prioritizing ensemble. Additionally, the enacted plan splits one municipality which is the most common number of splits in the municipality prioritizing ensemble.

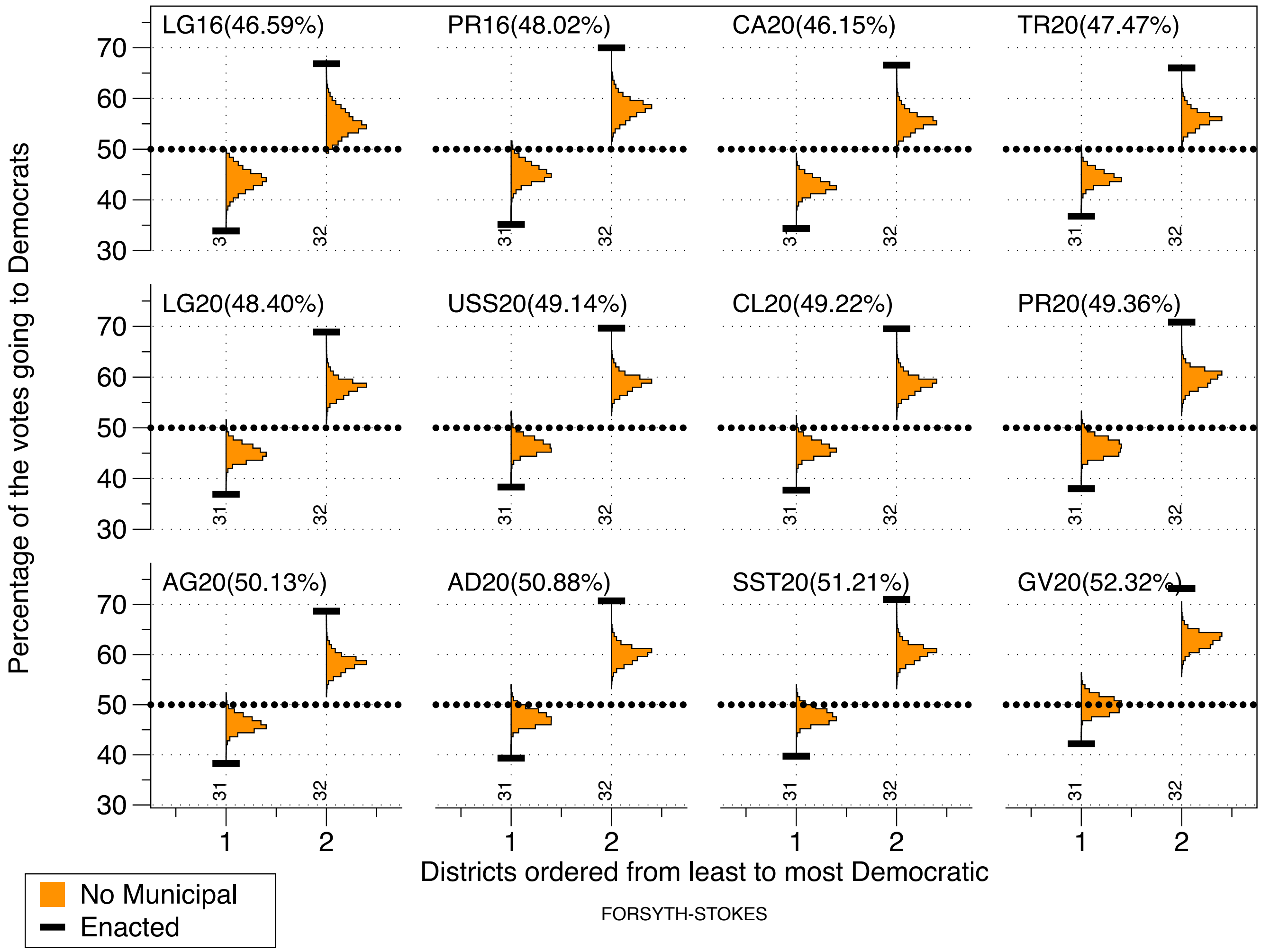


Figure 6.2.8: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the NC Senate Secondary ensemble which does not explicitly preserves municipalities.

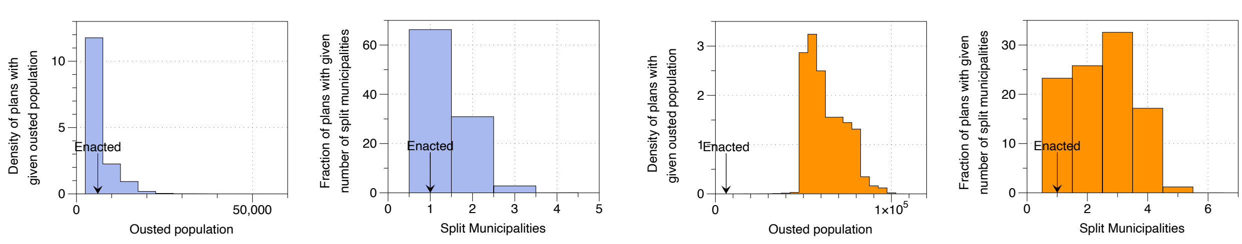


Figure 6.2.9: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2.4 Cumberland-Moore

There are only two districts in this cluster. The districts in the enacted are chosen to maximize the number of Democrats in the more democratic district and the number of republicans in the most Republican district. The map is an extreme outlier in both of these regards. The effect is a maximally non-responsive map. The effect is shown in Figure 6.2.10 across the 12 elections. In each of the elections considered, no more than 0.06% of the ensemble plans have a lower fraction of Democrats in the most Republican districts. Also no more than 0.06% of the ensemble plans have a higher fraction of Democrats in the most Democratic districts.

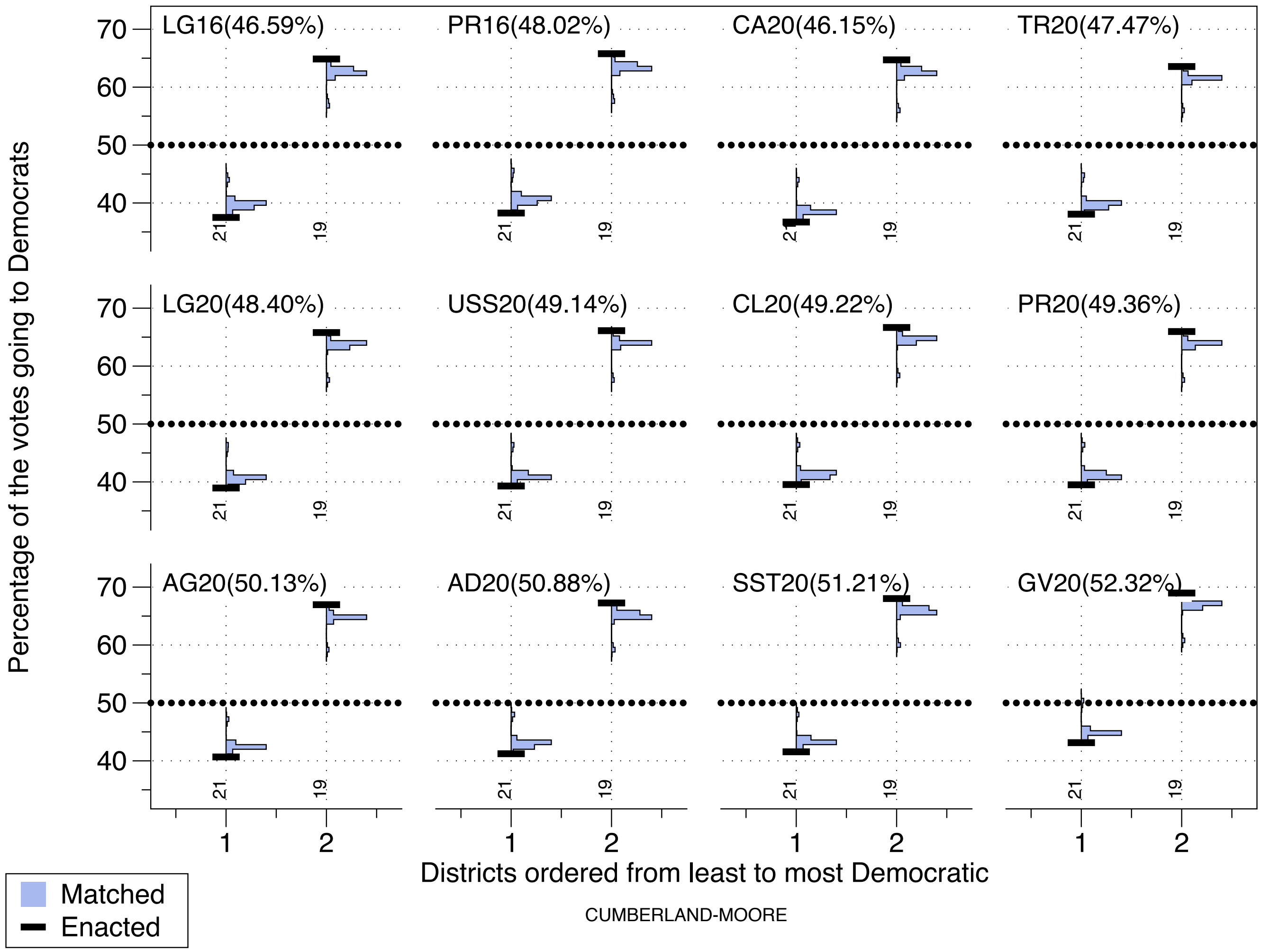


Figure 6.2.10: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

The prioritization of municipal preservation leads a dramatically less responsive pair of districts. When municipalities are less prioritized, both district have politically more centrist make up. Additionally, the more Republican district would regularly lean democratic without the prioritization of municipal preservation. The effect is show in Figure 6.2.11 across the 12 elections.

Municipal Splits and Ousted Population: In the Cumberland-Moore cluster, the enacted plan ousts a number of people close to the minimum number of ousted people seen in the ensemble prioritization municipal preservation. The enacted plan splits two municipalities which is the most common number of splits found in the ensemble prioritization municipal preservation.

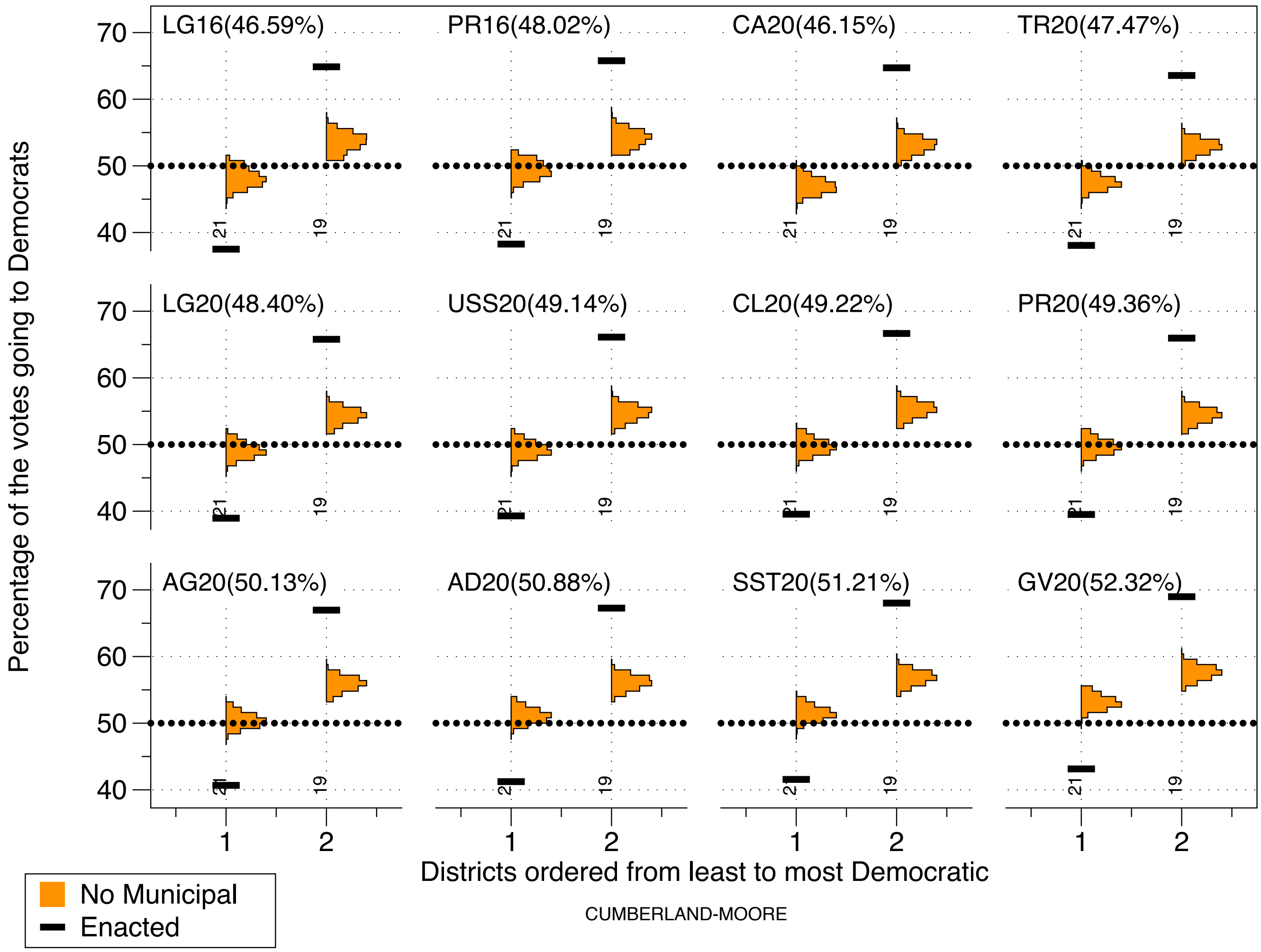


Figure 6.2.11: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the NC Senate Secondary ensemble which does not explicitly preserves municipalities.

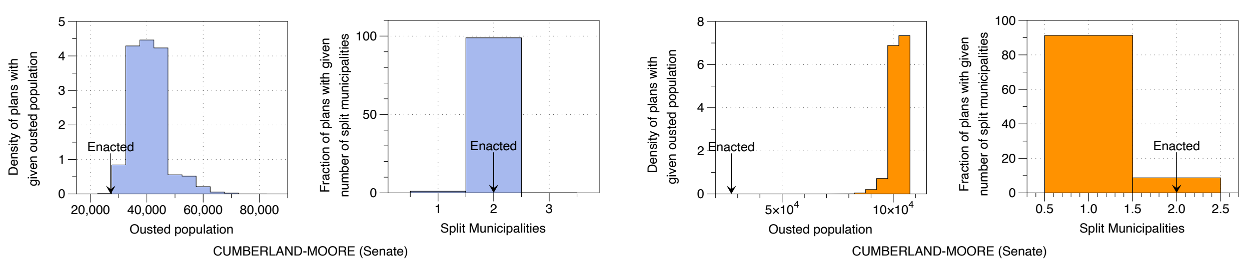


Figure 6.2.12: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2.5 Guilford-Rockingham

The three districts in the Guilford-Rockingham cluster are constructed to pack an exceptional number of democrats in the most democratic district (district 28) and exceptionally few Democrats in the most Republican district (district 26). The effect is to ensure a Republican victory in the district 26, when in some elections the most republican district would be at risk of going to the Democratic Party. The effect is shown in Figure 6.2.13 across the 12 elections. In the Guilford-Rockingham

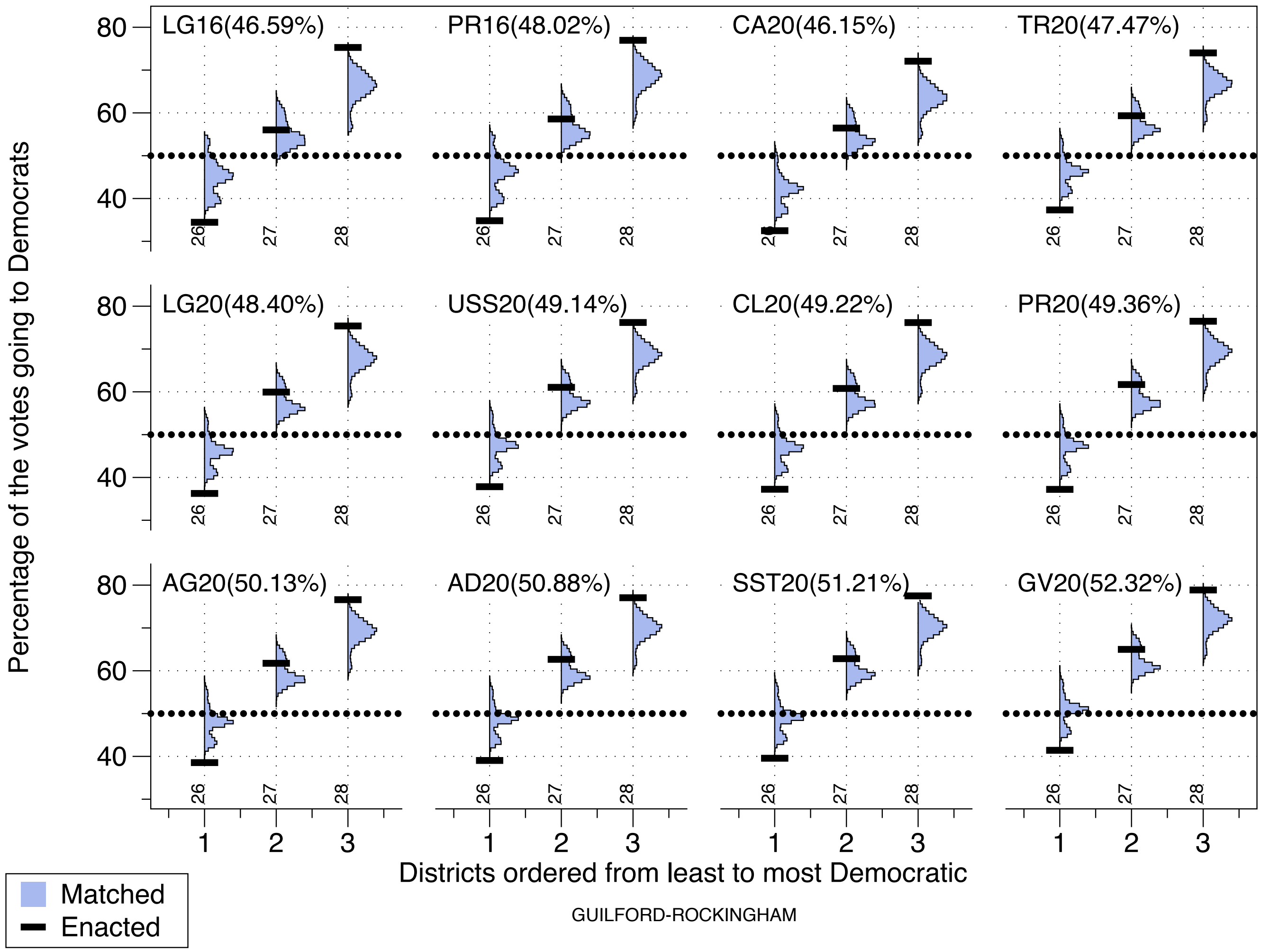


Figure 6.2.13: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the Primary ensemble which was tuned to match the municipal preservation of the enacted plan.

across all of the elections considered, none of the plans have lower fraction of Democrats in the most Republican district than the enacted plan. Conversely, in none of the elections considered do more than 0.08% of the plans have more Democrats packed in the most Democratic district than the enacted plan.

When municipalities are prioritized less, the effect is even more dramatic. In that setting, the extreme number of Democrats packed into the most democratic district and Republicans into the most Republican distinct is even more extreme. The effect is shown in Figure 6.2.14 across the 12 elections.

Municipal Splits and Ousted Population: In the Guilford-Rockingham cluster, the enacted plan splits one municipality and ousts a number of people which is typically found in the ensemble prioritizing municipality preservation which has an average ousted population which is slightly higher than the enacted plan.

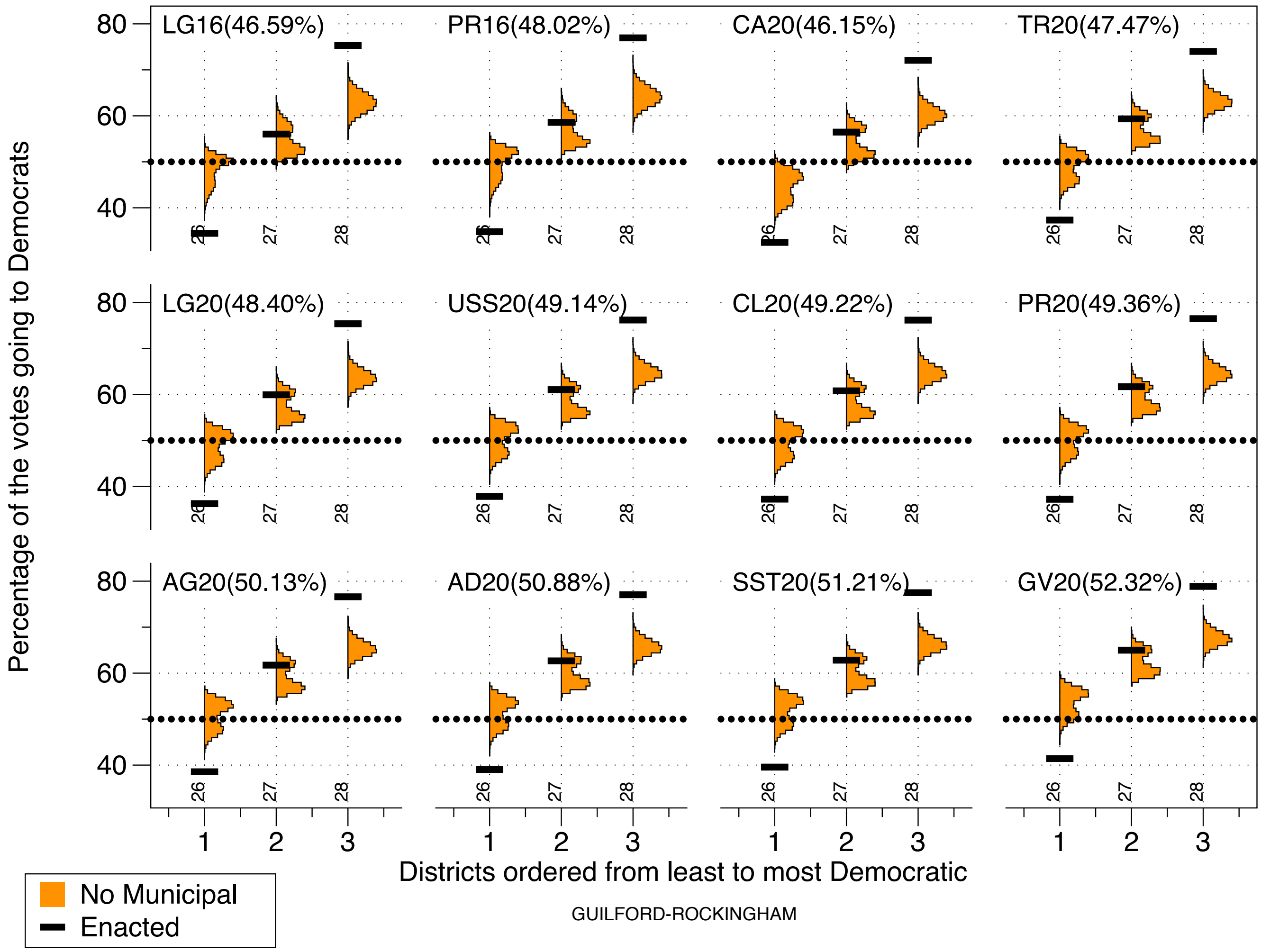


Figure 6.2.14: Shown are the distributions of the Democratic vote fraction of the districts in the plan when ordered from most Republican (on the left) to most Democrat (on the right). The " – " on each marginal histogram denotes the vote fraction of the corresponding district in the enacted plan. The numbers along the horizontal axis give the district numbers in the enacted plan corresponding to the " – ". This plot uses the NC Senate Secondary ensemble which does not explicitly preserves municipalities.

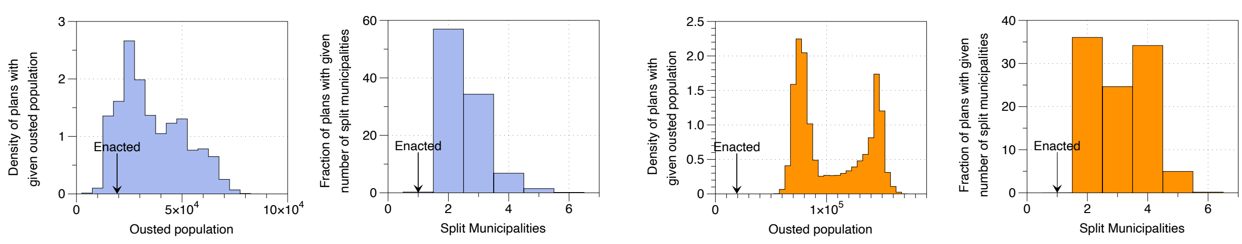


Figure 6.2.15: Plots showing the distribution of the number of people ousted from municipalities in this cluster under the primary and secondary ensemble. The amount of people ousted by the enacted map is also shown.

6.2.6 Northeastern County Cluster

In the NC Senate, there is more than one possible group of county clusters in the northeast corner of the state. As described in Figure 4.3.1 from Section 4.3, there is a choice between two different groups of county clusters. Each group consists of two different county clusters. Based on their population, each of these clusters has only one district. Thus, there is no choice on how to redistrict this region once the county grouping is set. We now explore partisan implications of choosing one county grouping over the other. As shown in the table below, under the enacted county groupings, Republicans win both districts in every election we consider. By contrast, under the alternative county grouping, each party won one of the two districts under every election we consider.

	Enacted Cluster 1	Enacted Cluster 2	Alternative Cluster 1	Alternative Cluster 2
County Clusters	Martin, Warren, Halifax, Hyde, Pamlico, Chowan, Washington, Carteret	Gates Currituck Pasquotank Dare Bertie Camden Perquimans Hertford Tyrrell Northampton	Pasquotank, Dare, Perquimans, Hyde, Pamlico, Chowan, Washington, Carteret	Gates, Currituck, Camden, Bertie, Warren, Halifax, Hertford, Tyrrell, Northampton, Martin
Democratic Vote %(LG16)	46.07%	47.74%	38.51%	55.42%
Democratic Vote % (PR16)	45.60%	46.70%	37.83%	54.59%
Democratic Vote % (CA20)	42.28%	44.47%	36.48%	50.75%
Democratic Vote % (USS20)	45.31%	45.36%	38.45%	52.75%
Democratic Vote % (TR20)	44.12%	44.58%	37.61%	51.59%
Democratic Vote % (GV20)	46.79%	47.56%	40.75%	54.12%
Democratic Vote % (AD20)	47.79%	47.72%	41.02%	54.99%
Democratic Vote % (SST20)	47.56%	47.85%	41.03%	54.89%
Democratic Vote % (AG20)	45.88%	46.11%	39.15%	53.40%
Democratic Vote % (PR20)	44.09%	45.54%	38.30%	51.84%
Democratic Vote % (LG20)	43.80%	45.12%	37.74%	51.69%
Democratic Vote % (CL20)	45.23%	46.42%	39.12%	52.00%

Table 3: Voting History for the two different choices of county grouping northeast corner in the NC Sente.

7 State Legislature: Additional Details

7.1 State Legislature: Details on the Sampling Method

To effectively generate a representative ensemble of maps from the desired non-partisan distributions, we use the well-established method of parallel tempering. It allows one to effectively sample from a possibly difficult to sample distribution by connecting it to an easy to sample distribution through a sequence of intermediate "interpolating" distributions.

We connect our desired distributions to a distribution on redistricting plans that favors plans with a larger number of spanning trees. This alternative distribution satisfies the same constraints, however, it does not consider compactness nor municipal preservation. We make this choice because it can be effectively sampled using a variation on the Metropolized Multiscale Forest RECOM sampling algorithm outlined in [1, 2] coupled with the Metropolis-Hasting algorithm. Using Parallel Tempering, we interpolate between the desired distribution on redistricting and a distribution which is chosen so that the Markov Chain Monte Carlo algorithm converges to its target distribution quickly.

In sampling the interpolating ladder of distributions between the easier-to-sample distribution and our target distribution with the needed policy considerations, we use parallel tempering with a classical Metropolis-Hasting sampling scheme to sample each level of the interpolating ladder of distributions. As proposals in the Metropolis-Hasting sampling scheme, we use a mixture of the Multiscale Forest RECOM proposals and single node flip proposals, depending on what is appropriate for the distribution associated with the given level in the interpolation. The Multiscale Forest RECOM has a number of advantages. Its multiscale nature seems to provide improvements in computational efficiency and the global moves of RECOM lead empirically to faster mixing. Additionally, it can efficiently preserve counties and other groupings. Lastly, it can be effectively combined with the Metropolis-Hasting algorithm to produce an algorithm that samples from the specified distribution.

To facilitate mixing and for computational practicality, we often split the interpolating groups of manageable size, typically between 10 and 30 interpolating levels. Each grouping is then run to produce an ensemble at the top level which approaches; which is closer to the desired ensemble. This ensemble is then used as an *independent sample reservoir* to generate independent samples for the next group of interpolating levels. This process is repeated until the desired level is reached. We typically use between 60 and 100 interpolating levels in our sampling schemes. The number of plans sampled differs from cluster to cluster. We also sometimes group clusters together for sampling. Usually the number of samples in around 80,000 but in all cases we have check various empirical measure to evaluate if the sampling has converged and is well mixed.

7.2 State Legislature: Mathematical Description of Ensemble Distribution

In designing our distributions, we have chosen to define explicit distributions and then use an implementation of the Metropolis-Hastings algorithm to generate the ensemble. We feel this choice promotes transparency because an explicit distribution can better be discussed and critiqued. It also allows us to more explicitly translate the policy considerations into the ensemble.

In order to formally define our distributions, we consider the labeling ξ of the precincts of the map of NC with the number $\{1, \ldots, d\}$, where d is the total number of districts. So for the i-th precinct, $\xi(i)$ gives the district to which the precinct belongs. If we let $A_j(\xi)$ and $B_j(\xi)$ be respectively the surface area and perimeter (or length of the boundary) of the j-district then our compactness score is

$$J_{\text{compact}}(\xi) = \sum_{j=1}^{d} \frac{A_j(\xi)}{B_j^2(\xi)}.$$

Then the probability of drawing the redistricting ξ is

$$\text{Prob}(\xi) = \begin{cases} \frac{1}{Z} e^{-w_{\text{compact}} J_{\text{compact}}(\xi)} & \text{for } \xi \text{ which is allowable} \\ 0 & \text{for } \xi \text{ which is not allowable} \end{cases}$$

Here Z is a number that makes the sum of Prob(ξ) over all redistricting plans are equal to one.

The collection of allowable redistricting plans ξ is defined to be all redistricting plans which satisfy the following conditions:

1. all districts are connected
2. the populations of each district is within %5 of the ideal district population unless the district in the wake county cluster in the senate or the Craven-Carteret county cluster in the house.[2]
3. The number of split counties is minimized.
4. We minimize the occurrence of districts traversing county boundaries.

The second distribution includes a municipality score, $J_{MCD}(\xi)$. This score describes the number of people who have been displaced from a district that could have preserved the voters within their municipality, and is defined as

$$J_{MCD}(\xi) = \sum_{m \in M} \text{pop}_{oust}(\xi, m),$$

where M is the set of all MCDs, and $\text{pop}_{\text{oust}}(\xi, m)$ is the number of displaced people from the municipality m under the redistricting plan ξ. We define pop_{oust} in one way if the population of the municipality is less than the size of a district and another if it is greater.

[2] In the two exceptional clusters, it is impossible to draw districts that preserve precincts and also achieve population balance within 5%. For Wake in the senate, we sample with a deviation of 6% and generate an associated ensemble; past experience has shown that this does not create a partisan effect and we will be confirming this in follow on analyses. In Craven-Carteret, precinct 02 in Craven is the only precinct that connects the bulk of Craven with Carteret and it must be split to achieve population balance between the two districts within this cluster. We have examined the voting patterns when assigning this precinct to the district with the bulk of Craven or with all of Carteret and found minimal effects on the outcome.

If m has a population that is less than the population of a district, we consider the district that holds the most people from the municipality m as the representative district for that municipality. Any person within municipality m, but not within the representative district is considered to have been displaced.

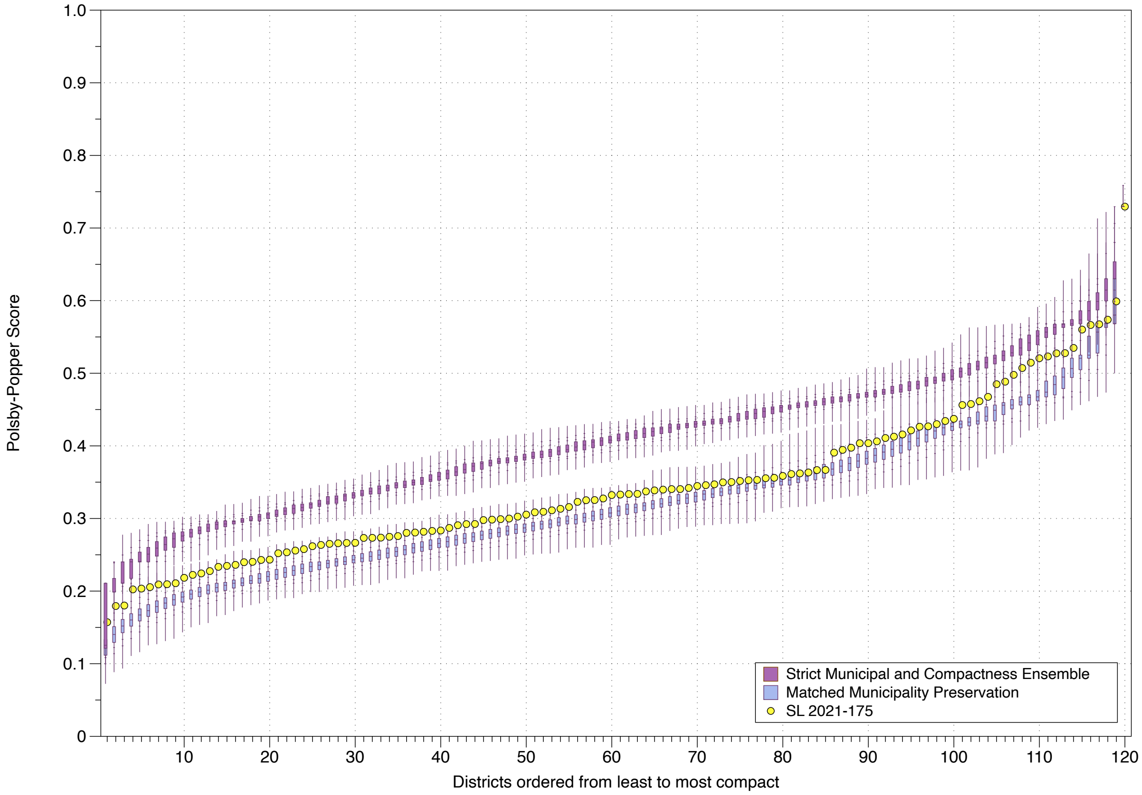
If m has a population that is greater than the population of a district, we consider the number of districts that could fit within m to be $d(m) = \lfloor \text{pop}(m)/\text{pop}_{\text{ideal}} \rfloor$, where $\text{pop}(m)$ is the population of the MCD m and $\text{pop}_{\text{ideal}}$ is the ideal district population. We also consider the remaining population in the municipality that cannot fit within a whole district to be $r(m) = \text{pop}(m) - d(m) \times \text{pop}_{\text{ideal}}$. To determine the displaced population, we look at the $d(m)$ districts that contain the largest populations from the municipality m. Hypothetically, everyone in these districts could live in the municipality m. Therefore, anyone who is in one of these districts and that does not live in the municipality m could be replaced by someone who does live in the municipality. Thus, we sum the number of people not in m in the $d(m)$ districts that contain the largest populations of m. We also note that the remaining population $r(m)$ could hypothetically be kept intact when drawing a $(d(m)+1)$th district. We, therefore, look at the number of people in the municipality m who are living in the district with the $(d(m)+1)$th most population of the municipality. If the number of people in m is less than $r(m)$, then we add this difference to the number of ousted people (since each of these people in the municipality could have conceivably been placed in the district).

Formally, we let the $|M| \times d$ matrix, $MCD(\xi)_{m,j}$ represent the number of people who are in the municipality m and the district ξ_j. Then

$$\text{pop}_{\text{oust}}(\xi, m) \begin{cases} \sum_j MCD(\xi)_{m,j} - \max_j(MCD(\xi)_{m,j}) & \text{pop}(m) < \text{pop}_{\text{ideal}} \\ \sum_{j \in D(m)} (\text{pop}(\xi_j) - MCD(\xi)_{m,j}(\xi)) & \text{pop}(m) \geq \text{pop}_{\text{ideal}} \\ \quad + \max(0, MCD(\xi)_{m,N(m)} - r(m)) & \end{cases},$$

where $\text{pop}(\xi_j)$ is the population of district ξ_j, $D(m)$ is the set of district indices that represent the $d(m)$ districts with the largest populations of municipality m, and $N(m)$ represents the district index with the $d(m)+1$ most population of municipality m.

7.3 State Legislature: Additional Ensemble Statistics



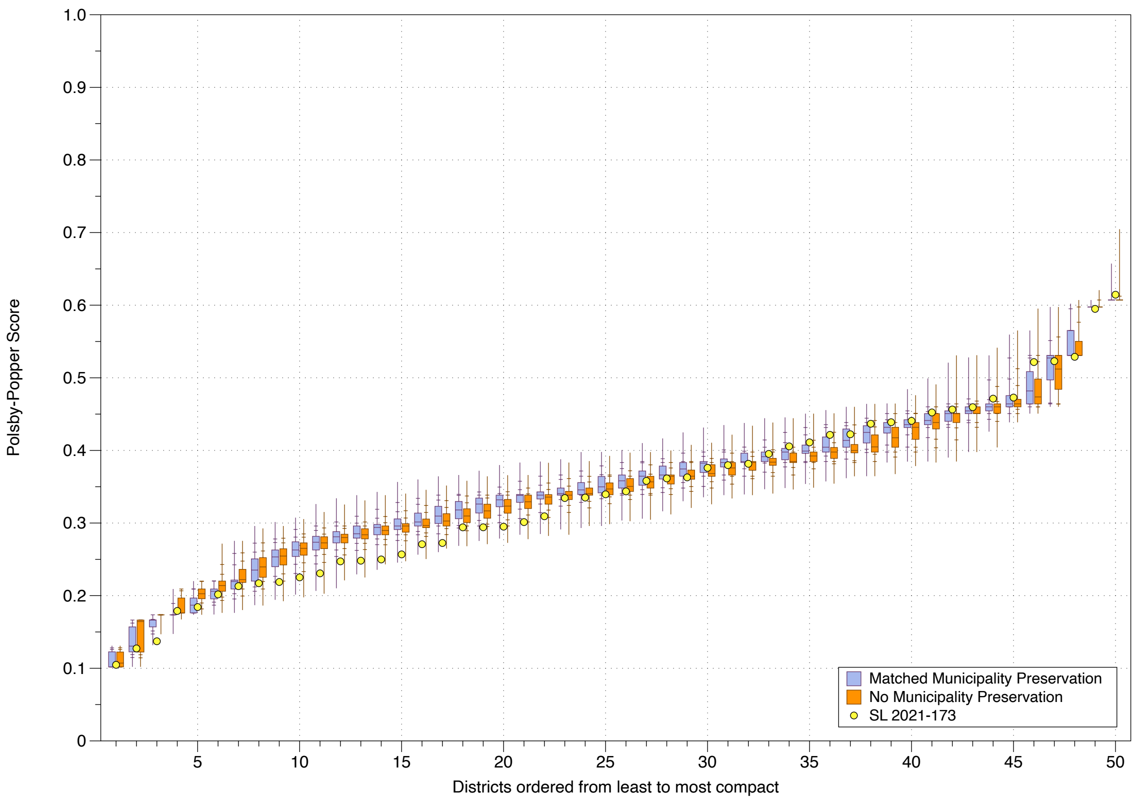


Figure 7.3.1: These plots compare the Polsby-Popper Score of the enacted maps (shown we the yellow dots) with the marginal histograms of the primary and secondary ensembles.

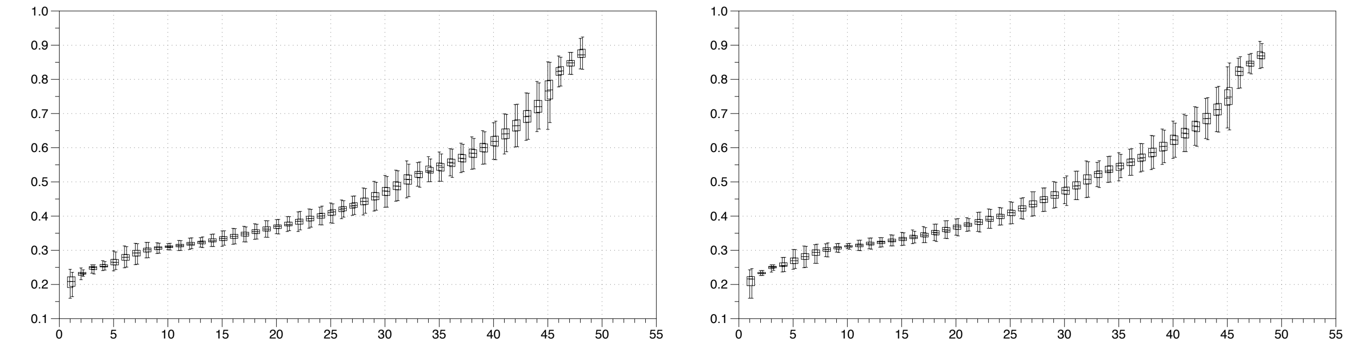


Figure 7.4.1: We compare a subset of the threads to the remaining threads. Each thread represents a different initial condition, and thus takes a different trajectory through the phase space. We compare our standard observables, such as the ranked ordered marginal distributions and confirm that they yield equivalent results. On the left we show an example of comparing one thread with all threads in a parallel tempering run; on the right we show an example of comparing half of the thread with the other half of the threads in a parallel tempering run.

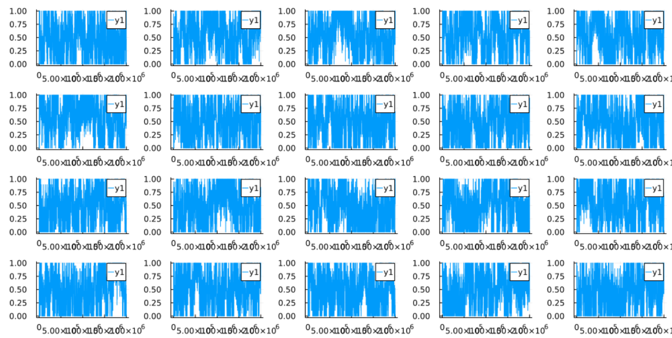


Figure 7.4.2: We examine how each of the parallel tempering threads swaps as a function of the proposal number. The vertical axis represents different measures and the horizontal axis represents the proposal in the Markov Chain. When the thread (or redistricting) is near the bottom of the vertical axis it mixes quickly when drawing from the reservoir; when it is at the top of the vertical axis it is at the desired measure which is either the desired measure we are sampling from or an intermediate measure that will act as a subsequent reservoir.

7.4 State Legislature: Convergence Tests

We performed a number of tests to assess if our sampling of the desired distribution was sufficient to provide an accurate representation of the desired distribution. Sometimes many samples are needed, yet in other cases a much smaller number is sufficient. We use a number of different methods to assess convergence.

Many of our runs were generated with an implementation of the *parallel tempering algorithm* with an *independent sample reservoir*. The use of parallel tempering provides a number of different threads that can be grouped and then compared against each other. As each thread starts from a different initial condition, if the distributions look similar then there is evidence that the system is mixing. Similarly, if a subset of the threads has a similar distribution to all of the threads, then there is evidence that enough samples were used.

The following plots show representative ranked ordered histograms for some NC House and NC Senate runs where different threads in a parallel tempering run are compared.

Each time a thread exchanges its state with the independent sample reservoir, it receives a new configuration that is independent of the previous state of the system. Additionally, if the thread then progresses up to the parameter level of interest, then we have strong evidence that we are producing decorated samples. The following plots show the current level of each for the different threads in a parallel tempering run. Switching regularly from the highest level (the desired sample distribution) to the lowest level (the level with the independent sample reservoir) is a strong indication that the system will be well mixed and converged.

In some cases, we run two or more complete sampling runs for the same target distribution. If the ensembles generated are close then we have strong evidence that the ensembles are converged as each run started from different initial conditions and used different randomness.

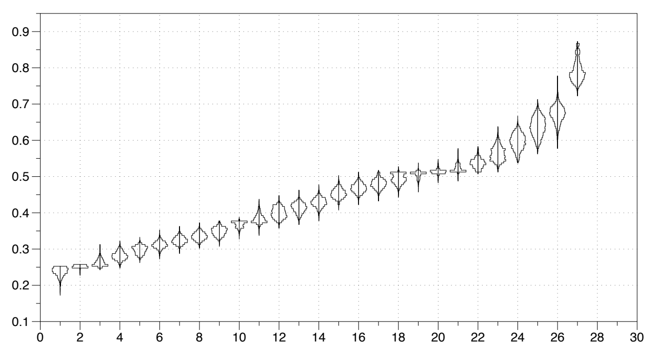


Figure 7.4.3: We compare the ranked ordered marginals on two independent parallel tempering runs.

8 Congressional Plan

As with the NC House and NC Senate plans, we place a probability distribution on Congressional plans for North Carolina. The distributions embody different policy choices. With each distribution, we produce representative ensembles of maps to serve as benchmarks against which to compare specific maps. The ensembles are generated by using the Metropolis-Hasting Markov Chain Monte Carlo Algorithm in a parallel tempering framework which employs the proposal from the Multiscale Forest RECOM algorithm [2, 1].

This analysis parallels the analysis already presented for the NC House and NC Senate with the simplification that we no longer need to consider County Clusters and that some of the criteria are modified. The details are given in Sections 8.1 and 7.2.

8.1 Congressional: Ensemble Overview

Similarly to the distribution placed on the NC Legislative redistricting plans in Section 4.4, we consider a distribution (and hence an ensemble) satisfying the following constraints:

- The maps split no more than 14 counties.
- The maps split no county into more than two districts.
- Districts traverse counties as few times as possible.
- All districts are required to consist of one contiguous region.
- The deviation of the total population in any district is within 1% of the ideal district population. We have verified in previous work in related settings that the small changes needed to make the districting plan have perfectly balanced populations do not change the results. (See [7] and the expert report in Common Cause v. Rucho).
- **Compactness**: The distributions on redistricting plans are constructed so that a plan with a larger total isoperimetric ratio is less likely than those with a lower total isoperimetric ratio. The total isoperimetric ratio of a redistricting plan is simply the sum of the isoperimetric ratios over each district. The isoperimetric ratio is the reciprocal of the Polsby-Poper score; hence, smaller isoperimetric ratio corresponds to larger Polsby-Poper scores. As the General Assembly stated in its guidance that the plans should be compact according to the Polsby-Popper score [9], we tuned the distribution so that it yields plans of a similar compactness to those of the legislature. (See Figure 10.2.1 in Section 10.2.) We further limited our distribution only to include those with an Isoparametric score less than 80.

  The legislature also listed the Reock score as another measure of compactness which one could consider. However, we have found Polsby-Popper/isoperimetric score to be a better measure when generating districts computationally. In our previous work, we have seen that this choice did not qualitatively change our conclusions (see [7] and the expert report in Common Cause v. Rucho).

8.2 Congressional Plan: Sampling Method

We have chosen the distribution from which to draw our ensemble to comply with the desired policy and legal considerations. It is well accepted that not all distributions on possible redistricting plans are equally easy to sample from.

As discussed in Section 7.1 to effectively generate a representative ensemble of maps from these distributions, we use the well-established method of parallel tempering. It allows one to effectively sample from a possibly difficult to sample distribution by connecting it to an easy to sample distribution through a sequence of intermediate “interpolating” distributions.

We connect our desired distributions, which includes a compactness score, to a measure on redistricting plans which is uniform on spanning forests which satisfy the population and county constants. Furthermore, the enacted plan can be effectively sampled using a variation on the Metropolized Multiscale Forest RECOM sampling algorithm outlined in [1, 2].

In sampling the interpolating ladder of distributions between the easier-to-sample measure and our target measure which includes a compactness score, we use parallel tempering with a classical Metropolis-Hasting sampling scheme to sample each level of the interpolating ladder of distributions. As proposals in the Metropolis-Hasting sampling scheme, we use Multiscale Forest RECOM proposals. We sample around 80,000 plans have confirmed that the distribution seems well mixed and than it has been sufficiently sampled to provide stable statistics.

8.3 Election Data Used in Analysis

The same historic elections and abbreviations were use to analyze the congressional plan and ensemble as were used for the NC legislative maps and ensemble. See Section 4.6.

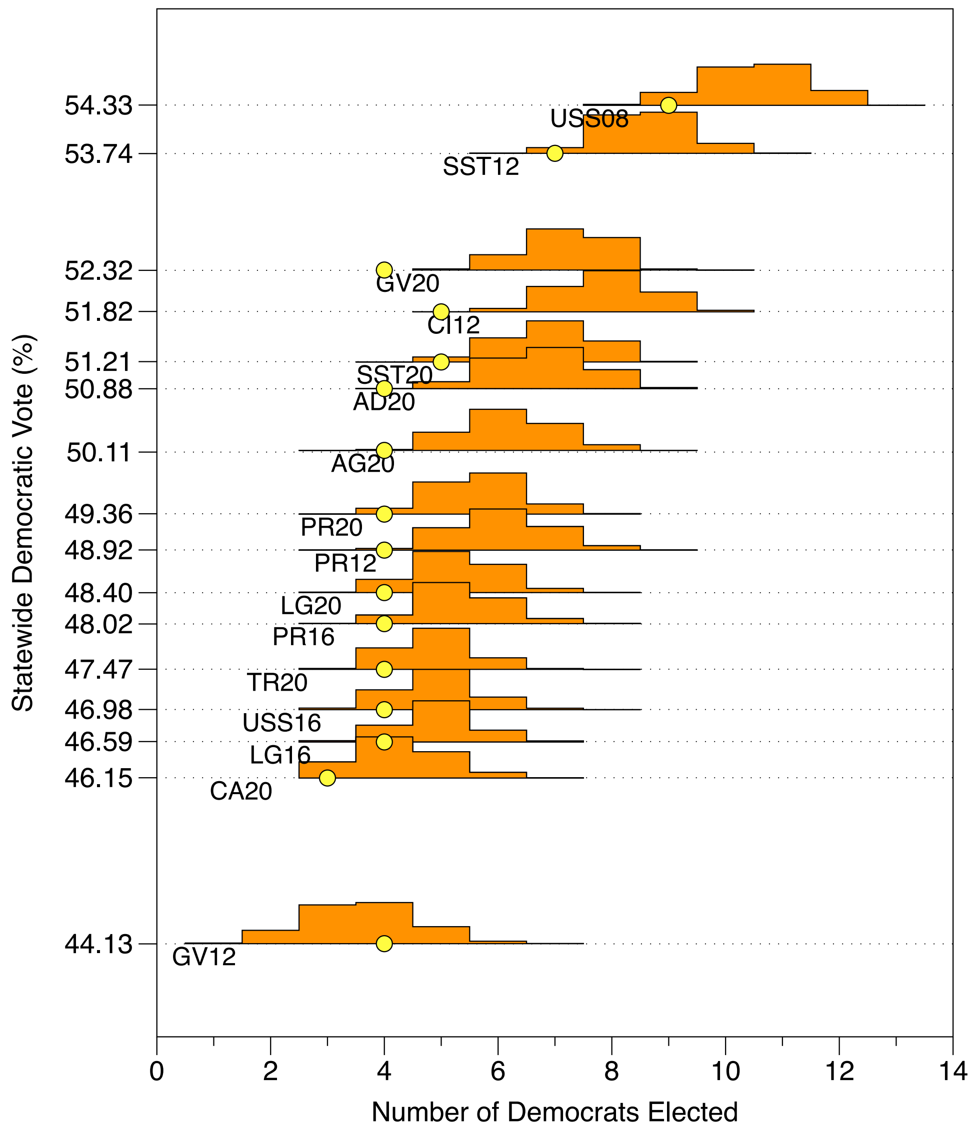


Figure 9.0.1: Each histogram represents the range and distribution of possible Democratic seats won in the ensemble of plans; the height is the relative probability of observing the result. The yellow dots represent the results from the enacted congressional plan under the various historic votes.

9 Congressional Plan: Main Analysis

Figure 9.0.1 gives the *Collected Seat Histograms* for the ensemble sampled from the distribution. This figure also shows how many Democrats the enacted congressional plan would have elected under the votes from a variety of historic elections.

Without reference to a particular ensemble, a primary message of this plot is that the enacted congressional plan is largely stuck electing 4 of 14 Democrats despite large shifts in the statewide vote fraction and across a variety of election structures. Over the statewide vote Democratic partisan vote range of 46.59% to 52.32%, the enacted map only twice changes the number of Republicans elected. The outcome of the election is largely stuck at 4 Democrats. This shows the enacted map to be highly non-responsive to the changing opinion of the electorate. Without holding the election one largely knows that the result will be 10 Republicans and 4 Democrats.

This non-responsiveness is not observed in the ensemble. The ensemle shows that a typical map drawn without political considerations gradually shift from 4-5 Democrats typically being elected at one end of this regime to 7-8 being elected at the other end. Hence, under historic elections in which Democrats win 46% to 53% of the statewide vote, a typical map would gradually shift from around 4 Democrats in the NC congressional delegation to around 8 Democrats as the electorate changed is vote. This does not happen under the enacted plan with the elections considered. Instead, as described above, the enacted map sticks at only 4 Democrats in North Carolina's congressional delegation under nearly all of these elections.

To better illuminate the structure responsible for making the enacted map an extreme outlier, we turn to the Rank Ordered Box plots already discussed in general in Section 3.4 and in the context of the state legislative maps in the previous sections. The plots show extreme packing of Democrats in the three most Democratic districts and depletion of Democrats from the next 7 to 9 most Democratic districts. The effect of this cracking and packing is the non-responsiveness seen in Figure 9.0.1.

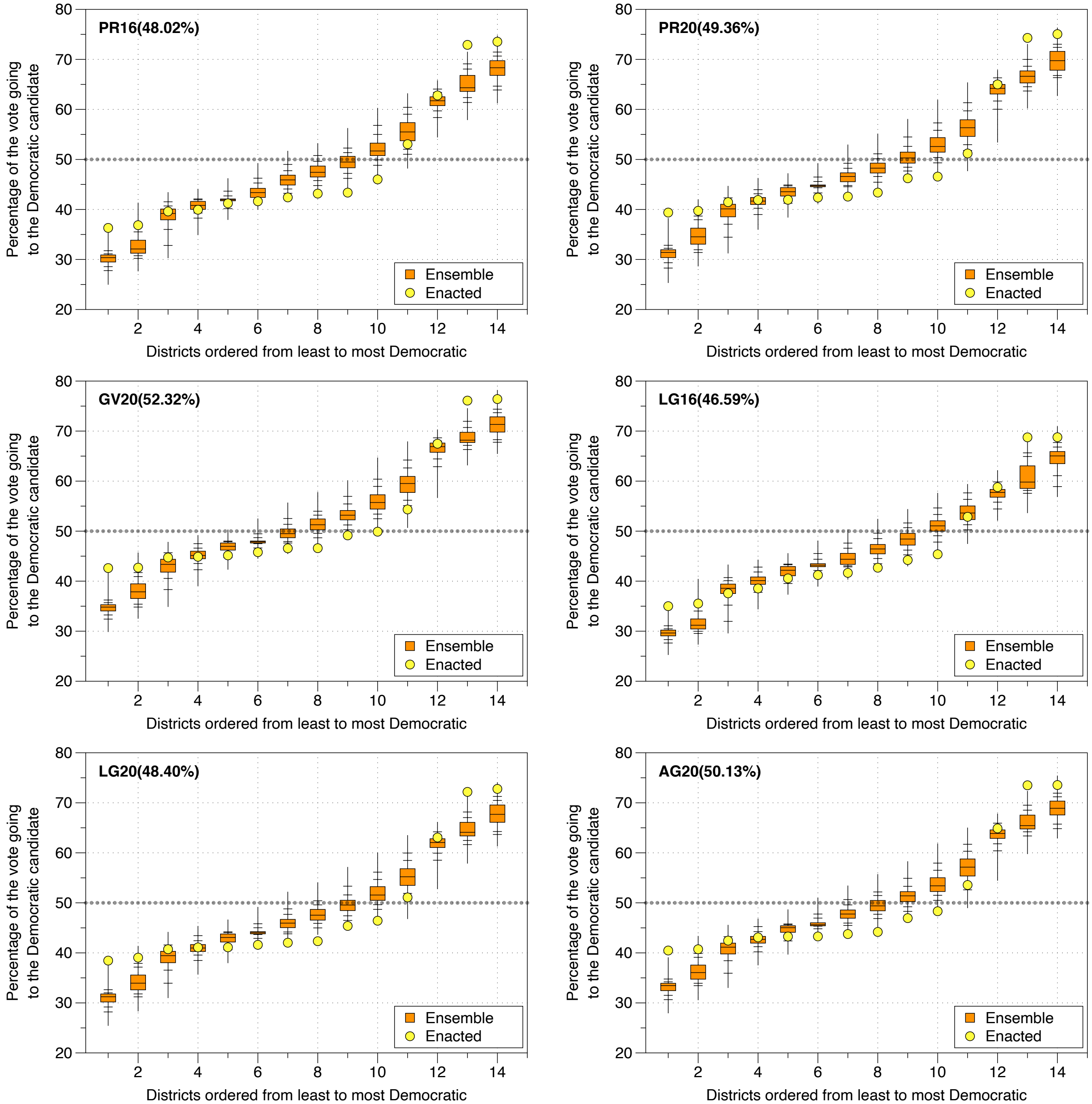


Figure 9.0.2: The Ranked Marginal Box-plots for the NC Congressional Plan. The ranked ordered marginals for the enacted map are shown in yellow. 50% of the ensemble is contained within the box. Inside the first pair of tick marks is 80% of the data and inside the second set is 95% of the points.

Motivated by the cracking and packing of Democrats shown in Figure 9.0.1, we ask how common is such a highly polarized districts in our non-partisan ensemble of maps. The results are summarized in Table 4. They show that the Congressional map is not only non-responsive to the changing preferences of the electorate but it is also an extreme partisan gerrymander. Maps which lock in such an extreme partisan outcome do not occur in our ensemble.

Election	Plans with the same or more Dem (1-2)	Plans with the same or more Rep (5-11)	Plans with the same or more Dem (12-14)	Total Plans
LG16	18	0	0	79997
PR16	0	0	0	79997
CA20	0	0	0	79997
TR20	0	0	0	79997
LG20	0	0	0	79997
USS20	0	0	0	79997
CL20	0	0	0	79997
PR20	0	0	0	79997
AG20	0	0	0	79997
AD20	0	0	0	79997
SST20	0	0	0	79997
GV20	0	0	0	79997
CI20	0	0	0	79997
USS16	0	0	0	79997
GV16	1	0	0	79997
AG16	15	0	0	79997

Table 4: Over the approximately 80,000 plans in our ensemble, we ask how many plans have (1) as high Democratic fraction in the two most Republican districts, (2) as small a fraction of Democrats in the 5th through 11th most Republican districts, and (3) have as high a Democratic fraction in the 12th through 14th most Republican districts. The answer is given in this table along with the total number of plans in our ensemble.

10 Congressional: Additional Details

10.1 Congressional Plan: Mathematical Description of Ensemble Distribution

In specifying our distribution, we have chosen to define explicit distributions and then use an implementation of the Metropolis-Hastings algorithm to generate the ensemble. We feel this choice promotes transparency because an explicit distribution can better be discussed and critiqued. It also allows us to more explicitly translate the policy considerations into the ensemble.

In order to formally define our distributions, the partition of the precinct adjacency graph into a spanning forest $\mathcal{T}$ with 14 district trees $\{\mathcal{T}_1, \cdots, \mathcal{T}_{14}\}$ corresponding to each district. Hence $\mathcal{T} = \{\mathcal{T}_1, \cdots, \mathcal{T}_{14}\}$ completely specifies the redistricting.

If we let $A_j(\mathcal{T})$ and $B_j(\mathcal{T})$ be respectively the surface area and perimeter (or length of the boundary) of the j-district then our compactness score is

$$J_{\text{compact}}(\mathcal{T}) = \sum_{j=1}^{14} \frac{A_j(\mathcal{T})}{B_j^2(\mathcal{T})}.$$

Then the probability of drawing the spanning forest $\mathcal{T}$ is

$$\text{Prob}(\mathcal{T}) = \begin{cases} \frac{1}{Z} e^{-w_{\text{compact}} J_{\text{compact}}(\mathcal{T})} & \text{for } \mathcal{T} \text{ which is allowable} \\ 0 & \text{for } \mathcal{T} \text{ which is not allowable} \end{cases}$$

Here Z is a number which makes the sum of Prob($\mathcal{T}$) over all spanning forests with 14 trees equal to one.

The collection of allowable spanning forests $\mathcal{T}$ is defined as those which produce redistricting plans which satisfy the following conditions:

1. all districts are connected
2. the populations of each district is within %1 of the ideal district population.
3. No more than 14 counties are split with no county split more once.
4. We minimize the occurrence of districts traversing county boundaries.

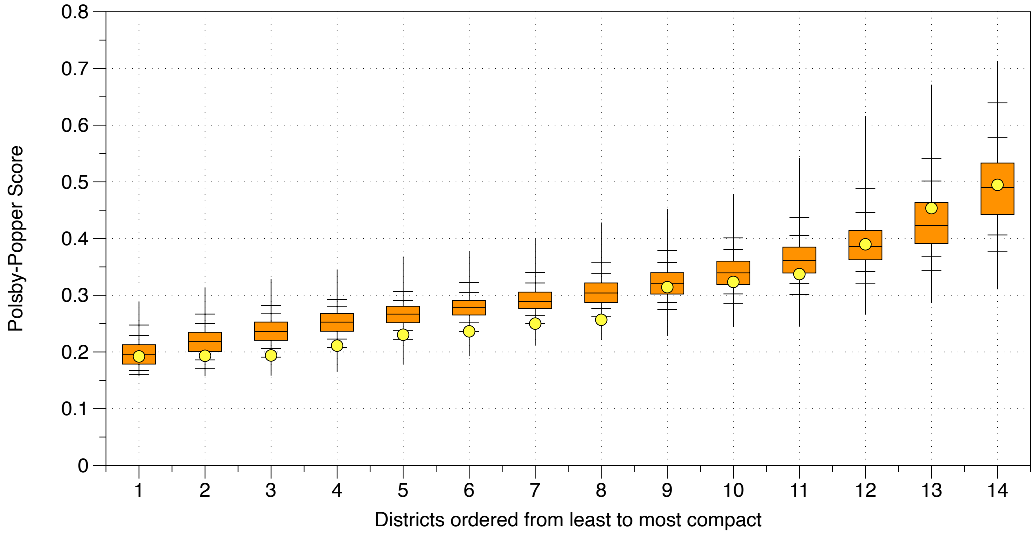


Figure 10.2.1: The yellow dots display the ordered Polsby-Popper score of the 14 districts in the enacted plan.

10.2 Congressional Plan: Additional Ensemble Statistics

In Figure 10.2.1, we give the box-plots for the ranked ordered marginal distribution for the compactness score, namely the Polsby-Popper score (see companion methods document). We compare the ensemble of plans with the enacted plan.

10.3 Congressional Plan: Convergence Tests

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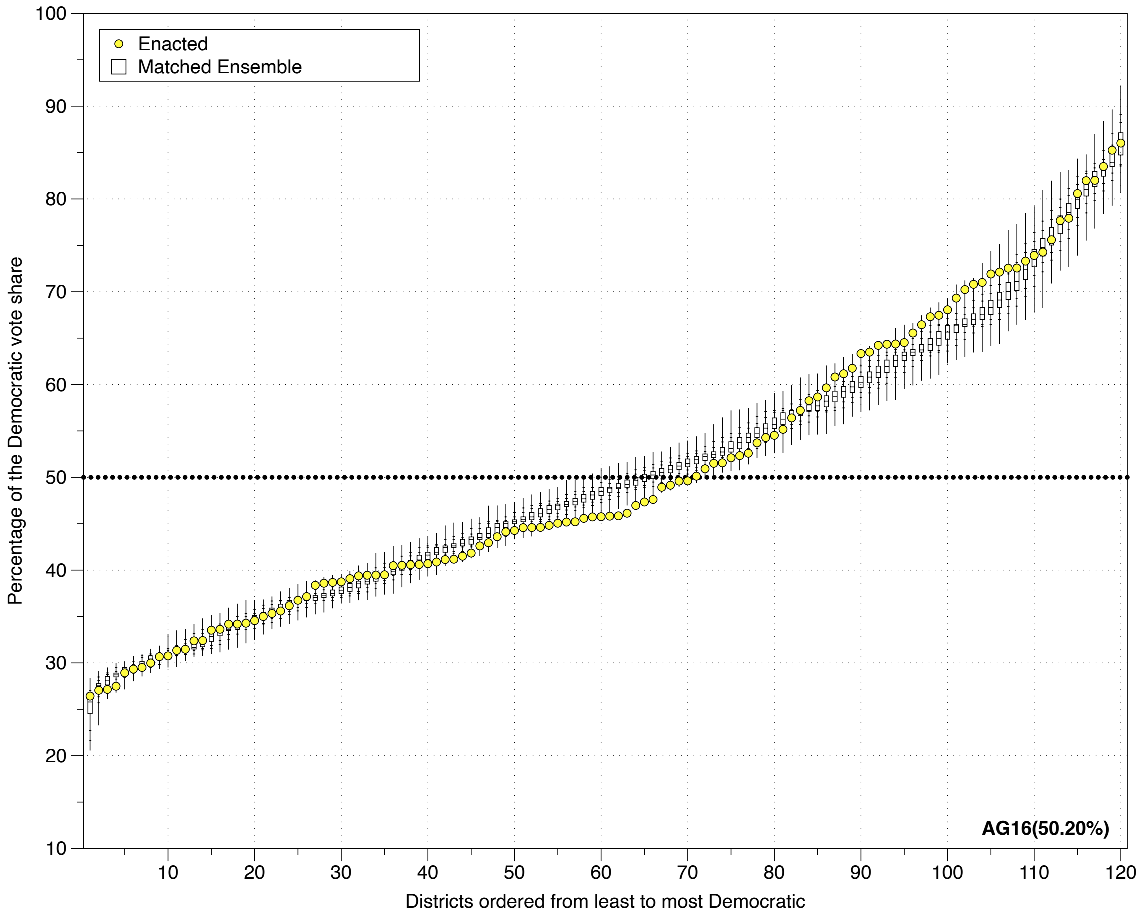
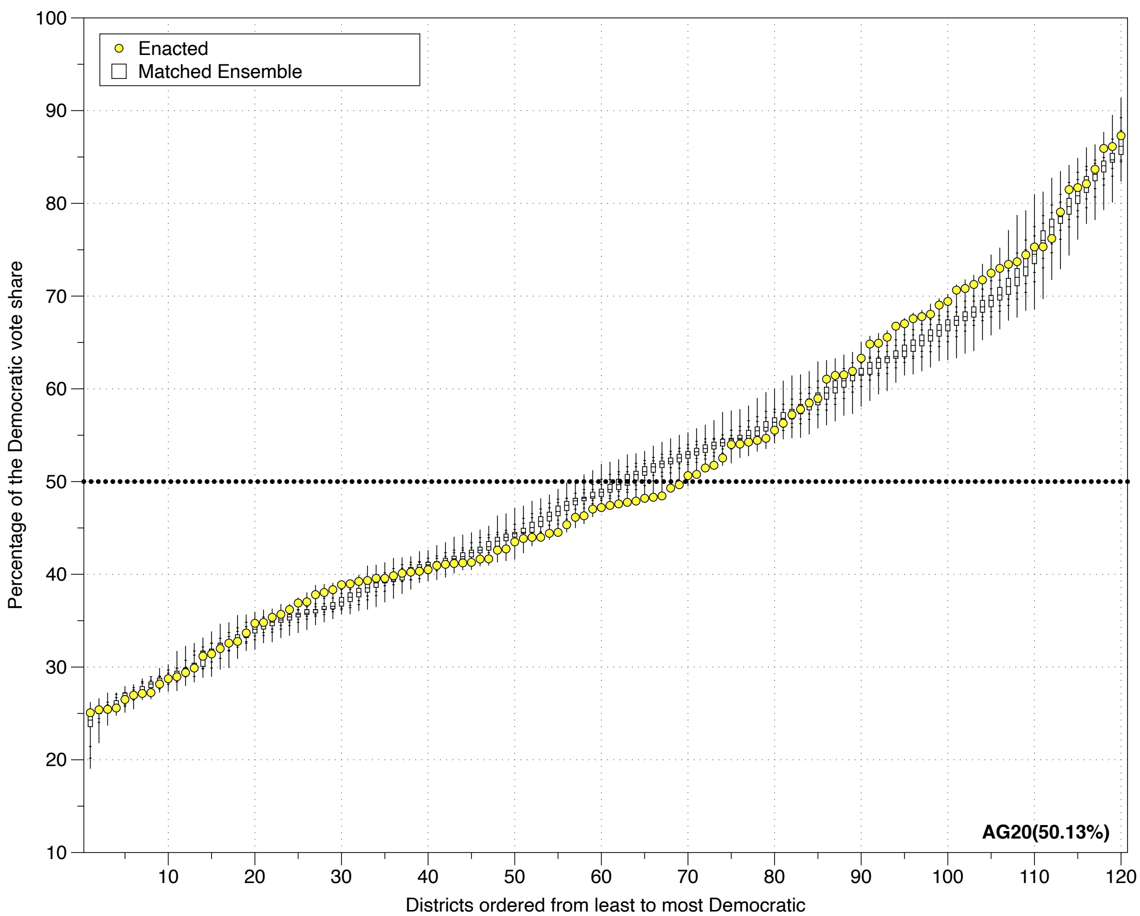
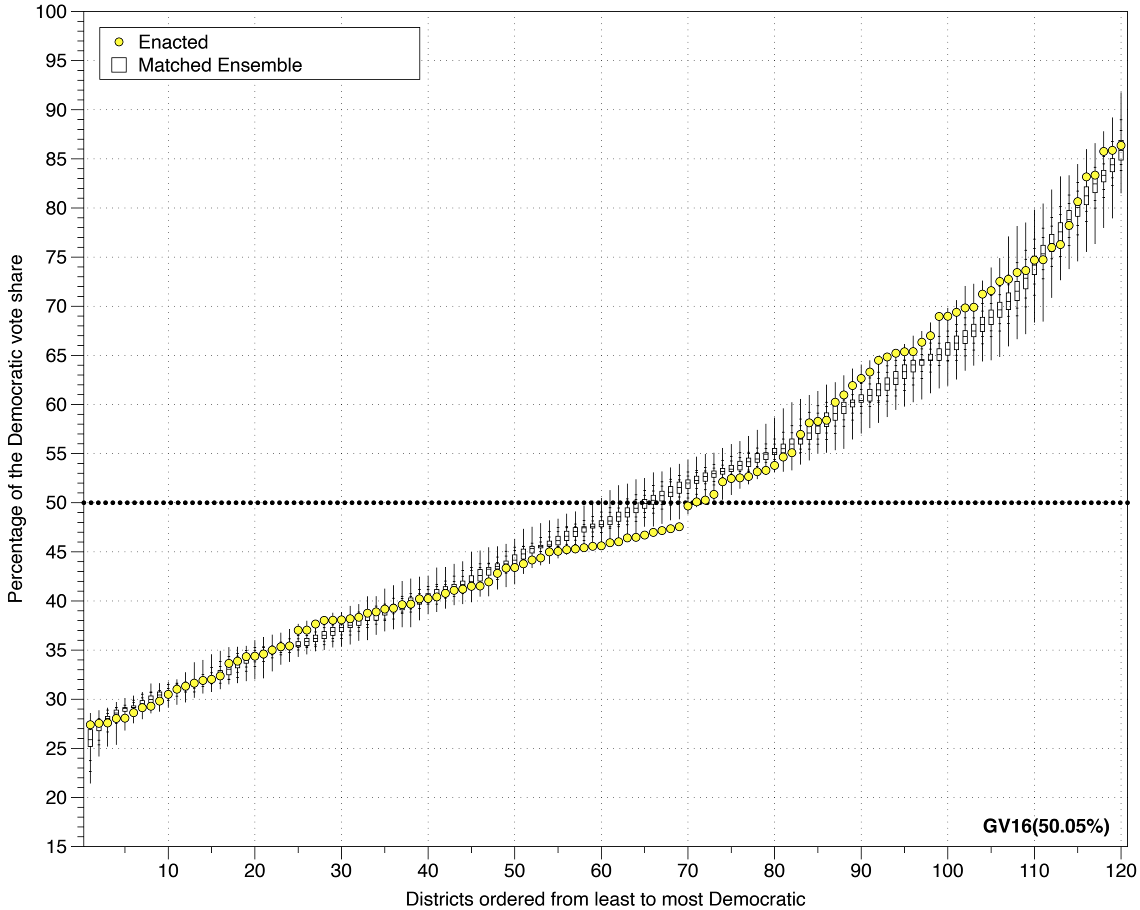
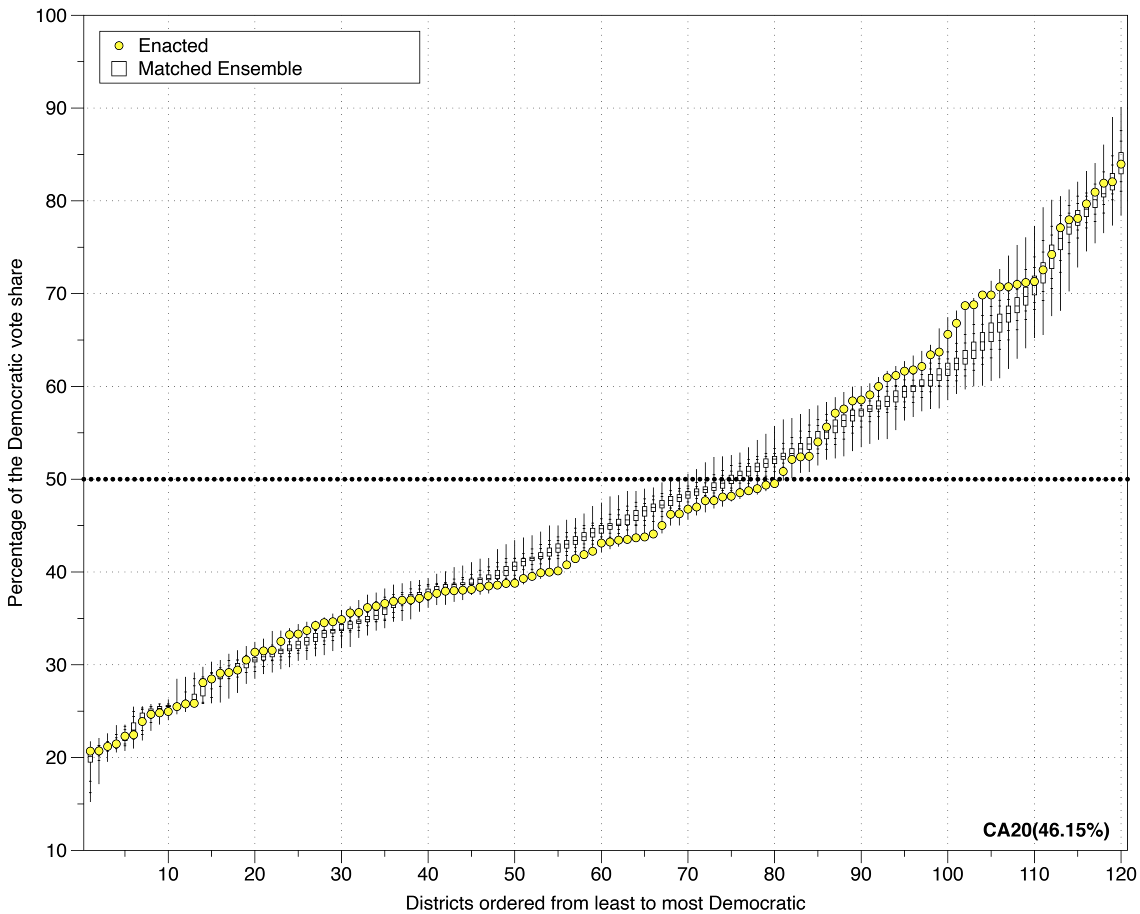
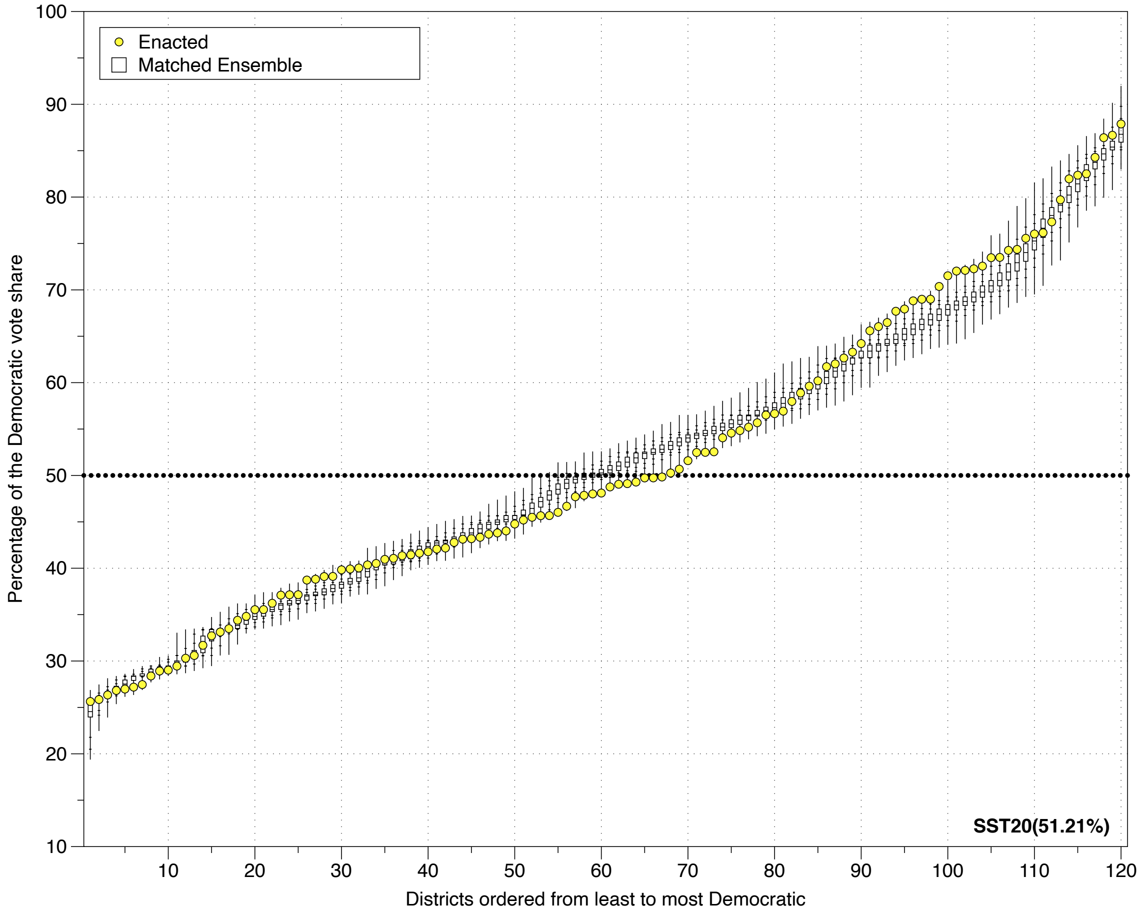
A NC House: Ranked-Ordered Marginal Boxplots

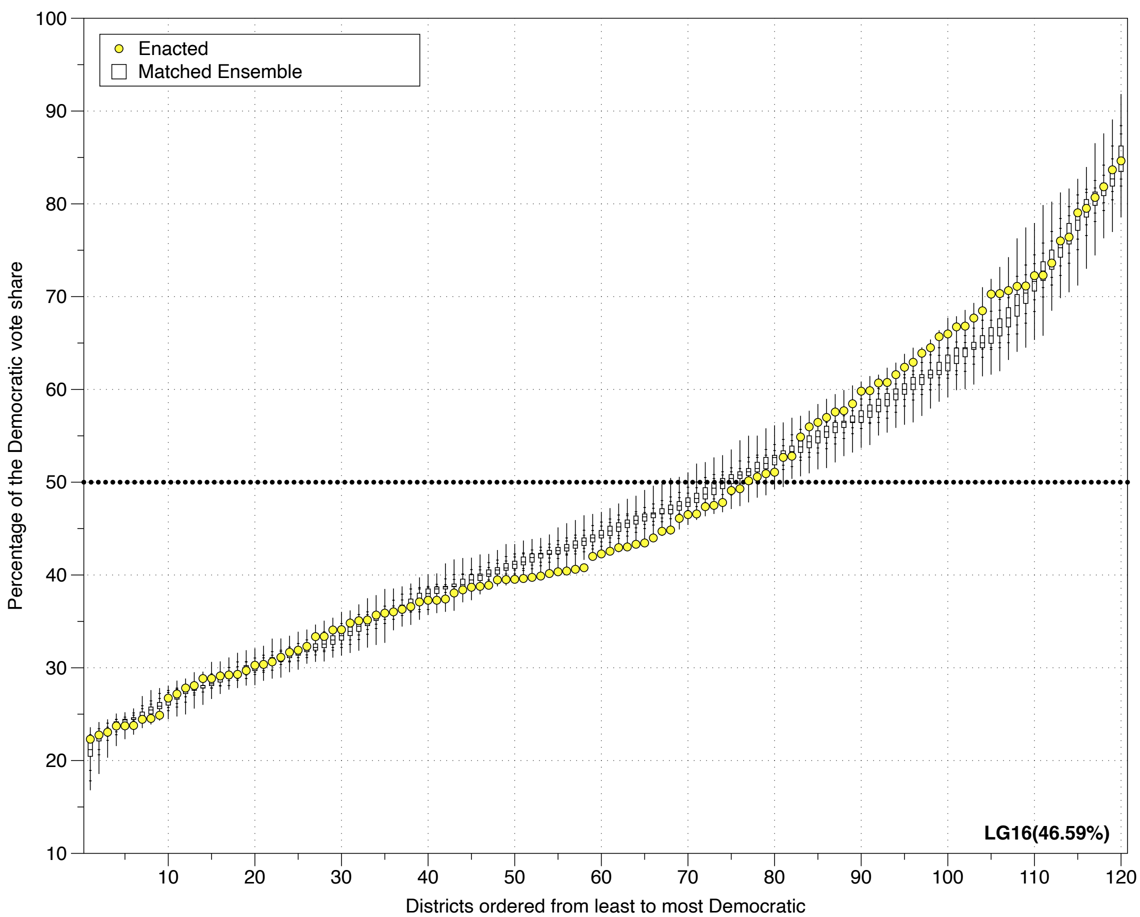


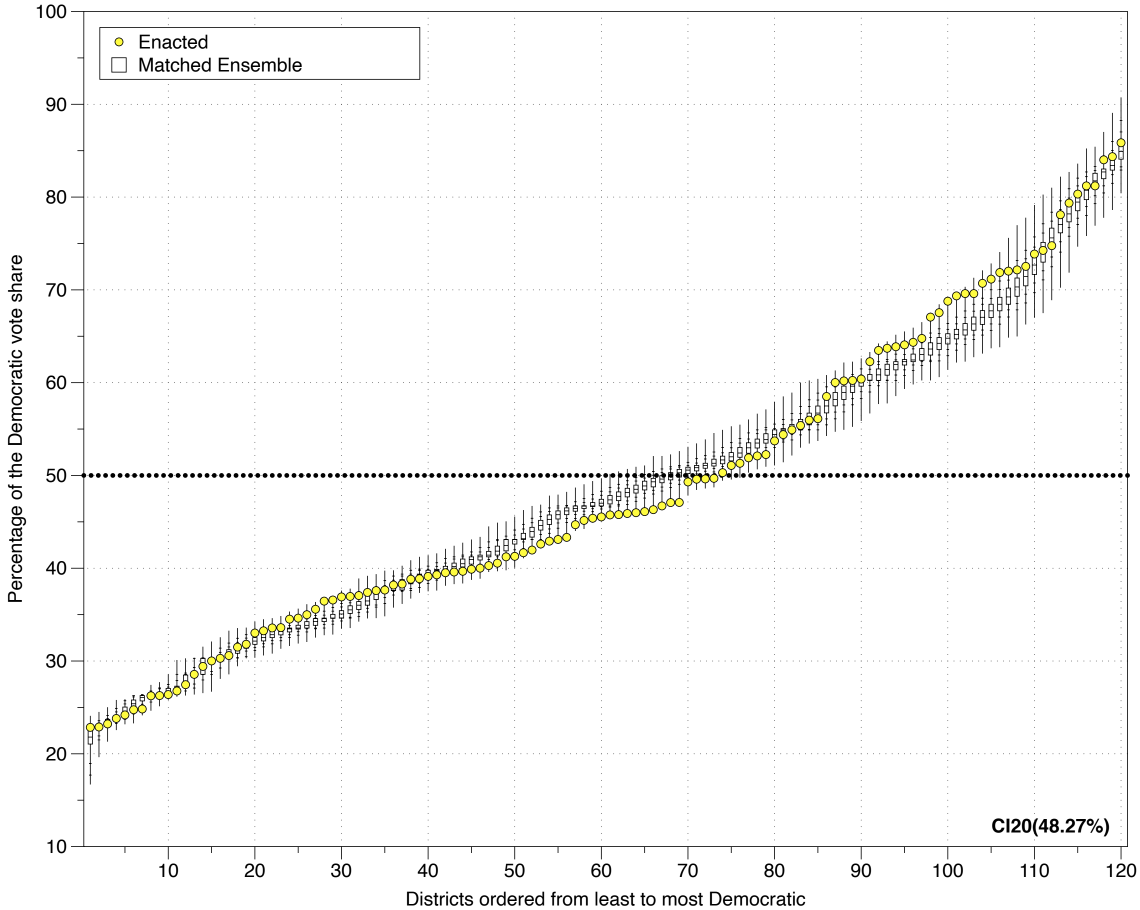



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
GV16(50.05%)

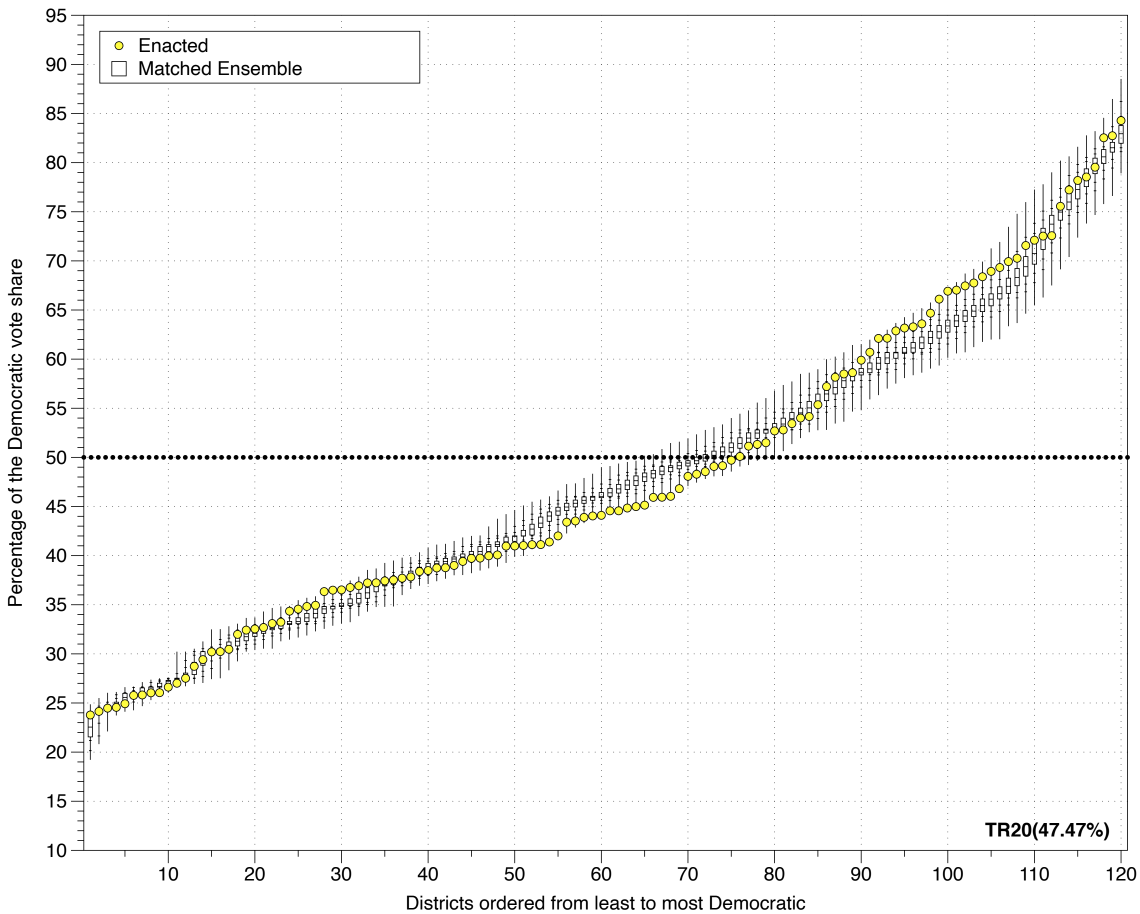

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
CA20(46.15%)


Enacted
Matched Ensemble
SST20(51.21%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
110
120

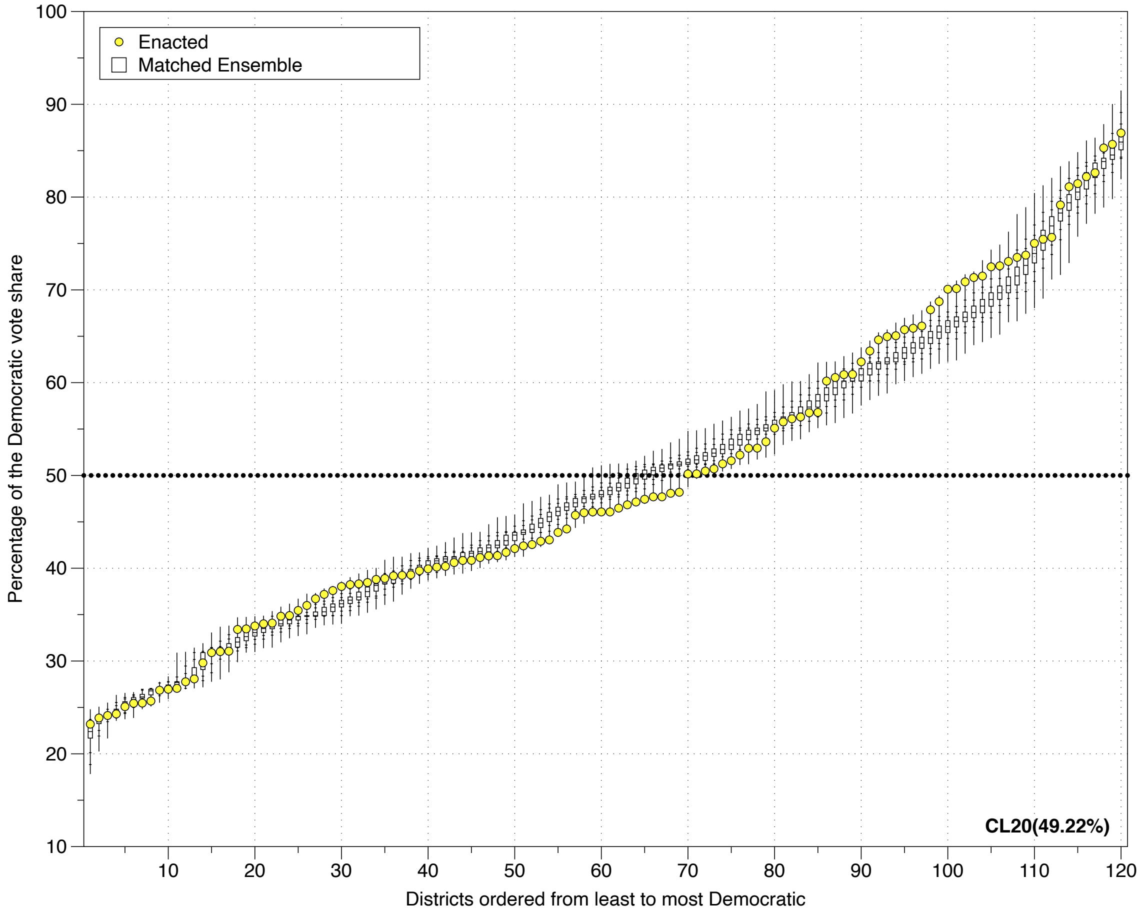

Enacted
Matched Ensemble
LG16(46.59%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
110
120



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
CI20(48.27%)



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
TR20(47.47%)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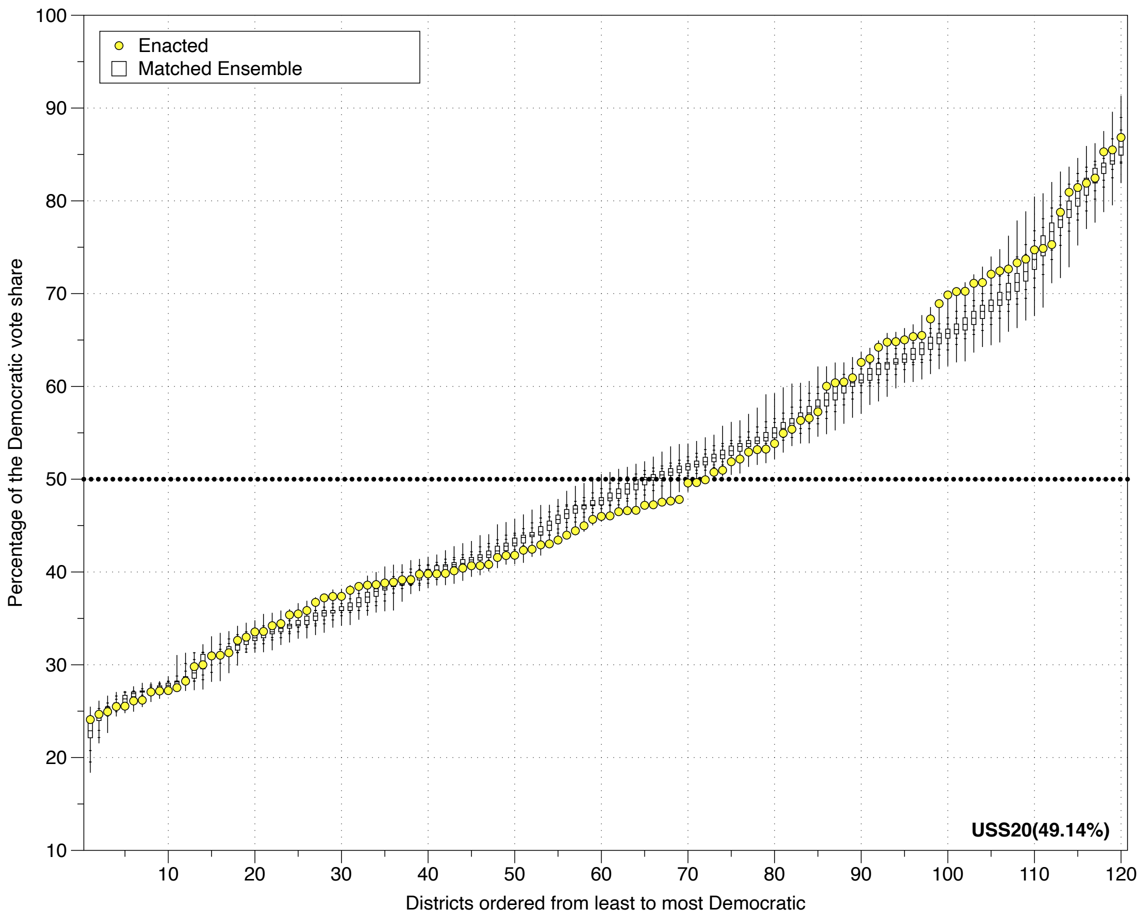
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
CL20(49.22%)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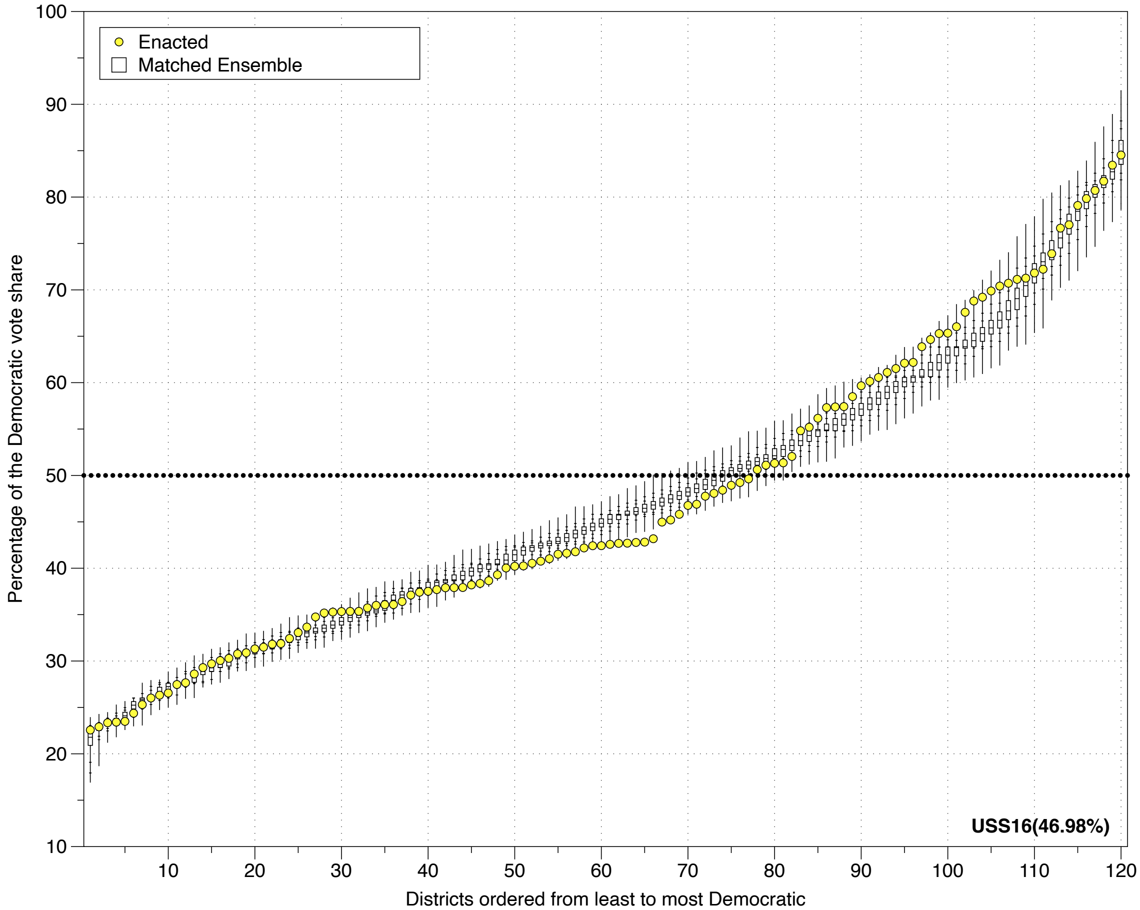
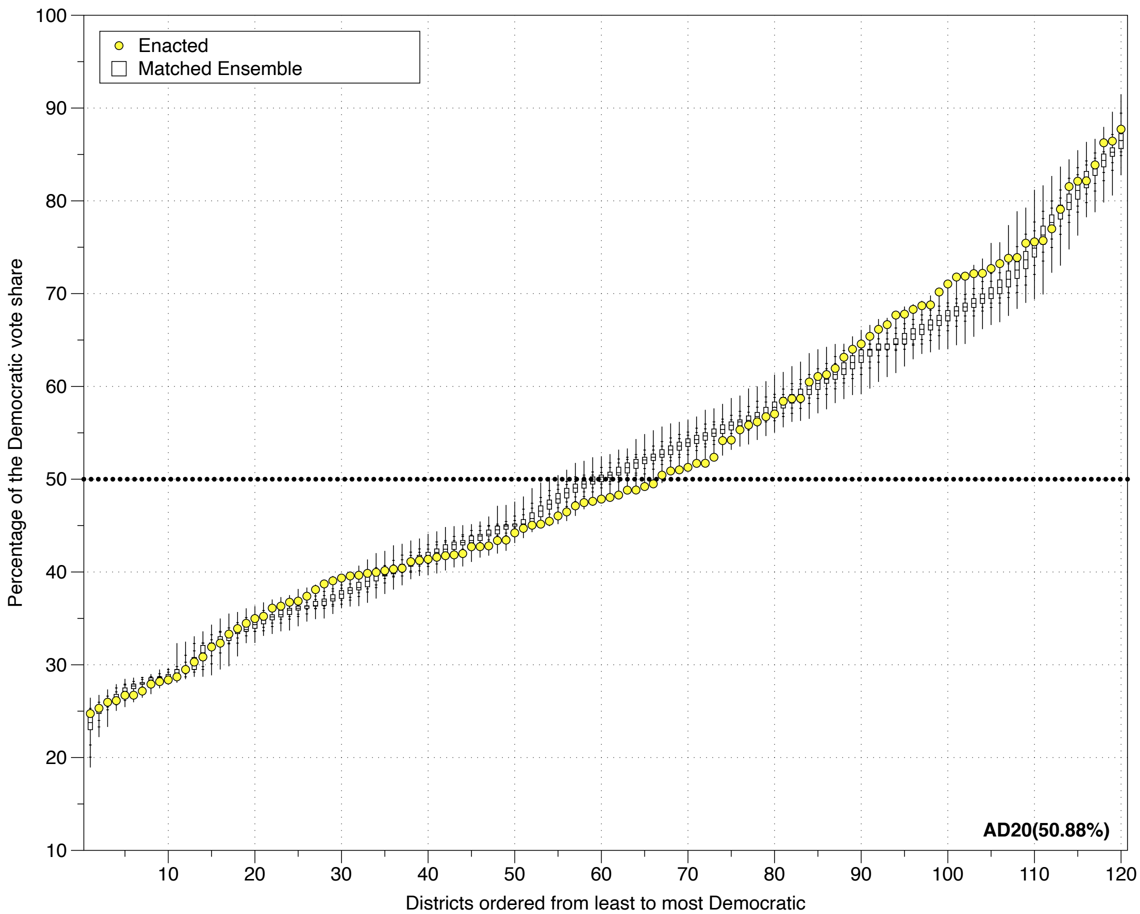
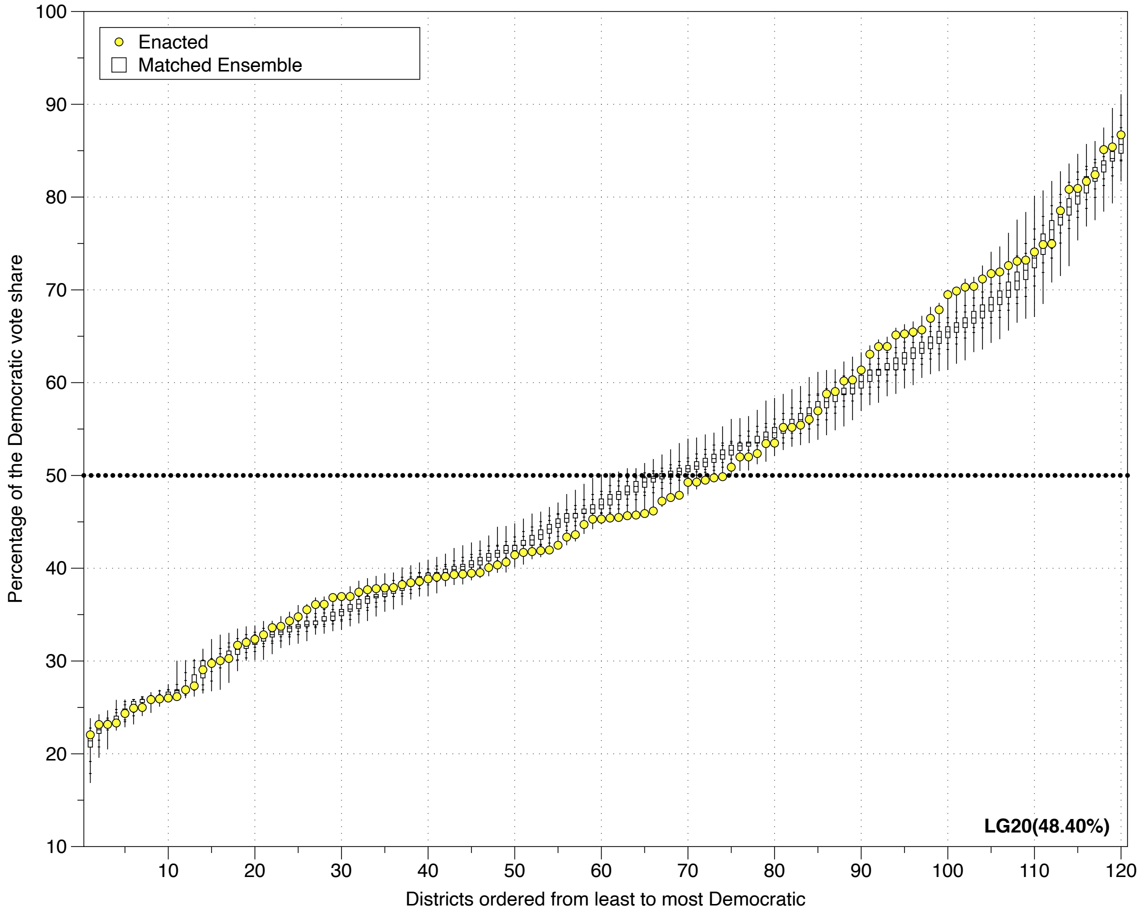
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
USS20(49.14%)



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
USS16(46.98%)



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
AD20(50.88%)



Enacted
Matched Ensemble
LG20(48.40%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
100
90
80
70
60
50
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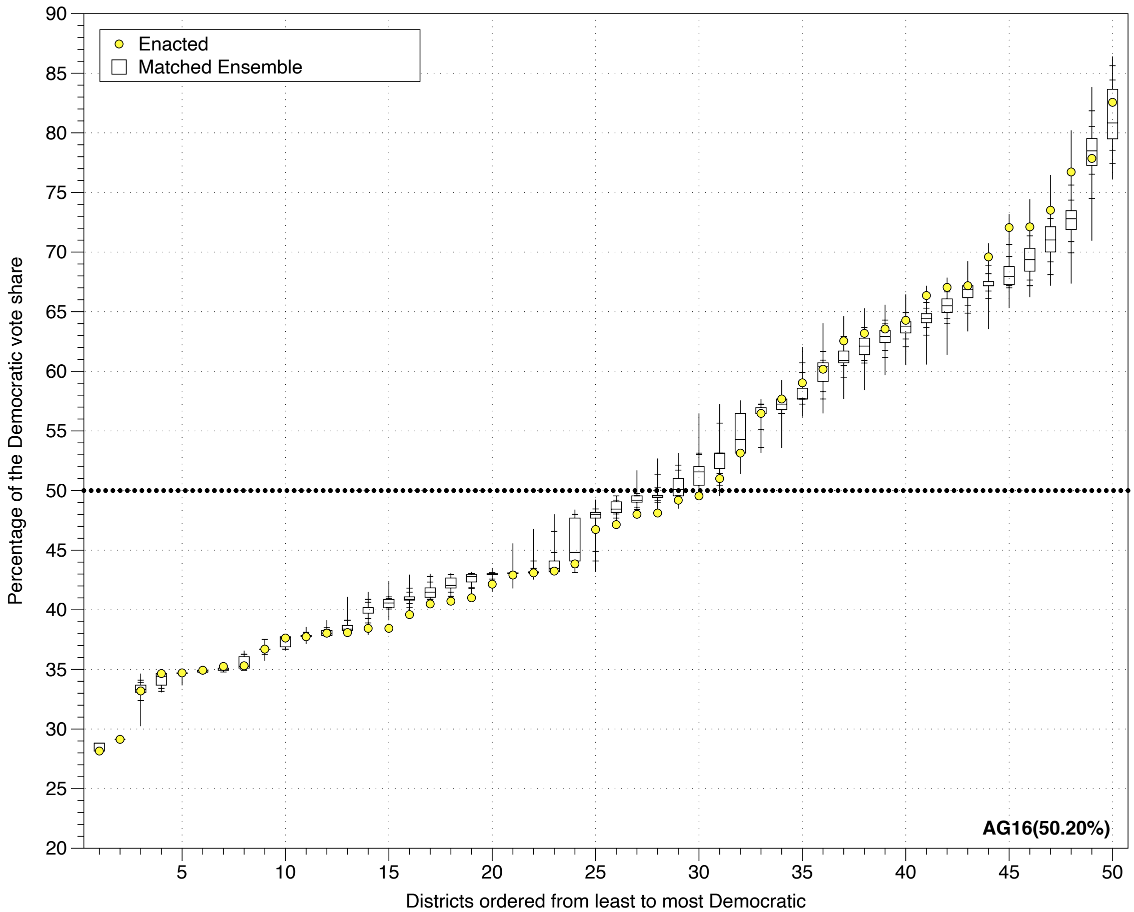
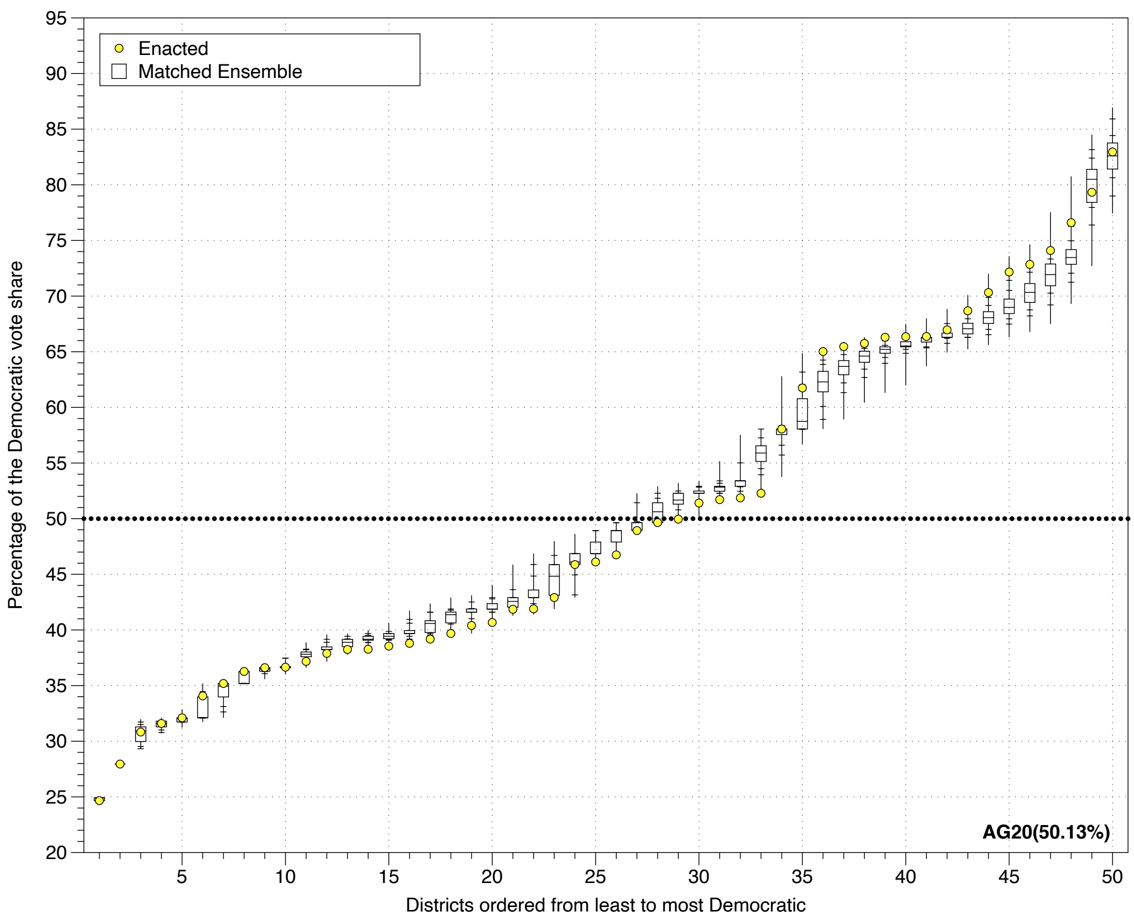
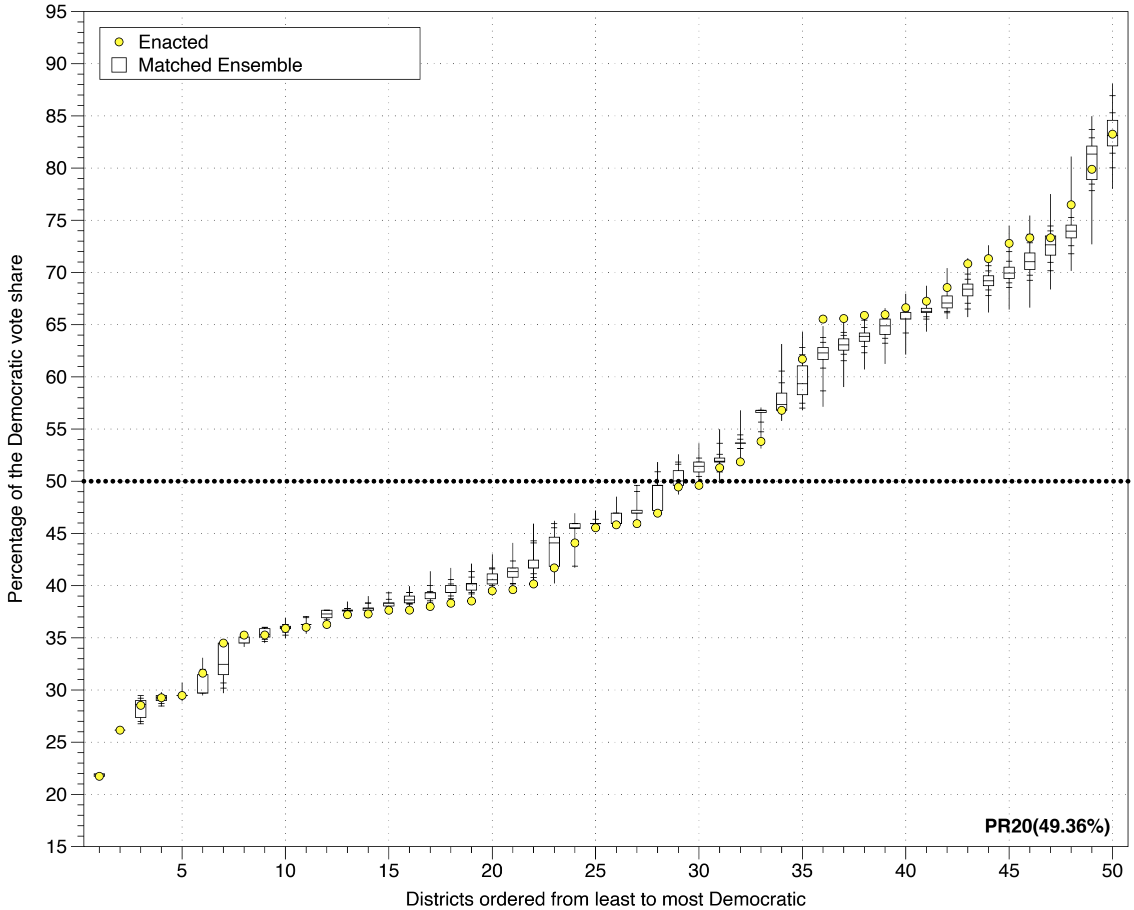
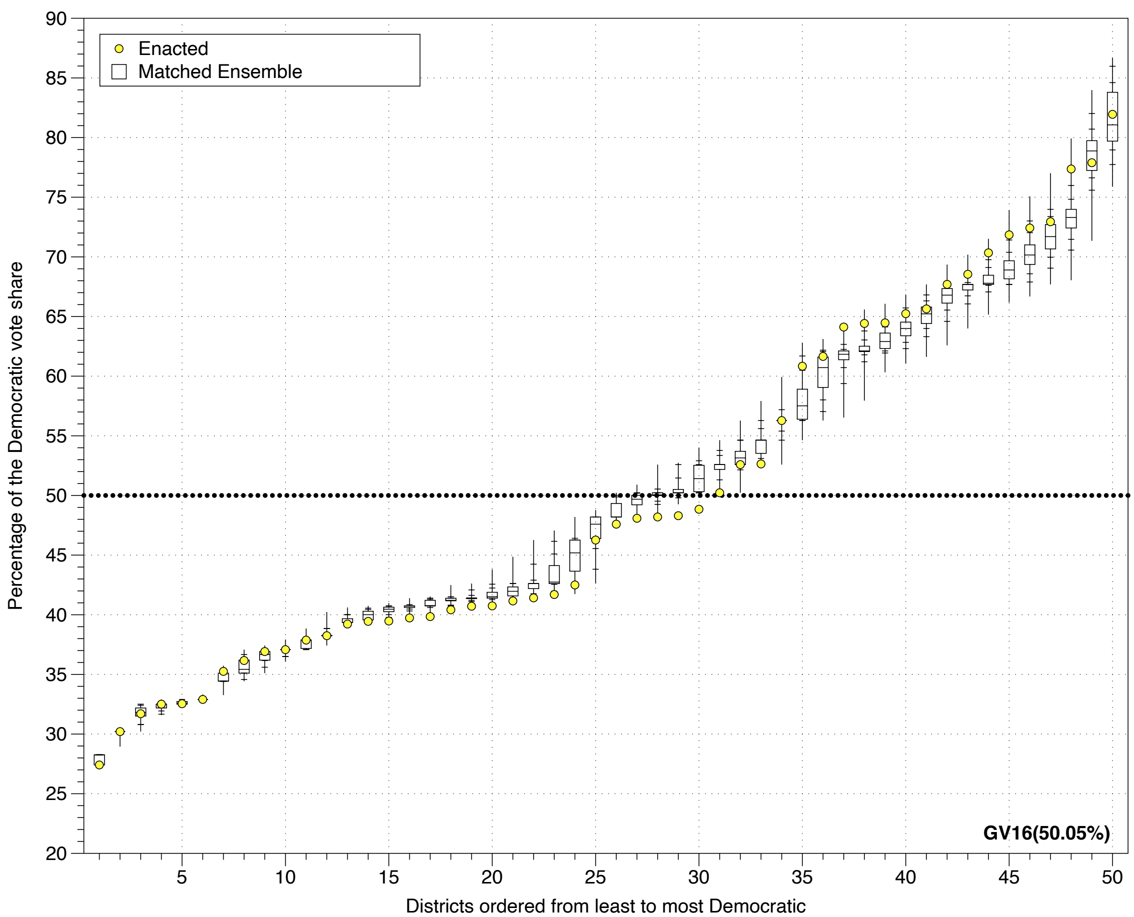
60
70
80
90
100
110
120

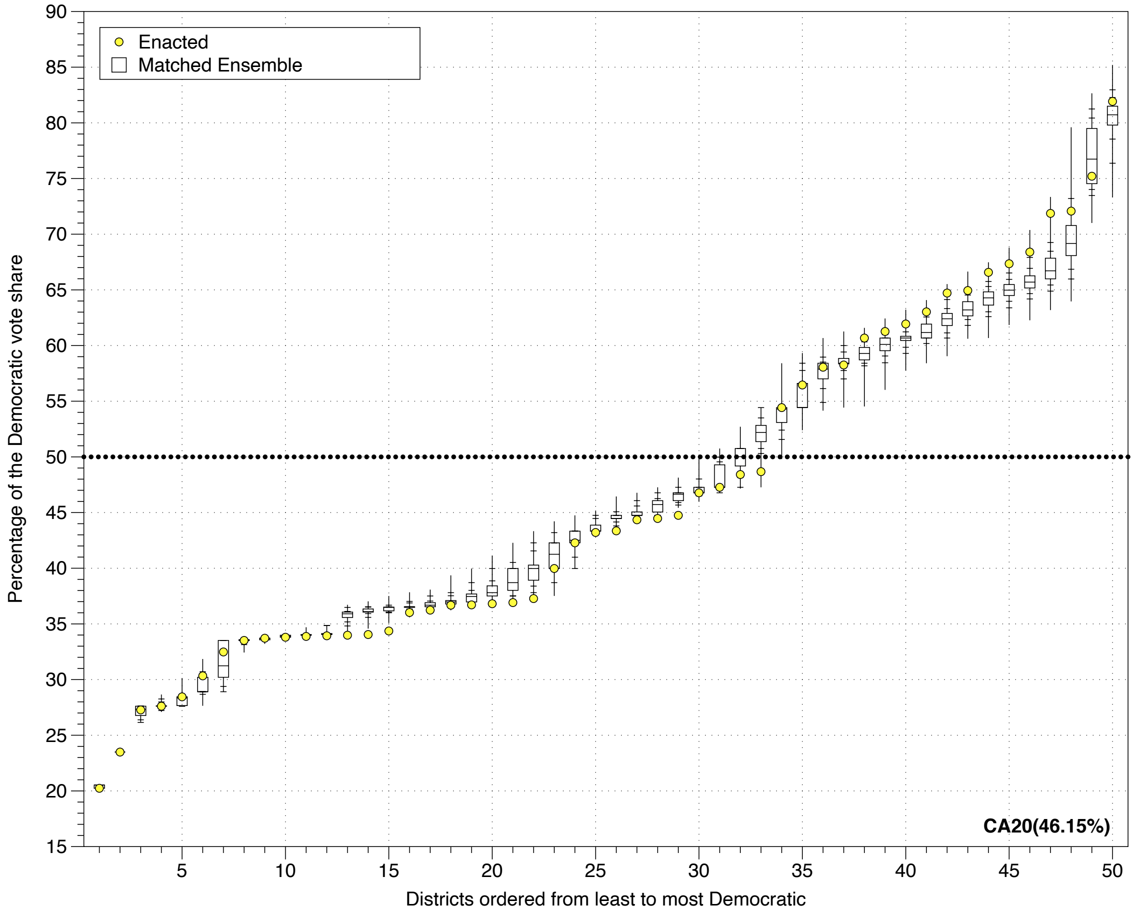
B NC Senate: Ranked-Ordered Marginal Boxplots

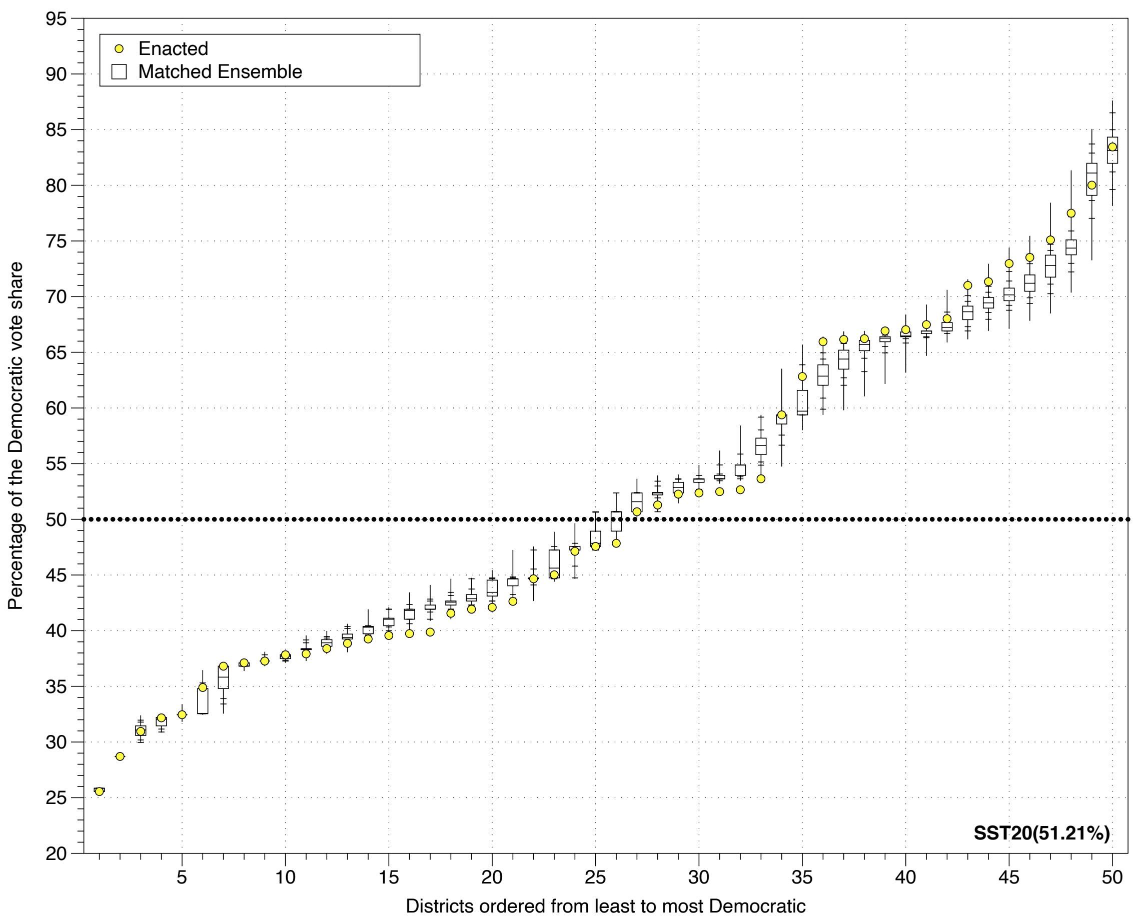


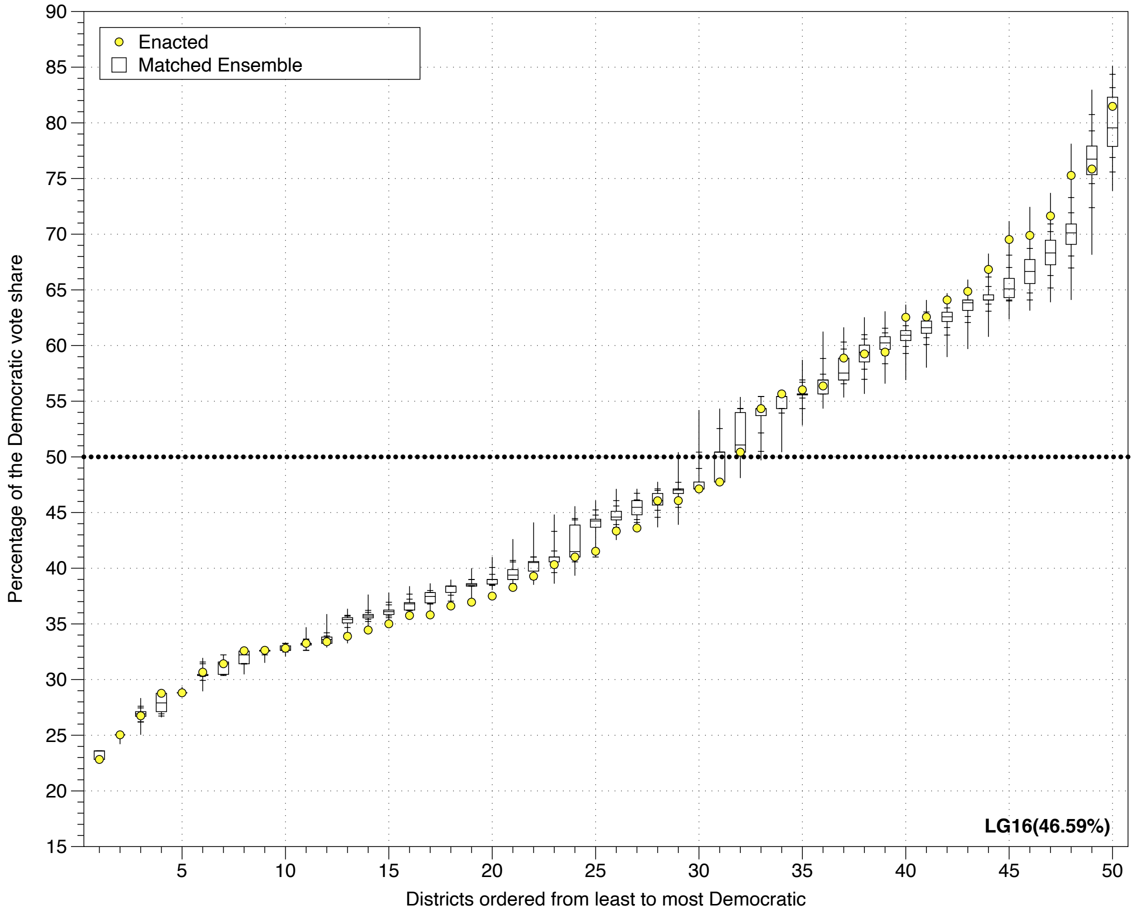


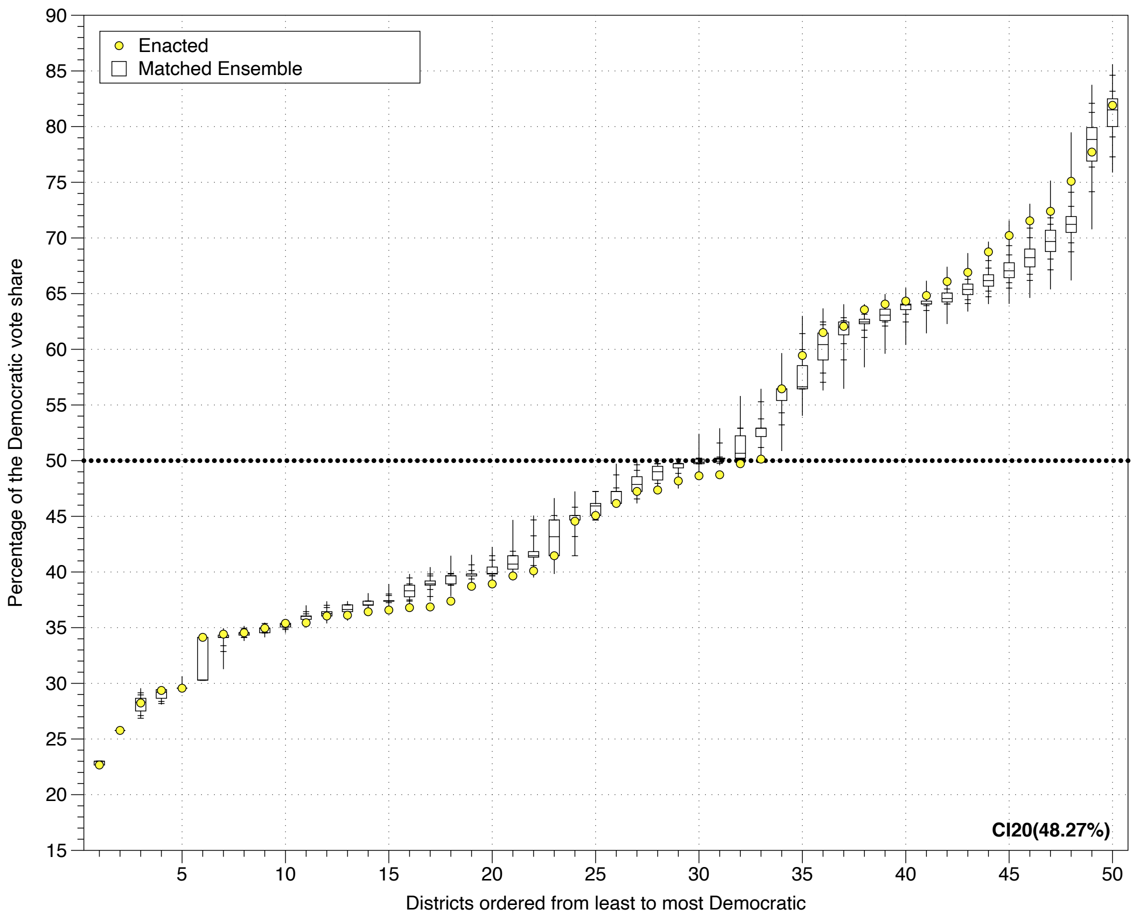

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
PR20(49.36%)

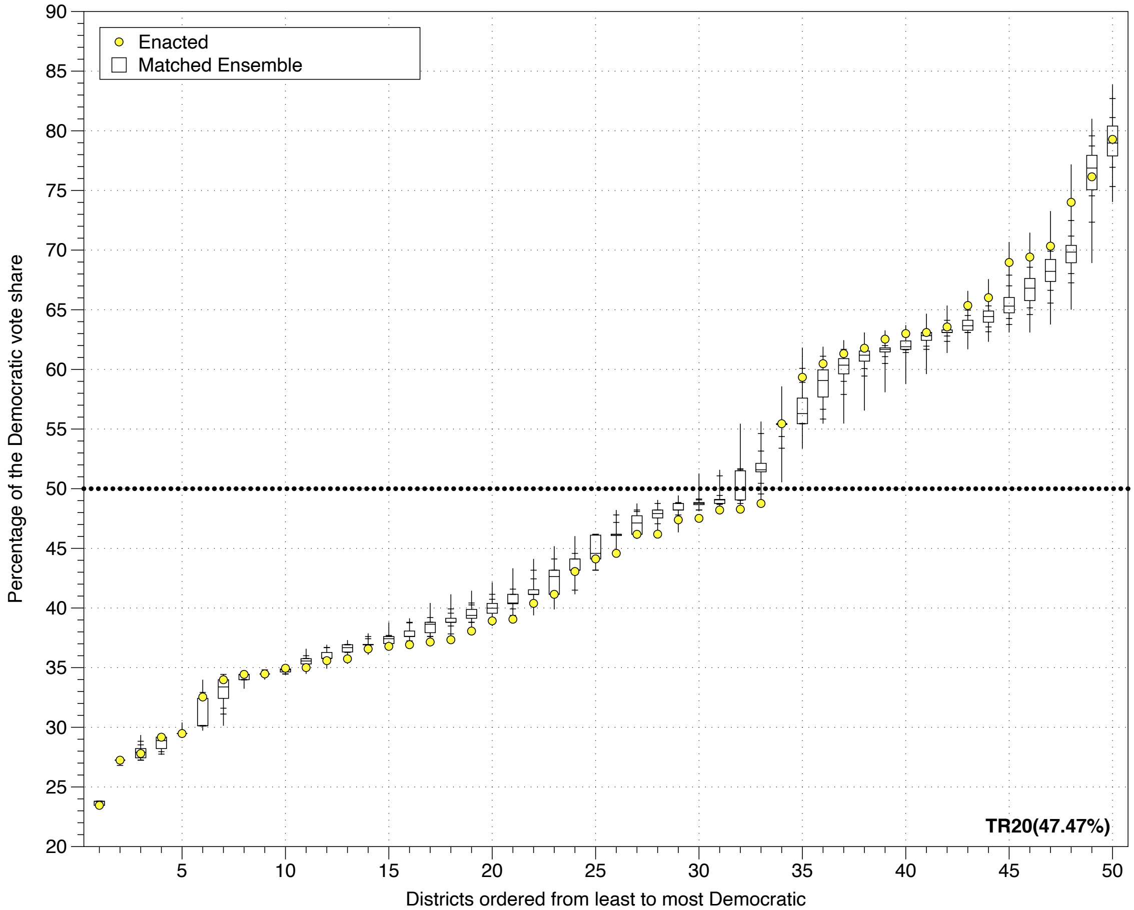

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
GV16(50.05%)

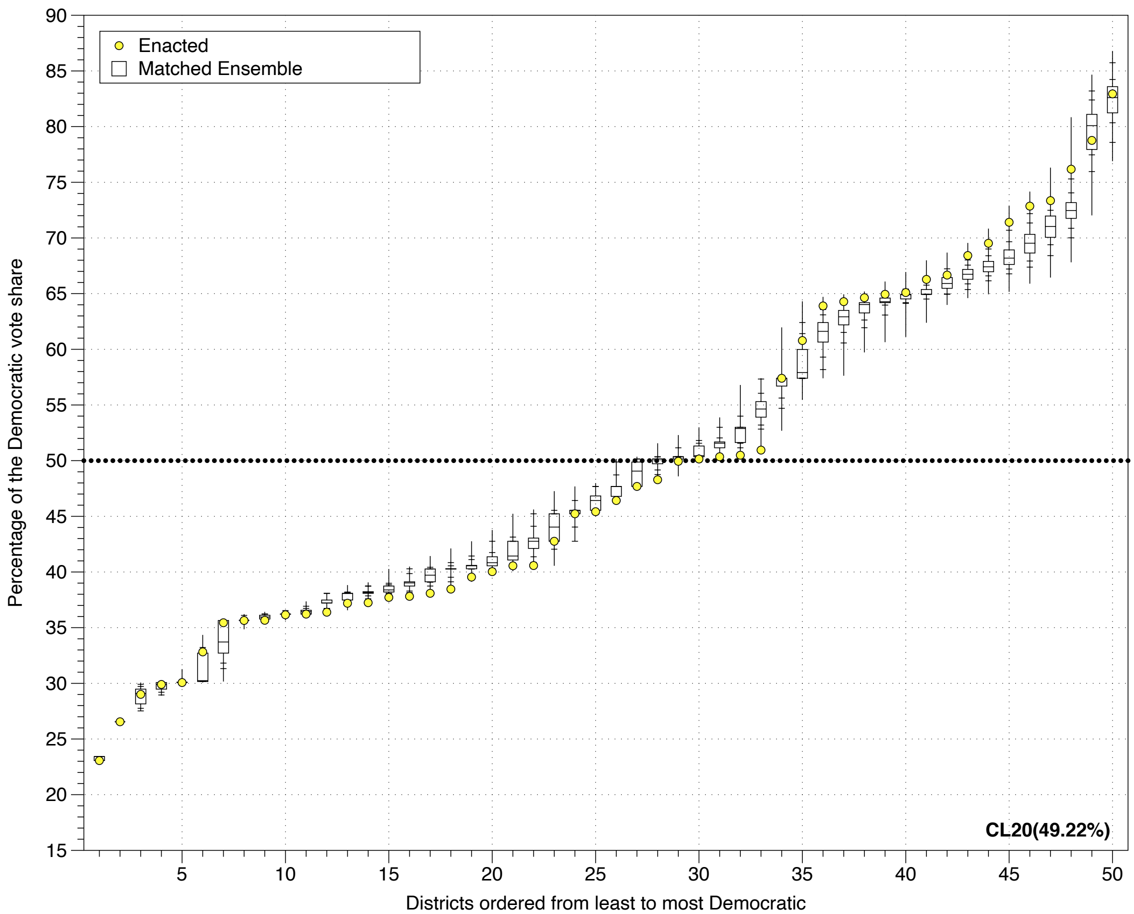

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
CA20(46.15%)

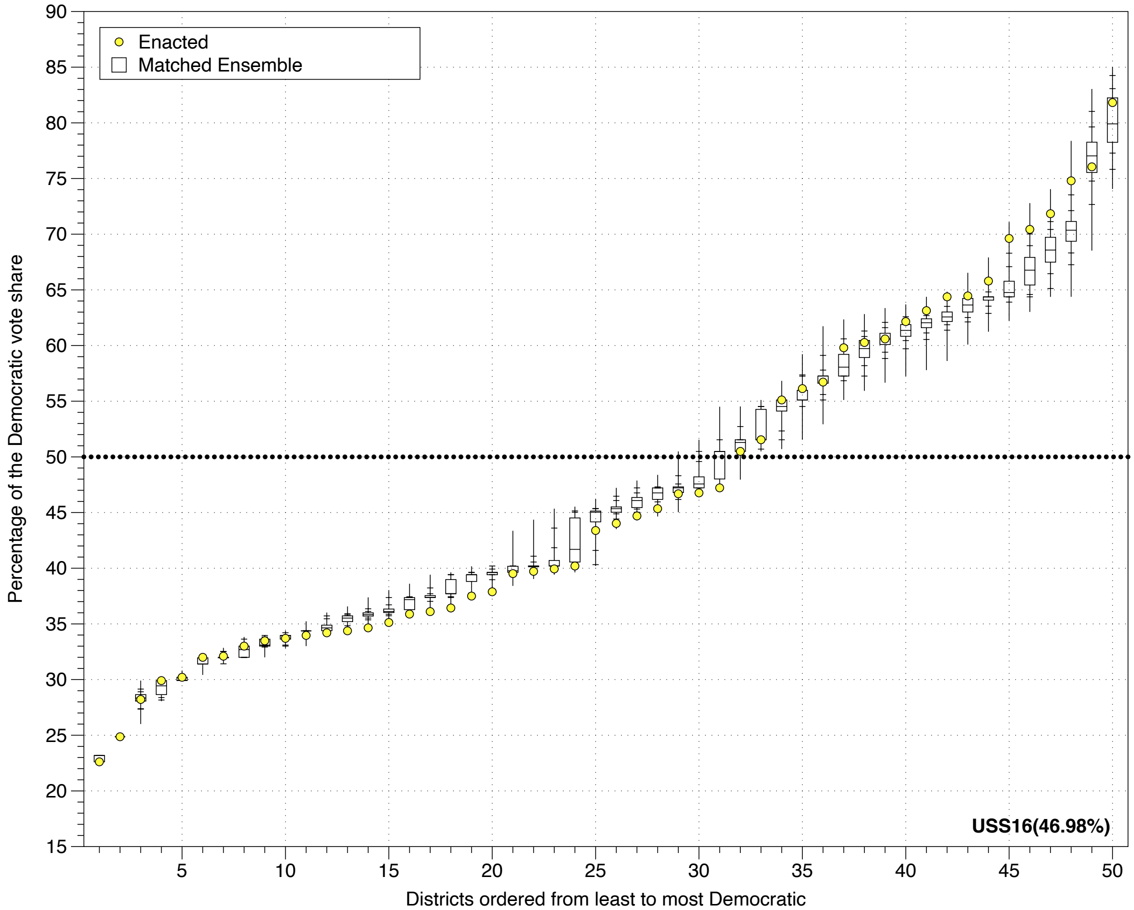

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
SST20(51.21%)


Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
LG16(46.59%)

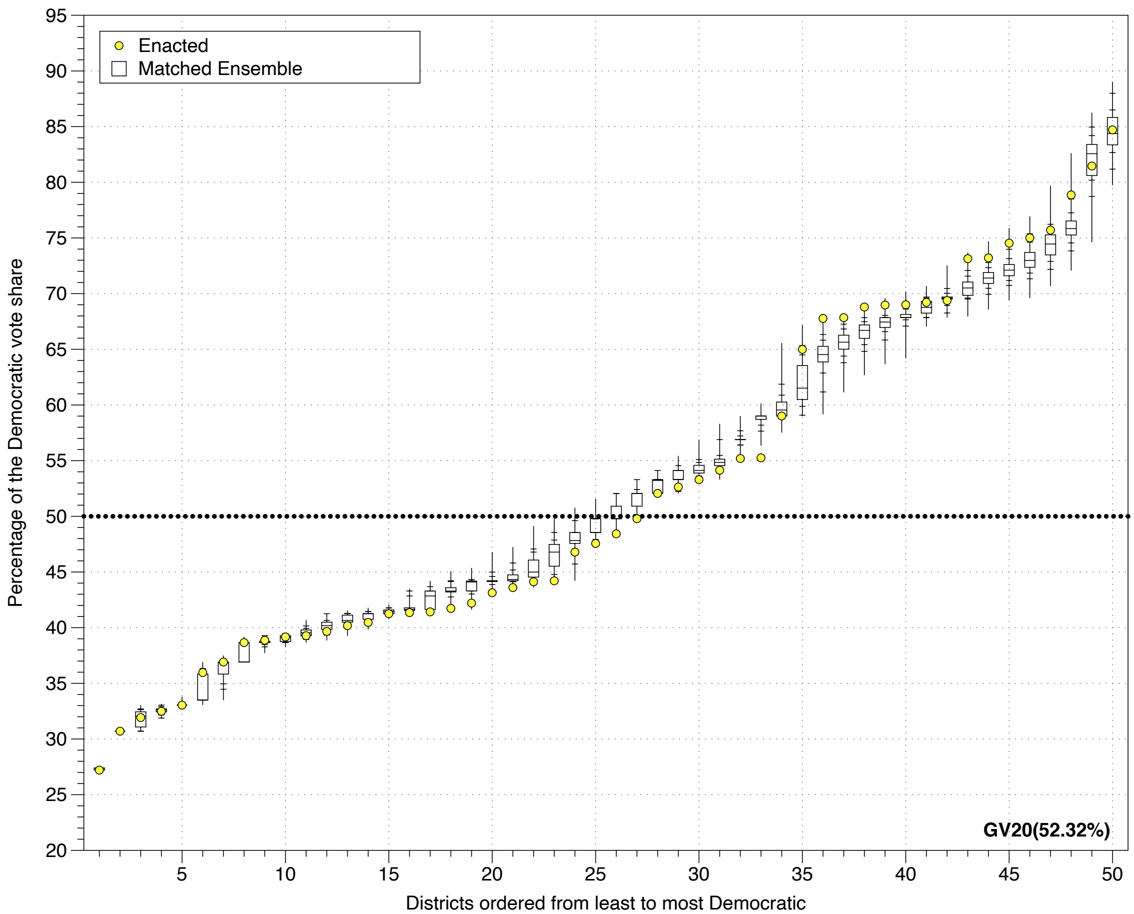

Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
CI20(48.27%)


Enacted
Matched Ensemble
TR20(47.47%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic

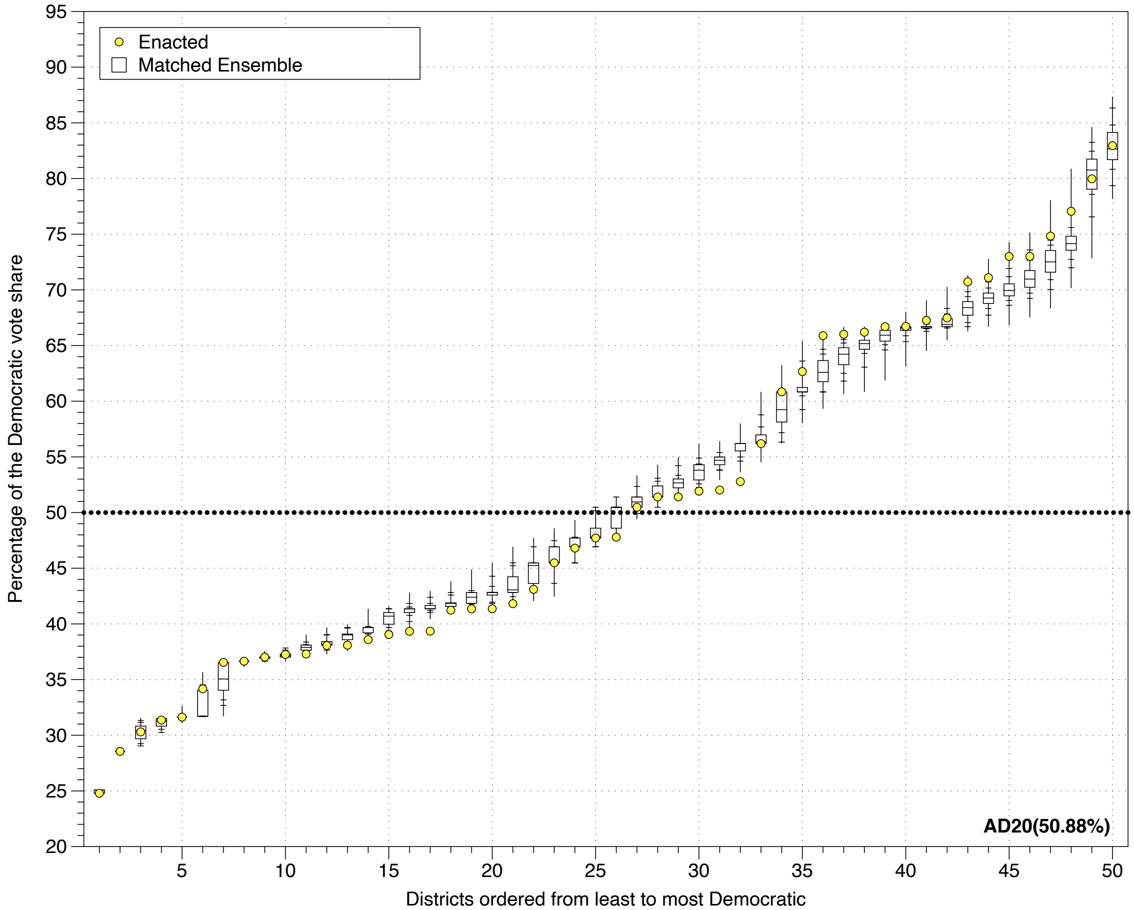

Enacted
Matched Ensemble
CL20(49.22%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
USS16(46.98%)



Enacted
Matched Ensemble
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
GV20(52.32%)


Enacted
Matched Ensemble
AD20(50.88%)
Percentage of the Democratic vote share
Districts ordered from least to most Democratic
95
90
85
80
75
70
65
60
55
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25
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50

C NC House: Additional Plots

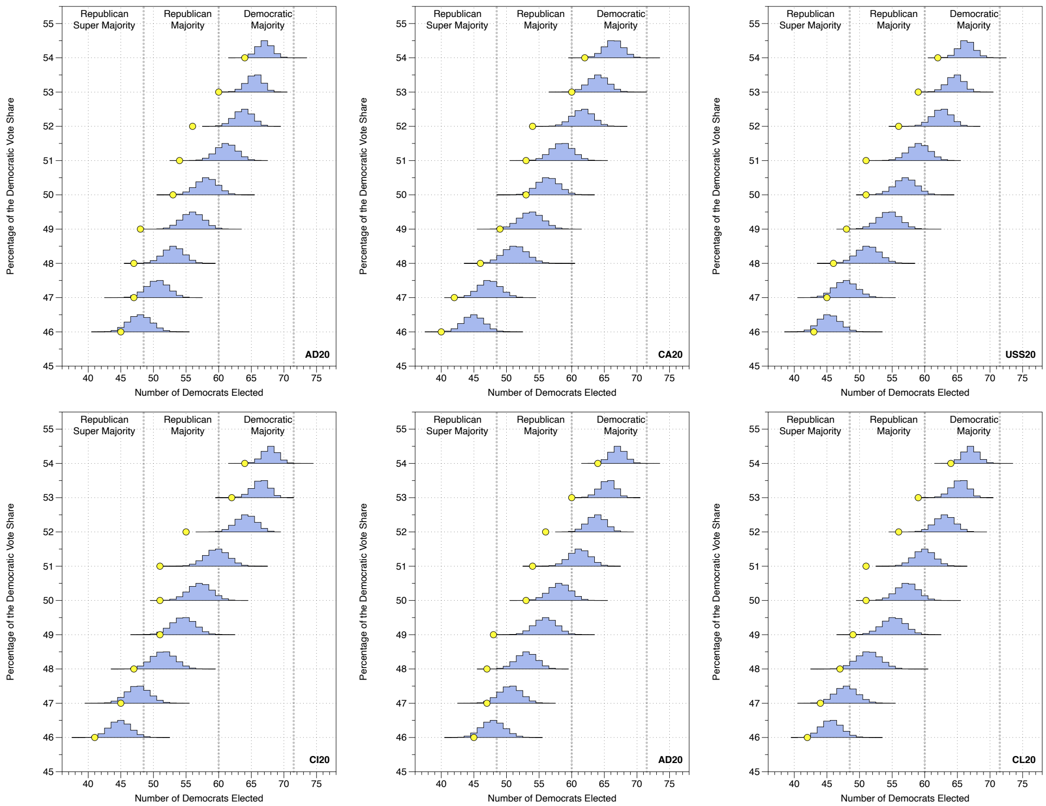

Figure C.0.1: The Collected Seat Histograms for the Primary Ensemble on the NC House built from a collection of voting data generated via uniform swing.

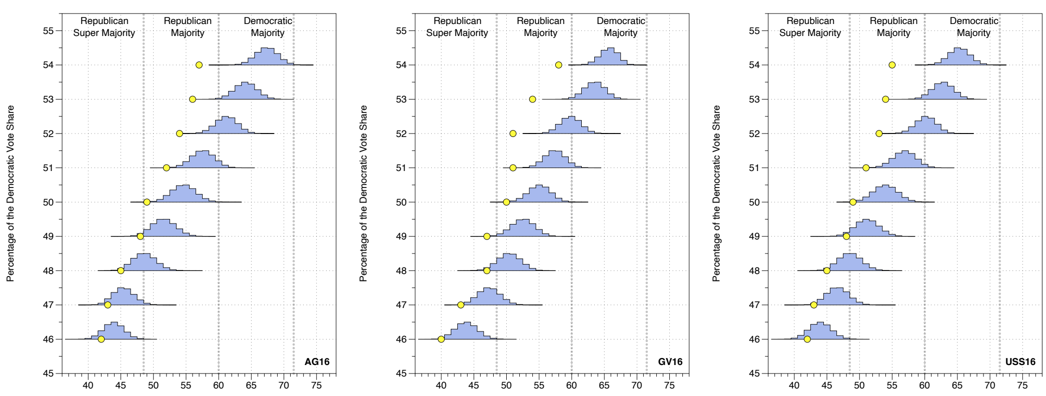

Figure C.0.2: The Collected Seat Histograms for the Primary Ensemble on the NC House built from a collection of voting data generated via uniform swing.

D NC Senate: Additional Plots

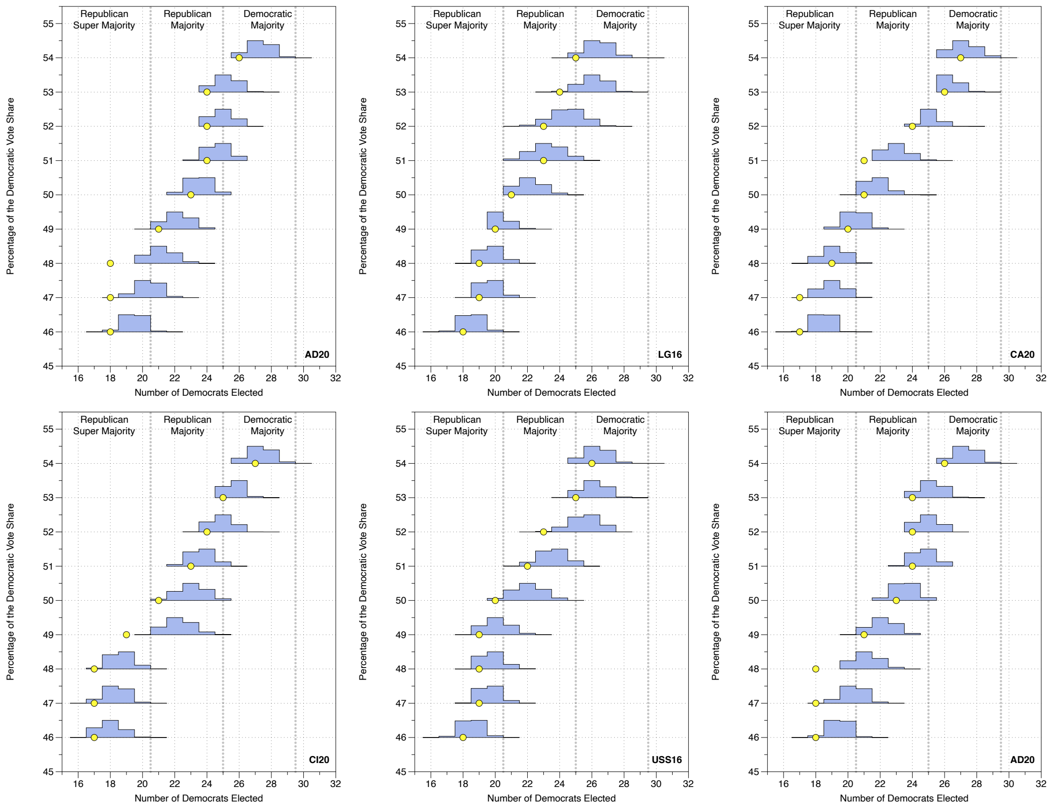

Figure D.0.1: The Collected Seat Histograms for the Primary Ensemble on the NC Senate built from a collection of voting data generated via uniform swing.

E NC Congressional: Ranked-Ordered Marginal Boxplots

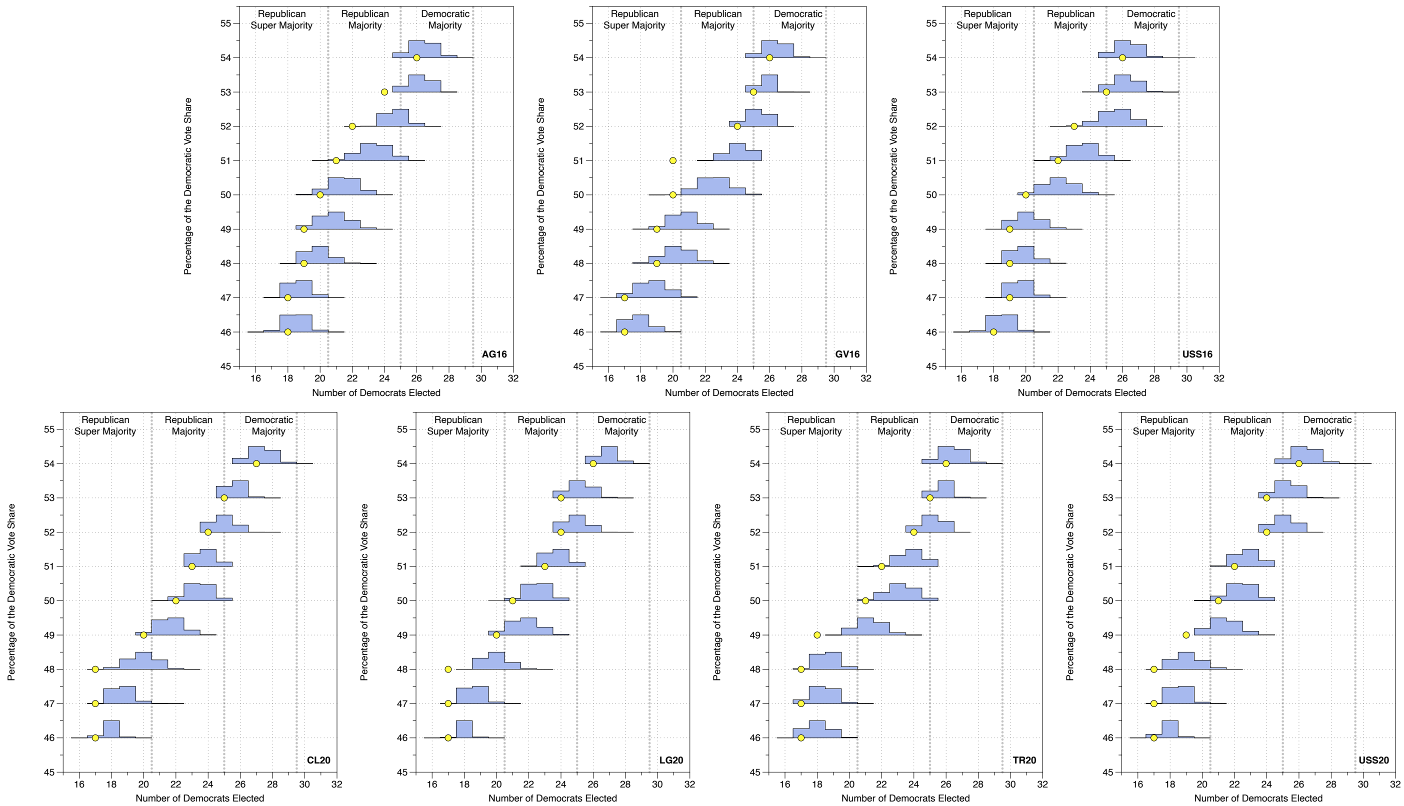

Figure D.0.2: The Collected Seat Histograms for the Primary Ensemble on the NC Senate built from a collection of voting data generated via uniform swing.

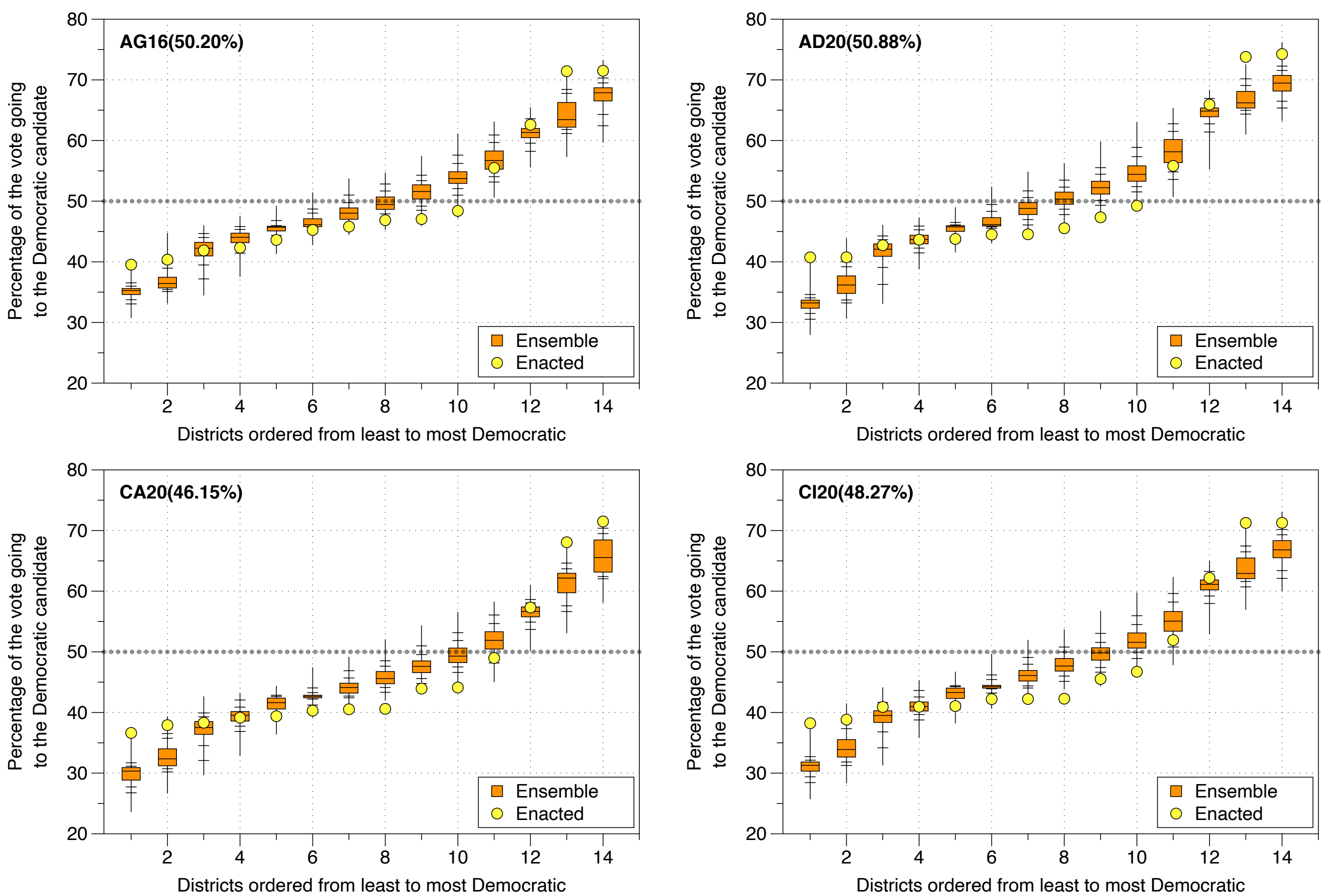

Figure E.0.1: something

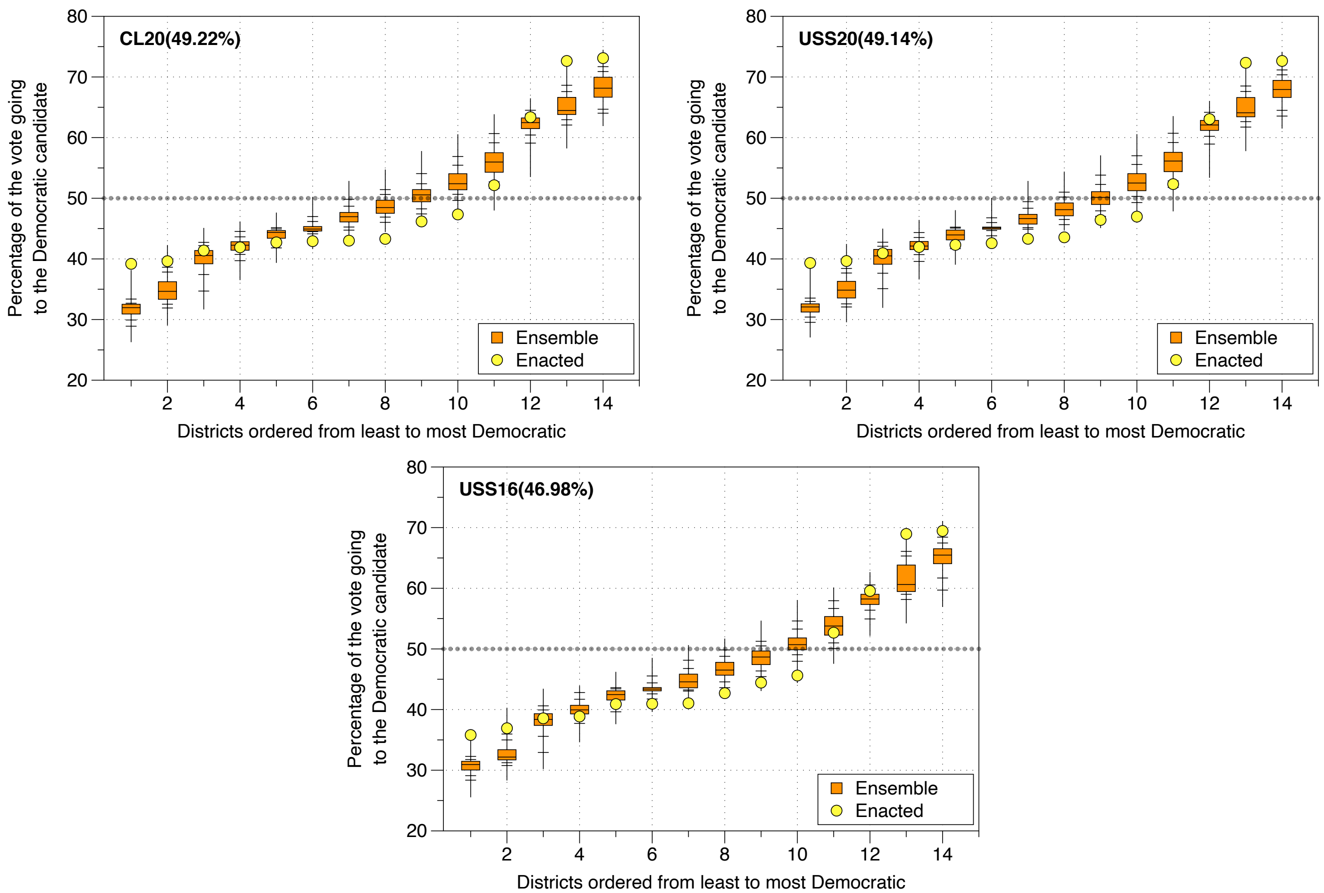

Figure E.0.2: something

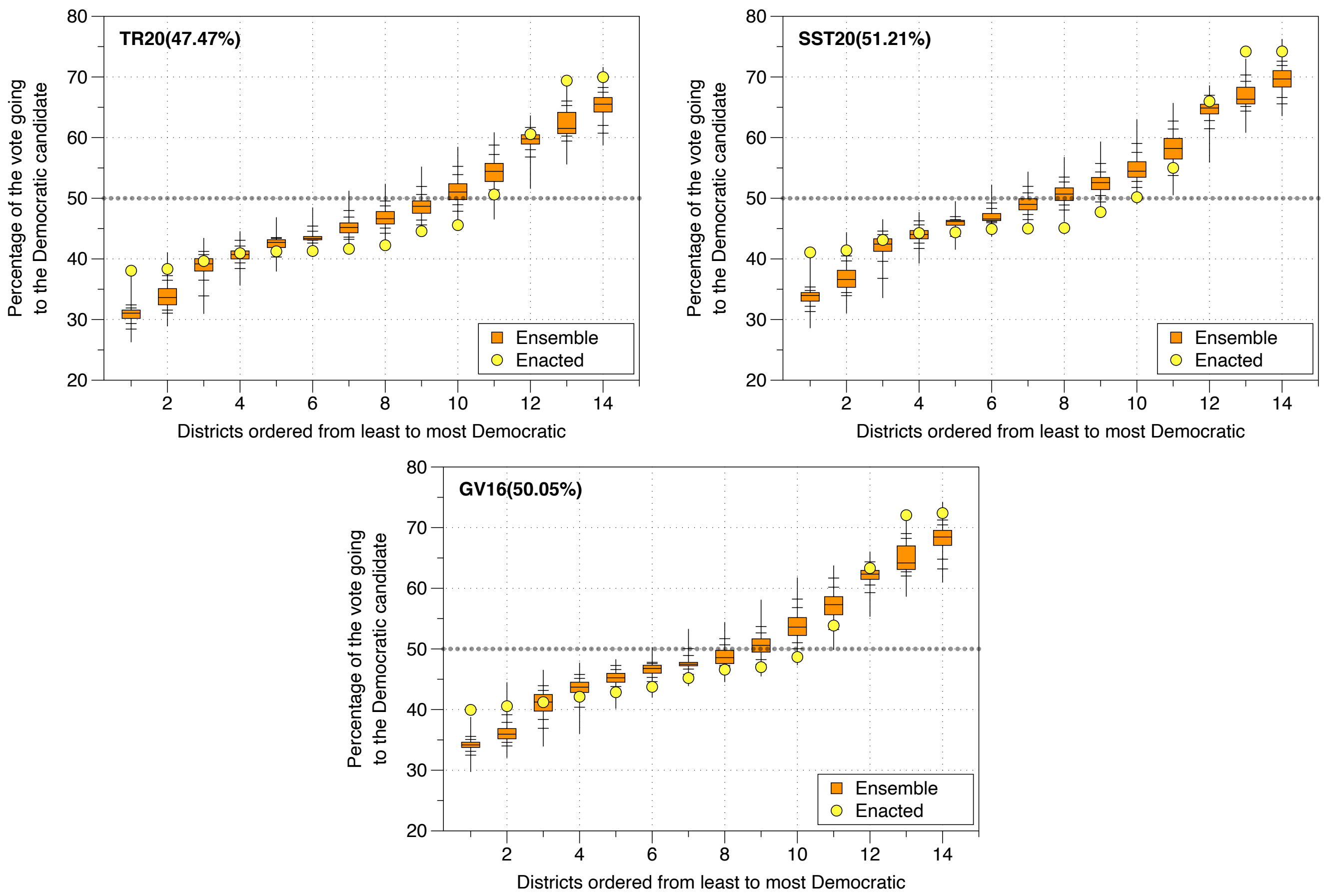

Figure E.0.3: something

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	13507	16.9	16380	20.5	79997	1	2
PR16	23688	29.6	25268	31.6	79997	1	2
AD20	7579	9.47	13561	17.0	79997	1	2
AG20	8831	11.0	14968	18.7	79997	1	2
CA20	7818	9.77	12779	16.0	79997	1	2
CL20	8308	10.4	14272	17.8	79997	1	2
GV20	14684	18.4	19730	24.7	79997	1	2
LG20	10040	12.6	15902	19.9	79997	1	2
PR20	15099	18.9	19674	24.6	79997	1	2
SST20	9265	11.6	15681	19.6	79997	1	2
TR20	10164	12.7	16049	20.1	79997	1	2
USS20	11197	14.0	16428	20.5	79997	1	2

Table 5: Alamance; house

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	384	0.48	2281	2.85	79997	2 3	4
PR16	288	0.36	4743	5.93	79997	2 3	4
AD20	72	0.09	5122	6.4	79997	2 3	4
AG20	64	0.08	5154	6.44	79997	2 3	4
CA20	48	0.06	4227	5.28	79997	2 3	4
CL20	56	0.07	4995	6.24	79997	2 3	4
GV20	200	0.25	6254	7.82	79997	2 3	4
LG20	80	0.1	5107	6.38	79997	2 3	4
PR20	128	0.16	5842	7.3	79997	2 3	4
SST20	72	0.09	5418	6.77	79997	2 3	4
TR20	80	0.1	4755	5.94	79997	2 3	4
USS20	56	0.07	4334	5.42	79997	2 3	4

Table 6: Brunswick-New Hanover; house

F Cluster-by-cluster outlier analysis

We quantify the visual trends seen in the cluster-by-cluster ordered marginal vote distributions. Similar to the analysis in Table 4, we group ranked districts and inquire how many plans in the ensemble have an average Democratic vote fraction that is more toward the extremes than the enacted plan. In general, lower numbers in the tables below signify more atypical clusters.

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	288	0.36	2406	3.01	79997	1	3
PR16	848	1.06	3910	4.89	79997	1	3
AD20	578	0.723	3738	4.67	79997	1	3
AG20	657	0.821	3711	4.64	79997	1	3
CA20	506	0.633	3072	3.84	79997	1	3
CL20	573	0.716	3578	4.47	79997	1	3
GV20	892	1.12	4803	6.0	79997	1	3
LG20	642	0.803	3699	4.62	79997	1	3
PR20	960	1.2	4790	5.99	79997	1	3
SST20	546	0.683	3305	4.13	79997	1	3
TR20	555	0.694	3295	4.12	79997	1	3
USS20	541	0.676	3404	4.26	79997	1	3

Table 7: Buncombe; house

Election	No. plans w/ $\geq$ Dems (First Cluster)	% of plans w/ $\geq$ Dems (First Cluster)	No. plans w/ $\leq$ Dems (Second Cluster)	% of plans w/ $\leq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	12935	16.2	12183	15.2	79997	3 4	5
PR16	13057	16.3	5371	6.71	79997	3 4	5
AD20	12585	15.7	1657	2.07	79997	3 4	5
AG20	12230	15.3	2081	2.6	79997	3 4	5
CA20	12445	15.6	1573	1.97	79997	3 4	5
CL20	12411	15.5	1785	2.23	79997	3 4	5
GV20	12167	15.2	1489	1.86	79997	3 4	5
LG20	12312	15.4	1789	2.24	79997	3 4	5
PR20	12320	15.4	921	1.15	79997	3 4	5
SST20	12059	15.1	1709	2.14	79997	3 4	5
TR20	12102	15.1	1537	1.92	79997	3 4	5
USS20	11901	14.9	1669	2.09	79997	3 4	5

Table 8: Cabarrus-Davie-Rowan-Yadkin; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	3767	4.71	13593	17.0	79997	2	3 4
PR16	5414	6.77	13064	16.3	79997	2	3 4
AD20	970	1.21	11880	14.9	79997	2	3 4
AG20	899	1.12	11149	13.9	79997	2	3 4
CA20	833	1.04	11167	14.0	79997	2	3 4
CL20	341	0.426	10790	13.5	79997	2	3 4
GV20	517	0.646	11339	14.2	79997	2	3 4
LG20	346	0.433	10829	13.5	79997	2	3 4
PR20	579	0.724	11315	14.1	79997	2	3 4
SST20	1206	1.51	12333	15.4	79997	2	3 4
TR20	587	0.734	10981	13.7	79997	2	3 4
USS20	360	0.45	10674	13.3	79997	2	3 4

Table 9: Cumberland; house

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	46063	57.6	46238	57.8	79997	1	2
PR16	43010	53.8	43894	54.9	79997	1	2
AD20	41097	51.4	41193	51.5	79997	1	2
AG20	38601	48.3	38516	48.1	79997	1	2
CA20	39051	48.8	39158	48.9	79997	1	2
CL20	38891	48.6	39038	48.8	79997	1	2
GV20	38179	47.7	38073	47.6	79997	1	2
LG20	38313	47.9	38392	48.0	79997	1	2
PR20	38660	48.3	38492	48.1	79997	1	2
SST20	41059	51.3	40686	50.9	79997	1	2
TR20	38891	48.6	39342	49.2	79997	1	2
USS20	38430	48.0	38734	48.4	79997	1	2

Table 10: Duplin-Wayne; house

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	0	0.0	2768	3.46	79997	1	3 4
PR16	0	0.0	409	0.511	79997	1	3 4
AD20	0	0.0	274	0.343	79997	1	3 4
AG20	0	0.0	312	0.39	79997	1	3 4
CA20	0	0.0	929	1.16	79997	1	3 4
CL20	0	0.0	417	0.521	79997	1	3 4
GV20	0	0.0	232	0.29	79997	1	3 4
LG20	0	0.0	328	0.41	79997	1	3 4
PR20	0	0.0	96	0.12	79997	1	3 4
SST20	0	0.0	296	0.37	79997	1	3 4
TR20	0	0.0	280	0.35	79997	1	3 4
USS20	0	0.0	497	0.621	79997	1	3 4

Table 11: Durham-Person; house

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	1	0.00125	659	0.824	79997	1 2 3	4 5
PR16	0	0.0	543	0.679	79997	1 2 3	4 5
AD20	8	0.01	952	1.19	79997	1 2 3	4 5
AG20	11	0.0138	1025	1.28	79997	1 2 3	4 5
CA20	11	0.0138	1032	1.29	79997	1 2 3	4 5
CL20	9	0.0113	995	1.24	79997	1 2 3	4 5
GV20	8	0.01	982	1.23	79997	1 2 3	4 5
LG20	8	0.01	980	1.23	79997	1 2 3	4 5
PR20	8	0.01	893	1.12	79997	1 2 3	4 5
SST20	0	0.0	912	1.14	79997	1 2 3	4 5
TR20	9	0.0113	944	1.18	79997	1 2 3	4 5
USS20	16	0.02	1106	1.38	79997	1 2 3	4 5

Table 12: Forsyth-Stokes; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	0	0.0	0	0.0	79997	1 2	3 4 5 6
PR16	0	0.0	0	0.0	79997	1 2	3 4 5 6
AD20	0	0.0	0	0.0	79997	1 2	3 4 5 6
AG20	0	0.0	0	0.0	79997	1 2	3 4 5 6
CA20	0	0.0	0	0.0	79997	1 2	3 4 5 6
CL20	0	0.0	0	0.0	79997	1 2	3 4 5 6
GV20	0	0.0	0	0.0	79997	1 2	3 4 5 6
LG20	0	0.0	0	0.0	79997	1 2	3 4 5 6
PR20	0	0.0	0	0.0	79997	1 2	3 4 5 6
SST20	0	0.0	0	0.0	79997	1 2	3 4 5 6
TR20	0	0.0	0	0.0	79997	1 2	3 4 5 6
USS20	0	0.0	0	0.0	79997	1 2	3 4 5 6

Table 13: Guilford; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	661	0.826	2	0.0025	79997	1 2 3 4	5 6 7 8
PR16	168	0.21	6	0.0075	79997	1 2 3 4	5 6 7 8
AD20	569	0.711	32	0.04	79997	1 2 3 4	5 6 7 8
AG20	763	0.954	35	0.0438	79997	1 2 3 4	5 6 7 8
CA20	1363	1.7	84	0.105	79997	1 2 3 4	5 6 7 8
CL20	1146	1.43	72	0.09	79997	1 2 3 4	5 6 7 8
GV20	396	0.495	40	0.05	79997	1 2 3 4	5 6 7 8
LG20	700	0.875	36	0.045	79997	1 2 3 4	5 6 7 8
PR20	202	0.253	19	0.0238	79997	1 2 3 4	5 6 7 8
SST20	496	0.62	29	0.0363	79997	1 2 3 4	5 6 7 8
TR20	975	1.22	88	0.11	79997	1 2 3 4	5 6 7 8
USS20	1082	1.35	69	0.0863	79997	1 2 3 4	5 6 7 8

Table 14: Mecklenburg; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	1194	1.49	899	1.12	79997	1	2
PR16	2115	2.64	1829	2.29	79997	1	2
AD20	8230	10.3	4317	5.4	79997	1	2
AG20	4434	5.54	2326	2.91	79997	1	2
CA20	2295	2.87	1334	1.67	79997	1	2
CL20	4069	5.09	2163	2.7	79997	1	2
GV20	6311	7.89	3379	4.22	79997	1	2
LG20	4123	5.15	2222	2.78	79997	1	2
PR20	6573	8.22	3564	4.46	79997	1	2
SST20	5386	6.73	2656	3.32	79997	1	2
TR20	4243	5.3	2177	2.72	79997	1	2
USS20	3799	4.75	2074	2.59	79997	1	2

Table 15: Pitt; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	209	0.261	6107	7.63	79997	1 2	3 4 5 6 7 8 9
PR16	160	0.2	4317	5.4	79997	1 2	3 4 5 6 7 8 9
AD20	240	0.3	4968	6.21	79997	1 2	3 4 5 6 7 8 9
AG20	230	0.288	4728	5.91	79997	1 2	3 4 5 6 7 8 9
CA20	1151	1.44	15113	18.9	79997	1 2	3 4 5 6 7 8 9
CL20	337	0.421	6643	8.3	79997	1 2	3 4 5 6 7 8 9
GV20	225	0.281	3777	4.72	79997	1 2	3 4 5 6 7 8 9
LG20	298	0.373	5552	6.94	79997	1 2	3 4 5 6 7 8 9
PR20	241	0.301	4462	5.58	79997	1 2	3 4 5 6 7 8 9
SST20	291	0.364	4572	5.72	79997	1 2	3 4 5 6 7 8 9
TR20	377	0.471	7229	9.04	79997	1 2	3 4 5 6 7 8 9
USS20	354	0.443	6912	8.64	79997	1 2	3 4 5 6 7 8 9

Table 16: Wake; house

Election	No. plans w/ $\leq$ Dems (First Cluster)	% of plans w/ $\leq$ Dems (First Cluster)	No. plans w/ $\geq$ Dems (Second Cluster)	% of plans w/ $\geq$ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	48	0.06	0	0.0	79997	1	2
PR16	48	0.06	48	0.06	79997	1	2
AD20	48	0.06	48	0.06	79997	1	2
AG20	48	0.06	48	0.06	79997	1	2
CA20	48	0.06	48	0.06	79997	1	2
CL20	48	0.06	48	0.06	79997	1	2
GV20	48	0.06	48	0.06	79997	1	2
LG20	48	0.06	48	0.06	79997	1	2
PR20	48	0.06	48	0.06	79997	1	2
SST20	48	0.06	48	0.06	79997	1	2
TR20	48	0.06	48	0.06	79997	1	2
USS20	48	0.06	48	0.06	79997	1	2

Table 17: Cumberland-Moore; senate

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	855	1.07	3472	4.34	79997	1	2
PR16	600	0.75	1822	2.28	79997	1	2
AD20	506	0.633	1745	2.18	79997	1	2
AG20	595	0.744	2455	3.07	79997	1	2
CA20	570	0.713	2521	3.15	79997	1	2
CL20	550	0.688	2191	2.74	79997	1	2
GV20	471	0.589	1496	1.87	79997	1	2
LG20	485	0.606	1967	2.46	79997	1	2
PR20	447	0.559	1392	1.74	79997	1	2
SST20	515	0.644	1827	2.28	79997	1	2
TR20	646	0.808	2696	3.37	79997	1	2
USS20	498	0.623	2174	2.72	79997	1	2

Table 18: Forsyth-Stokes; senate

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	0	0.0	6	0.015	39991	1 2	3 4 5 6
PR16	0	0.0	3	0.0075	39991	1 2	3 4 5 6
AD20	0	0.0	0	0.0	39991	1 2	3 4 5 6
AG20	0	0.0	0	0.0	39991	1 2	3 4 5 6
CA20	0	0.0	9	0.0225	39991	1 2	3 4 5 6
CL20	0	0.0	4	0.01	39991	1 2	3 4 5 6
GV20	0	0.0	0	0.0	39991	1 2	3 4 5 6
LG20	0	0.0	0	0.0	39991	1 2	3 4 5 6
PR20	0	0.0	0	0.0	39991	1 2	3 4 5 6
SST20	0	0.0	0	0.0	39991	1 2	3 4 5 6
TR20	0	0.0	5	0.0125	39991	1 2	3 4 5 6
USS20	0	0.0	4	0.01	39991	1 2	3 4 5 6

Table 19: Granville-Wake; senate

Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
LG16	0	0.0	13	0.0163	79997	1	3
PR16	0	0.0	13	0.0163	79997	1	3
AD20	0	0.0	54	0.0675	79997	1	3
AG20	0	0.0	33	0.0413	79997	1	3
CA20	0	0.0	15	0.0188	79997	1	3
CL20	0	0.0	23	0.0288	79997	1	3
GV20	0	0.0	56	0.07	79997	1	3
LG20	0	0.0	22	0.0275	79997	1	3
PR20	0	0.0	59	0.0738	79997	1	3
SST20	0	0.0	32	0.04	79997	1	3
TR20	0	0.0	20	0.025	79997	1	3
USS20	0	0.0	23	0.0288	79997	1	3

Table 20: Guilford-Rockingham; senate

Election	**No. plans w/ ≤ Dems (First Cluster)**	**% of plans w/ ≤ Dems (First Cluster)**	**No. plans w/ ≥ Dems (Second Cluster)**	**% of plans w/ ≥ Dems (Second Cluster)**	**Total Plans**	**First Cluster**	**Second Cluster**
LG16	0	0.0	0	0.0	79997	1 2	3 4 5 6
PR16	0	0.0	0	0.0	79997	1 2	3 4 5 6
AD20	0	0.0	0	0.0	79997	1 2	3 4 5 6
AG20	0	0.0	0	0.0	79997	1 2	3 4 5 6
CA20	0	0.0	8	0.01	79997	1 2	3 4 5 6
CL20	0	0.0	0	0.0	79997	1 2	3 4 5 6
GV20	0	0.0	0	0.0	79997	1 2	3 4 5 6
LG20	0	0.0	0	0.0	79997	1 2	3 4 5 6
PR20	0	0.0	0	0.0	79997	1 2	3 4 5 6
SST20	0	0.0	0	0.0	79997	1 2	3 4 5 6
TR20	0	0.0	8	0.01	79997	1 2	3 4 5 6
USS20	0	0.0	0	0.0	79997	1 2	3 4 5 6

Table 21: Iredell-Mecklenburg; senate

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I declare under penalty of perjury under the laws of the state of North Carolina that the foregoing is true and correct to the best of my knowledge.

Jonathan Mattingly, 12/23/2021